Manipulation, Coordination and Competition

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Abstract

We analyze how public disclosure of informed investors' trades results in manipulation, which in turn affects coordination and competition among informed investors in a duopolistic setting. Under disclosure requirement, an informed trader's order flow consists of two components: an information-based component to profit and a random component to manipulate. The random components from all informed traders collectively equals, in distribution, the random orders from all liquidity traders. Market is more efficient with disclosure. When each informed investor have very imprecise information, disclosure helps to coordinate trading among informed investors and they make more expected profits compared to what they expect in a market without disclosure. Moreover, an informed investor would prefer competition in the presence of disclosure as each informed investor makes more expected profits than he would obtain in a monopolistic market.

At first thought, one would have expected that public disclosure of informed investors' trades should reduce informed investors' expected profits. Indeed, many regulatory proposals and legislations have argued that disclosure would help level the playing field, reduce information asymmetry and benefit small investors. For example, corporate insiders are required to disclose their trades to the Securities and Exchange Commission (SEC). Section 16(a) of the SEC Act requires the insiders to report their trades to the Commission within ten days following the end of the month in which the trade occurs. Recently, SEC have made proposals to report high frequency trading on a timely basis. On April 14, 2013, SEC issued a release proposing that certain large-volume, high frequency traders (classified as "large traders") be required to self-identify to the SEC and that broker-dealers that effect transactions for "large trader" customers maintain and produce records of these customers' trades to the SEC.

While disclosure should reduce informed investors informational advantage when they all have the same information, it is less clear cut when investors have diverse information. In the latter case, not only the market can learn from the public disclosure, the informed investors can also learn more from the disclosure about each other's signals. With diversely informed investors, trade disclosure can act as a coordination device that allows informed investors to communicate with each other. How would disclosure of informed investors' trades affect market efficiency and market liquidity in a setting with heterogeneously informed investors? Under what conditions, would an informed investor prefer competition if they can learn more from each other through disclosure?

We consider a Kyle model of two informed investors each of whom is required to disclose his trade immediately after the trade is made. In discrete time, we derive a recursive formula for the equilibrium, which can be solved by numerical methods. In continuous time, we derive a closed-form formula for the equilibrium. To determine the impact of disclosure, we compare our closed-form equilibrium formula with that obtained in Back, Cao, and Willard (2000), whose model is the same except no disclosure is required there.

Disclosure of informed investors' trades creates incentives for informed investors to manipulate in that they sometimes trade against their own valuation to mislead the market, so that the market maker cannot perfectly infer information from their trades. As a result, the informed investors randomize to manipulate the market maker's belief until the last moment of trading. The mixed strategy allows the informed investors to maintain an informational advantage over the market for a longer period of time. We show that the combined random components in informed investors' trade equals in distribution to that of the liquidity traders. This is intuitively appealing as informed traders and liquidity traders will each contribute to half of the trading volume. Too much randomization will cause informed traders to lose a lot from randomized trade and too little randomization will cause informed traders to lose their informational advantage too early. To camouflage themselves, informed investors contribute to half of the trading volume in the market.

The effects of trade disclosure on market efficiency is unambiguous. Market is more efficient at all times after disclosure. As informed investors know more about each other's signal, their valuations converge more quickly and they trade more aggressively on their information, which in turn makes the market more efficient.

The effects on the expected profits of informed investors and market liquidity are more complicated.

Public disclosure has three effects on informed investors' expected profits. The first is the randomization effect. As informed investors manipulate and add noise to their own trades, they lose money from the noise trades. This will reduce informed investors' expected profits. The second is the coordination effect. With trade disclosure, informed investors learn more about each other than the market maker and they can coordinate their trades better which in turn increase their expected profits. The third is the market efficiency effect. Disclosure increases market efficiency and make informed investors trade more aggressively. The reduction of asymmetric information between informed investors and market makers will increase market liquidity and reduce the expected profits of informed investors.

When informed investors have very precise signals, they won't be able to learn from each other as much. In this case the coordination effect will be less important and disclosure will always decrease expected profits of informed investors. On the contrary, when investors have very noisy information, they tend to wait until they know more from each other before they trade aggressively. Trade disclosure can reduce the incentive to wait and make investors trade more aggressively. Informed investors learn more from disclosure than the market maker. The coordination effect could dominate other effects and result in higher expected profits of informed investors. Moreover, the coordination effect could be so strong such that an informed investor makes more profits in a duopolistic setting than what he would receive in a monopolistic setting. Therefore in the presence of disclosure, an informed investor could prefer to have competition. Indeed, an informed investor can even make more money in a duopolistic setting with disclosure than what he would expect in a monopolistic market without disclosure.

Similarly, randomization will reduce the informational content in the aggregate order flow and thus increase market liquidity. However, coordination among investors could reduce market liquidity. The reduction of asymmetric information would increase market liquidity. As a result, market liquidity can either increase or decrease depending on the parameters and the timing of the trades.

We extend the model to more than two informed investors and show that while competition reduces informed investors profits, it is still possible for informed investor to make more profits in a multiple players setting than what he would receive in a monopolistic setting.

The effect of disclosure rules on informed traders' trading has been studied by a number of authors including Fishman and Hagerty (1995) and John and Narayanan (1997). But these articles exclusively focus on the case of a single informed trader. Fishman and Hagerty (1995) study a two period model when an informed trader only possesses inside information with a certain probability. While an *informed* informed trader will never manipulate the market in their model, an *uninformed* informed trader can manipulate the market since the market may mistakenly believe that the *uninformed* informed trader is *informed*. John and Narayanan (1997) extend the Fishman-Hagerty model such that an informed trader receives good or bad signal with different probabilities, and they show that if such difference in probabilities is large enough, even an *informed* informed trader may manipulate the market. Here, the asymmetry in the likelihood of receiving different signals adds a new factor to induce an informed trader to manipulate: If the prior probability of good news is high, an informed trader with good news will sell initially and then reverse his trades in the next period.[1] While both FH and JN have found that it is possible for disclosure to increase informed trader's expected profits,

[1] John and Narayanan (1997) contains a brief an extension of their model to allow two informed traders. However,

the intuition is very different from our model. In FH, the result is driven by the assumption that the market does not know if the informed trader indeed has observed a signal or not while in JN the result is driven by the assumption in the asymmetry of the likelihood of receiving different signals. In our model, disclosure increases informed traders' profits because it can reduce the incentive to wait when informed investors have very noisy signals.[2]

The most related paper is by Huddart, Hughes, and Levine (2001) who study disclosure effects in a discrete-time Kyle model with a monopolistic informed trader. They show that the informed trader uses a mixed strategy in which the informed trader attaches a random order flow, for hiding information, to the information-based flow that is exactly the same as in Kyle's model. In addition, mandatory disclosure unambiguously reduces informed trader's profits, increases market liquidity, and improves market efficiency. However, they do not analyze how disclosure will affect informed traders' strategic trading behavior when there are more than one informed trader. Gong and Liu (2012) extend their results to multiple insiders. However they do not allow investors to have heterogeneous information and thus in continuous time, information will be revealed in opening trades and the expected profits for insiders go to zero. Zhang (2004) show that when the informed investor is risk averse, trade disclosure can reduce market efficiency as the risk averse investor will be facing less price risk in the future when he unloads his positions and thus will not trade in a hurry.

The rest of the paper is organized into sections as follows. The model is described in Section 1. Section 2 discusses the condition for equilibrium with public disclosure in a discrete-time framework. Section 3 offers a closed-form formula for the equilibrium in a continuous-time framework. Section 4 gives comparative statistics such as the effects of the number of informed traders and the correlation of their signals on the intensity of trading, the rate of information transmission, the depth of the market, and the expected profits of informed traders. Section 5 extends the model from a duopolistic setting to a general multiple players setting. Section 6 concludes. All proofs are left to the appendices.

1 The Model

In this section, we describe a model of two informed investors who are required to disclose their trades based on the classic model of Kyle (1985). In our model, there are one risk-free asset and one risky asset. An announcement is made at time 1 that reveals the liquidation value of the asset. The risk-free rate is taken to be zero. There are 2 risk neutral informed investors and many liquidity traders who trade for liquidity reasons. Trading takes place over time interval [0, 1). In the discrete-time version

their study on the two-informed trader case is limited to arguing that an informed trader's incentive to manipulate the market decreases when the number of informed traders rises.

[2] In models with disclosure but with multiple trading periods, Chakraborty and Yilmaz (2004) show that when the market faces uncertainty about the existence of the insider in the market and when there is a large number of trading periods before all private information is revealed, long-lived informed traders will manipulate in every equilibrium. Brunnermeier (2005) how disclosure of intermediary public information can cause investors with short term noisy information to manipulate the market.

of the model, there are M periods over time [0,1), and the time between any two consecutive trading periods is $\Delta t = 1/M$.

Let v denote the liquidation value of the risky asset at time 1. Before any trading starts, each informed trader i ($i = 1, 2$) receives a mean-zero signal s^i at time 0. We assume the signals and the liquidation value of the risky asset has a non-degenerate joint normal distribution that is symmetric in the signals.[3] More specifically, we have

$$s^1 = \frac{v + \epsilon}{2} \tag{1.1}$$

$$s^2 = \frac{v - \epsilon}{2} \tag{1.2}$$

$$v = \sum_{i=1}^{2} s^i. \tag{1.3}$$

The variances of v, ϵ are denoted σ_v^2 and σ_ϵ^2 respectively.

We use ρ to denote the correlation coefficient of s^1 with s^2.

$$\rho = \frac{\sigma_v^2 - \sigma_\epsilon^2}{\sigma_v^2 + \sigma_\epsilon^2} \tag{1.4}$$

In the special case of $\sigma_\epsilon^2 = 0$, $\rho = 1$, each informed trader has perfect information about v. For convenience, we also introduce the following notation

$$\delta_0 \equiv \frac{\mathrm{var}(v) - \mathrm{var}\,(v|s^1)}{\mathrm{var}(v)} = \frac{\mathrm{var}^{-1}(v|s^1) - \mathrm{var}^{-1}\,(v)}{\mathrm{var}^{-1}(v|s^1)}. \tag{1.5}$$

This is a measure of the quality of private information of informed investor 1 and by the argument of symmetry, informed investor 2 as well. Specifically, δ_0 is the "R-squared" in the linear regression of v on s^i for an arbitrary i, i.e., it is the percent of the variance in v that is explained by a single informed trader's information. Alternatively, it is also the percentage drop in precision of the informed investor to that of the market maker. It is easy to check that δ_0 is related to ρ by the following equation

$$\delta_0 = \frac{1}{2} + \frac{1}{2}\rho = \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\epsilon^2}. \tag{1.6}$$

Thus, when δ_0 is larger than, equal to, or smaller than1/2, informed investors signals are positively correlated, uncorrelated or negatively correlated respectively. When σ_ϵ is small, each informed investor has very precise information of the liquidation value. However, when ϵ is large, each informed investor has very coarse information about the liquidation value.

In each trading period m, a risk-neutral market maker receives the total order from all the informed investors and liquidity traders. Based on such order information, the market maker adjusts the price P_{m-1} to a new price P_m at which he buys or sells the risky security to clear the market in period m. Since the market maker is assumed to be risk neutral, price P_m must be the conditional expectation

[3]Symmetry means that the joint distribution of the asset value and the signals s^1, s^2 is invariant to a permutation of the indices.

given all public information. We use x_m^i to denote informed trader i's order, and use z_m^0 to denote the total order by all liquidity traders. We assume that z_m^0 are serially uncorrelated and normally distributed with mean zero and variance

$$E(z_m^0) = 0 \text{ and } \mathrm{var}(z_m^0) = \sigma_u^2 \Delta t \quad \text{for all } m.$$

For simplicity, we assume $\sigma_u = 1$. In addition, z_m^0 is independent of all other random variables in the model. Moreover, we assume that informed investors are prevented from any market making activities, and hence when they submit their orders in period m they have no information about the mth-period order flow from any other party.

The only difference between a model with disclosure and a model without disclosure is whether or not each informed investor is required to disclose his mth period trade immediately after all trades are completed in period m. Technically, this implies the following difference in how each of the involved parties behaves in the model. Without disclosure, (1) the market maker sets his price P_m by observing the history of the aggregate order flow $\{y_k : 1 \leq k \leq m\}$, where

$$y_k \equiv z_k^0 + \sum_{1 \leq i \leq N} x_k^i$$

and (2) each informed trader i decides his trade by observing his own past order flow $\{x_k^i : 1 \leq k < m\}$, his own signal s^i, and the past price history $\{P_k : 1 \leq k < m\}$. With disclosure, (1) the market maker sets his price by observing the breakdown of all traders' past order flow $\{x_k : 1 \leq k < m\}$ and $\{z_k^0 : 1 \leq k < m\}$ together with the current aggregate order flow $\{y_m\}$; and (2) each informed investor i decides his trade by observing all traders' past order flow $\{x_k : 1 \leq k < m\}$ and $\{z_k^0 : 1 \leq k < m\}$, in addition to his signal s^i and the past price history $\{P_k : 1 \leq k < m\}$. Note that in a model with disclosure, the breakdown of all the past order flow $\{x_k : 1 \leq k < m\}$ and $\{z_k^0 : 1 \leq k < m\}$ are made public through public disclosure and price history.

The above description has focused on the discrete-time version of the model. An intuitive way to think of the continuous-time version of the model is simply to take the limit of the discrete-time model with $M \to +\infty$. More technical details will be given when it comes to the derivation of our results in the continuous-time version of our model.

2 Informed Trading in Discrete Time with Public Disclosure

Under the disclosure requirement, informed traders announce their trades, $\{x_m^i\}, i = 1, 2$, immediately after the trade is executed. The market maker then adjusts his belief of the asset value from P_m (the market price for the risky asset in period m) to V_m which is defined to be the market maker's estimate of the fair value of the risky asset with all the information up to and including the disclosure made at the end of period m. We can think of V_m as the pseudo-price that market maker would have set for the mth period trading if he had observed the informed traders' order before the execution of trades in the mth period. Although V_m is only a pseudo-price at which no trade ever takes place, it is important since it will be the starting point for the market maker to set P_{m+1} for the $(m+1)$th period of trading.

In particular, in a linear equilibrium model that we will focus on, it is $P_{m+1} - V_m$ (as opposed to $P_{m+1} - P_m$) that will be linear to the total order flow submitted in the $(m+1)$th trading period.

Let $\underline{x}^i_m$ denote the history of trader i's trade in each past period before and including period m (i.e., $\{x^i_k : k = 1, \ldots, m\}$), let $\underline{y}_m$ denote the history of the net trade before and including period m (i.e., $\{z^0_k + \sum_{1 \le i \le 2} x^i_k : k = 1, \ldots, m\}$), and let $\underline{P}_m$ denote the price history before and including period m (i.e., $\{P_k : k = 1, \ldots, m\}$). With disclosure, informed trader i's private information prior to trading in period m includes his own signal s^i and the history of all past trades and prices $\underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{P}_{m-1}$. Let

$$x^i_m = x^i_m(s^i, \underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{P}_{m-1})$$

represent the optimal strategy of informed trader i. Let

$$P_m = P_m(\underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{y}_m)$$

represent the optimal strategy of the market maker given the history of all orders and the current aggregate order.

Let X^i and P denote the strategy functions for informed trader i and the market maker, respectively. Given the strategy functions for informed traders and the market maker, the profit of informed trader i from trading in period m and on can be written as:

$$\pi^i_m(X^1, X^2, P) = \sum_{k \ge m} (v - P_k) x_k.$$

An equilibrium of the trading game exists if there is an 3-dimension vector of strategies, (X^1, X^2, P) such that :

1. For any $i = 1, 2$ and for all $m = 1, ..., M$, if $\hat{X}^i \neq X^i$,

$$\begin{aligned} & E\left[\pi^i_m(\ldots, X^i, \ldots) | s^i, \underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{P}_{m-1})\right] \\ \ge \quad & E[\pi^i_m(\ldots, \hat{X}^i, \ldots) | s^i, \underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{P}_{m-1}] \end{aligned}$$

   i.e., the optimal strategy is the best no matter which past strategies informed trader i may have played.

2. For all $m = 1, \ldots, M$, we have

$$P_m = E[v | \underline{x}^1_{m-1}, \underline{x}^2_{m-1}, \underline{y}_m],$$

   i.e., the market maker sets prices equal to the conditional expectation of the asset value given the order-flow history.

In this model, since investor i's trade at period m will be disclosed afterwards, the pricing and trading strategies described earlier for the no-disclosure case cannot be an equilibrium in the new setting. To see this, suppose the informed trader follows a strategy of[4]

$$x^i_m = \beta_m \Delta t s^i_m + L_1(\underline{x}^i_{m-1}) + L_2(\underline{x}^1_{m-1}, \underline{x}^2_{m-1})$$

[4]We restrict our attention to symmetric linear equilibria.

where L_i is a linear function of all public information. Then the market maker would infer

$$v = \frac{\sum_{1\leq i\leq 2}[x_m^i - L_1(\underline{x}_{m-1}^i) - L_2(\underline{x}_{m-1}^1, \underline{x}_{m-1}^2)]}{\beta_m \Delta t}$$

and choose

$$P_{m+1} = \frac{\sum_{1\leq i\leq 2}[x_m^i - L_1(\underline{x}_{m-1}^i) - L_2(\underline{x}_{m-1}^1, \underline{x}_{m-1}^2)]}{\beta_m \Delta t}$$

in the next period. Hence, in the next period, the market depth would be infinity. Understanding this, the informed traders would have incentive to choose $\hat{x}_m^i \neq x_m^i$ which is inconsistent with the proposed equilibrium strategy.

We analyze a symmetric linear equilibrium. In particular, the informed trader's trade can be written as

$$x_m^i = \beta_m \Delta t s^i + L_1(\underline{x}_{m-1}^i) + L_2(\underline{x}_{m-1}^1, \underline{x}_{m-1}^2) + z_m^i, \tag{2.1}$$

where (1) $\beta_m \Delta t s^i$ represents a private-information based linear component, (2) $L_1(\underline{x}_{m-1}^i)+$ $L_2(\underline{x}_{m-1}^1, \underline{x}_{m-1}^2)$ is a public-information based linear component, and (3) z_m^i is a noise component with z_m^i being normally distributed with mean 0 and variance $\sigma_m^2 \Delta t$. Since informed traders are prevented from market making activities, we further assume that z_m^i are independently distributed across agents. The market maker also uses linear rules for setting prices before disclosure and for updating his value estimate after disclosure. In particular,

$$\begin{aligned} P_m &= V_{m-1} + \lambda_m \left(z_m^0 + \sum_{1\leq i\leq 2} x_m^i \right), \quad \text{and} \\ V_m &= V_{m-1} + \bar{\lambda}_m \left(\sum_{1\leq i\leq 2} x_m^i \right). \end{aligned}$$

The preceding equations imply that the random order from liquidity traders only has a temporary effect on price formation. In particular, liquidity traders' order in period m (i.e., z_m^0) only affects P_m but not P_k for any $k \geq m+1$: Once the mth-period disclosure is made, the market maker immediately abandons z_m^0 and adjusts his belief of asset value to V_m, which is not affected by z_m^0 and will be the base for forming future prices P_k ($k \geq m+1$).

Before stating our result, we first introduce some notation. Let F_m and F_m^i denote the information set of the market maker and informed trader i respectively *after* disclosure has been made in period m. Define

$$\begin{aligned} V_m^i &\equiv E[v|F_m^i], \\ V_m &\equiv E[v|F_m], \\ \Sigma_m &\equiv \text{Var}[v|F_m], \\ \Omega_m &\equiv \text{Var}[v|F_m^i], \quad \text{and} \\ \delta_m &\equiv \frac{\Sigma_m - \Omega_m}{\Sigma_m}. \end{aligned}$$

Theorem 2.1 *The necessary and sufficient conditions for a recursive linear symmetric equilibrium to exist are described below. For all* $m = 1, \cdots, M-1$ *and for all informed traders* $i = 1, 2$,

$$x_m^i = \frac{\beta_m \Delta t}{2\delta_{m-1}}(V_{m-1}^i - V_{m-1}) + z_m^i \tag{2.2}$$

$$P_m = V_{m-1} + \lambda_m \left(z_m^0 + \sum_{i=1}^{2} x_m^i \right) \tag{2.3}$$

$$V_m = V_{m-1} + \bar{\lambda}_m \sum_{i=1}^{2} x_m^i \tag{2.4}$$

$$\bar{\lambda}_m = \beta_m \Sigma_m / (2\sigma_m^2) \tag{2.5}$$

$$\lambda_m = \beta_m \Sigma_{m-1} / (\beta_m^2 \Delta t \Sigma_{m-1} + 1 + 2\sigma_m^2) \tag{2.6}$$

$$V_m^i - V_{m-1}^i = \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}} \left(v - V_{m-1}^i + \frac{z_m^j}{\beta_m \Delta t} \right) \tag{2.7}$$

$$V_m - V_{m-1} = \frac{\Sigma_{m-1} - \Sigma_m}{\Sigma_{m-1}} \left(v - V_{m-1} + \sum_{1 \le i \le 2} \frac{z_m^i}{\beta_m \Delta t} \right) \tag{2.8}$$

$$\Omega_m^{-1} = \Omega_{m-1}^{-1} + \beta_m^2 \Delta t / (\sigma_m^2) \tag{2.9}$$

$$\Sigma_m^{-1} = \Sigma_{m-1}^{-1} + \beta_m^2 \Delta t / (2\sigma_m^2) \tag{2.10}$$

$$E[\pi_m^i | F_{m-1}^i] = \alpha_{m-1}(V_{m-1}^i - V_{m-1})^2 + \zeta_{m-1} \tag{2.11}$$

$$\lambda_m = \alpha_m \bar{\lambda}_m^2 \tag{2.12}$$

$$\lambda_m = \frac{\bar{\lambda}_m}{2 - \bar{\lambda}_m \beta_m \Delta t (1 - 1/(2\delta_{m-1}))} \tag{2.13}$$

$$\alpha_{m-1} = \alpha_m \left(1 - \frac{\beta_m^2 \Delta t \Sigma_m}{2\sigma_m^2} \left(1 - \frac{1}{2\delta_{m-1}} \right) \right)^2 \tag{2.14}$$

$$\zeta_{m-1} = \zeta_m + \alpha_m \beta_m^2 \Delta t \left(\frac{\Omega_m}{\sigma_m^2} - \frac{\Sigma_m}{2\sigma_m^2} \right)^2 \left(\Omega_{m-1} \beta_m^2 \Delta t + \sigma_m^2 \right) \tag{2.15}$$

subjecting to the boundary conditions

$$\beta_M = \sqrt{\frac{2\delta_{M-1}}{\Sigma_{M-1}\Delta t}}, \tag{2.16}$$

$$\lambda_M = \frac{\sqrt{2\delta_{M-1}\Sigma_{M-1}/\Delta t}}{1 + 2\delta_{M-1}}, \tag{2.17}$$

$$\alpha_{M-1} = \frac{1}{\lambda_M (1 + 2\delta_{M-1})^2}, \tag{2.18}$$

$$\zeta_{M-1} = 0, \tag{2.19}$$

and the second order condition

$$\lambda_M > 0. \tag{2.20}$$

In general, the system of recursive equations can be solved by conjecturing an initial value of Ω_{M-1} and then solve recursively for $\Omega_{M-2}, ..., \Omega_0$. The initial value of Ω_{M-1} is then adjusted until the derived Ω_0 matches the given Ω_0. Details are given in Appendix A.

In the special case that $\sigma_\epsilon = \sigma_v$, the model can be solved in closed form:

$$\begin{aligned}
\lambda_m &= \frac{\sqrt{\Sigma_0}}{2} \\
\bar{\lambda}_m &= 2\lambda_m \\
\beta_m &= \frac{1}{2(M-m+1)\lambda_m} \\
\sigma_m^2 &= \frac{M-m}{2(M-m+1)} \\
\Sigma_m &= (1-m/M)\Sigma_0 \\
\Omega_m &= (1-m/M)\Omega_0 \\
\alpha_m &= \frac{1}{4\lambda_m} \\
\zeta_m &= 0
\end{aligned}$$

These results are exactly the same as the monopolistic case derived by Huddart, Hughes and Levine (2001). This is in sharp contrast to earlier results on imperfect competition of informed traders without disclosure. Foster and Viswanathan (1996), Cao (1995) and Back, Cao and Willard (2000) have shown that competition causes the market to be very illiquid and inefficient near the end of trade when there is no disclosure. With disclosure, we find that informed traders act in the aggregate as a monopolist when their signals are uncorrelated. This is because, with disclosure, informed traders will always know as much about others' signals as the market does. If informed traders' signals are uncorrelated to begin with, they remain uncorrelated due to public disclosure of trades after transaction is completed. Therefore, disclosure makes informed investors coordinate with each other to maximize their profits and they act like a monopolist in the aggregate. On the contrary, without disclosure, each informed trader gradually knows more about others' signals than the market maker since he knows what he traded in the past. Indeed, the conditional correlation coefficient of informed investors signals goes to -1 in a the setting without disclosure even when the initial correlation coefficient is zero.

In another special case where the number of trading periods goes to infinity, the model approaches to the continuous-time model. Ignoring higher order terms of Δt, we have the following:

$$\begin{aligned}
\bar{\lambda}(t) &= \beta(t)\Sigma(t) \\
\lambda(t) &= \beta(t)\Sigma(t)/2 \\
\sigma(t)^2 &= 1/2 \\
\frac{\Delta\Omega(t)^{-1}}{\Delta t} &= 2\beta(t)^2 \\
\frac{\Delta\Sigma(t)^{-1}}{\Delta t} &= \beta(t)^2 \\
2\alpha(t)\bar{\lambda}(t) &= 1 \\
\frac{\Delta\alpha(t)}{\Delta t} &= 2\alpha(t)\beta(t)^2\Sigma(t)\left(1 - \frac{1}{2\delta(t)}\right) \\
\frac{\Delta\zeta(t)}{\Delta t} &= -\alpha(t)\beta(t)^2[2\Omega(t) - \Sigma(t)]^2/2.
\end{aligned}$$

In the limit, these difference equations converge to the set of differential equations described in Theorem 3.1 and the lemmas in Section 3.

3 Informed Trading in Continuous Time with Public Disclosure

In this section, we derive closed-form formulae for the linear equilibrium of informed trader trading in a continuous-time framework. The section is divided into subsections as follows. Subsection 3.1 introduces necessary notations to state the main theorem. Subsection 3.2 contains the main theorem of the section. Subsections 3.3 and 3.4 outline the proofs of the main theorem by considering the value estimation processes and the informed traders' optimal trading strategy, respectively.

3.1 Model Setup

In this subsection, we introduce the basic notations and concepts for the continuous-time model. Most of these notations (e.g., $\beta(t)$ and $P(t)$) have already been used in the discrete-time model but will be redefined here for an identical or similar quantity in the continuous model.

Like in discrete time, we use s^i to denote the signal of informed trader i and assume $v = \sum_{1\le i\le 2} s^i$. We use $P(t)$ to denote the price set by the market maker for trading at time t, and we use $V(t)$ to denote the market maker's adjusted belief of the risky-asset value immediately after the disclosure of informed traders' trade at time t. Also, we use $x^i(s^i, t)$ to denote the total order of informed trader i up to time t, and we use $z^0(t)$ to denote the total order from all liquidity traders up to time t.[5]

For the price process, linearity means that there exist functions $\lambda(t)$ and $\bar{\lambda}(t)$ such that the market maker adjusts the risky asset's price and the post-disclosure value estimate by multiplying $\lambda(t)$ and $\bar{\lambda}(t)$ with the new orders from all traders and those from all informed traders, respectively. More precisely, we have

$$dV(t) = \bar{\lambda}(t) \sum_{1\le i\le 2} dx^i(t), \text{ and} \tag{3.1}$$

$$P(t+dt) - V(t) = \lambda(t)\left(dz^0(t) + \sum_{1\le i\le 2} dx^i(t)\right). \tag{3.2}$$

It should be noted that although at any time t, $P(t)$ and $V(t)$ only differ by an infinitesimal due to the liquidity traders' trades,[6] this infinitesimal will be important in calculating the profit of an informed trader, as we will see in the proof of Lemma 3.5.

[5] In contrast, in the discrete-time model, we have used x^i_m to denote informed trader i's instantaneous order at time m, rather than his cumulative order up to time m.

[6] It can be shown that $V(t) - P(t) = \bar{\lambda}(t)\sum_{1\le i\le 2} dx^i(t) - \lambda(t)\left(dz^0(t) + \sum_{1\le i\le 2} dx^i(t)\right)$, although we do not need this equation in deriving our equilibrium conditions.

We require that the trading strategy $x^i(s^i, t)$ depends only on the trade history up to time t (e.g., it is independent of future value of x^j for any $j = 1, 2$). We also require that the trading strategies to be such that Equation 3.1 with boundary condition $V(0) = 0$ has a unique solution V. Furthermore, we require the solution P to have a finite second moment and to have paths belonging to $\mathcal{C}$, where $\mathcal{C}$ denotes the set of continuous functions f: $[0, 1) \to R$ such that $\lim_{t\to 1} f(t)$ exists and is finite. This is a restriction on the strategy sets of the traders: given that agents $i \neq j$ follow linear strategies to be described in Equation 3.3, we require agent j to follow a strategy such that Equation 3.1 has a solution with the desired properties.[7]

For the trading strategy, linearity means that the rate of purchase for informed trader i can be specified as follows

$$dx^i(s^i, t) = \beta(t)s^i dt + f(t)dt + dz^i(t) \quad \text{for } 1 \le i \le 2 \tag{3.3}$$

where $f(t)$ is a certain function of all public information available up to time t and $z^i(t)$ is a (non-standard) Brownian motion with instantaneous variance

$$dz^i(t) = \sigma^i(t)dW^i(t) \quad \text{for } 1 \le i \le 2. \tag{3.4}$$

To be consistent with the discrete time model, we assume that $dz^0(t)$ is a standard Brownian.

$$\text{var}(dz^0(t)) = dt. \tag{3.5}$$

We restrict our attention to symmetric equilibria such that $\sigma^i(t) = \sigma(t)$ for all i and in equilibrium, we show that $\sigma(t) = 1/\sqrt{2}$.

Since informed investors are assumed not to participate in market making activities, each dz^i is uncorrelated with both noisy trader's trade dz^0 and all other informed traders trade dz^j for all $j \neq i$.

While we have only included t in our notation $f(t)$, it should be emphasized that $f(t)$ can be an arbitrarily complex function of all public information available before and including time t, such as the history of all the orders submitted by all the informed investors and the liquidity traders, which are revealed to the public through disclosures. We leave $f(t)$ in this very general form for now and will make it more explicit later.

In most previous studies in the literature, $f(t)$ is simply the asset price at time t multiplied by a certain function $\alpha(t)$, which solely depends on time t but no other information (see Kyle (1995), Back, Cao, and Willard (2000), and Huddart, Hughes, and Levine (2001)). In our current model, however, $f(t)$ has to depend on more public information other than price. Indeed, it can be shown in

[7] See, e.g., Protter (1990, §V.3) for conditions that guarantee the existence of unique solutions to stochastic differential equations. Our approach has the disadvantage of linking the feasible set for each trader to the strategies assumed to be chosen by the other traders and the market maker. In this respect, we are modeling a generalized game rather than a game. It would be better to define a feasible set for each trader and a set of $\bar{\lambda}$ functions for the market maker such that, given any vector of choices from these sets, the stochastic differential equation defining the price has a unique solution with the desired limits existing. However, this approach would lead us into a thicket of technicalities that we prefer to avoid.

the discrete-time model that informed traders' order flows of the form $\beta(t)s^i + \alpha(t)P(t)$ (where $\alpha(t)$ is a function of time t only) does not constitute an equilibrium.

It may be natural to consider trading that is linear in a trader's updated estimate of the asset value rather than linear in a trader's initial signal. One difficulty with such an approach arises in calculating each informed trader's dynamic estimates of the asset value, because each trader's estimate would depend on other agents' trades, which depend on their estimates of the asset value, which depend on other agents' trades, etc. This is what is called the "forecasting the forecasts of others" problem (see Foster and Viswanathan (1996)). By specifying the trading strategy as a linear function in a trader's initial signal, we can avoid this problem. In the end, Strategy 3.3 can be shown to be a linear functions of value estimates in equilibrium.

3.2 Equilibrium

We define a symmetric linear equilibrium to be functions $\beta(t)$ and $\bar{\lambda}(t)$ such that (1) they are positive and continuous on $[0, 1)$ and continuously differentiable on $(0, 1)$, (2) $P(t)$ and $V(t)$ calculated from Equations 3.1 and 3.2 are both rational expectations of the asset value at all time t, and (3) the trading strategy in Equation 3.3 for each informed trader i is feasible and maximizes his expected profit over the set of feasible strategies. The following theorem is our main result.

Theorem 3.1 *If $\sigma_\epsilon > 0$, i.e., informed investors' signals are not perfectly correlated, there is a unique symmetric linear equilibrium specified as follows*

$$\beta(t) = \frac{\sqrt{-\Sigma(t)'}}{\Sigma(t)} = \frac{1}{\sigma_\epsilon(1-t)},$$

$$\lambda(t) = \frac{\sqrt{-\Sigma(t)'}}{2} = \frac{\sigma_v^2 \sigma_\epsilon}{2[\sigma_v^2 t + \sigma_\epsilon^2(1-t)]},$$

$$\bar{\lambda}(t) = \sqrt{-\Sigma'(t)},$$

where $\Sigma(t)$ is specified as

$$\Sigma(t) = \frac{\sigma_\epsilon^2 \sigma_v^2 (1-t)}{\sigma_v^2 t + \sigma_\epsilon^2 (1-t)} \tag{3.6}$$

In equilibrium, the expected profit of each informed trader π_D is

$$\pi_D = \frac{1}{2}\int_0^1 \lambda(t)dt = \frac{\sigma_v^2 \sigma_\epsilon [\ln(\sigma_v) - \ln(\sigma_\epsilon)]}{2(\sigma_v^2 - \sigma_\epsilon^2)}. \tag{3.7}$$

Investors' aggressiveness trading β is proportional to $1/\sigma_\epsilon$. This is sensible as investors will trade more cautiously as they have noisier signals. Surprisingly, λ is finite through the trading period remains constant as long as $\sigma_v = \sigma_\epsilon$. This is in sharp contrast to the result in Back, Cao, and Willard (2000) who show that λ goes to infinity near the end of trading in the absence of disclosure.

3.3 Value Estimates and Variances

In this subsection, we consider the filtering problems of the traders and market maker in detail. Throughout this section, we assume $\beta(t)$ used in Strategy 3.3 is a continuous and non-negative function.

Let $\mathbf{F} \equiv \{\mathcal{F}(t)|0 \leq t < 1\}$ denote the filtration generated by the aggregate informed traders' order process

$$\sum_{i=1}^{2} x^i(t).$$

We interpret $\mathbf{F}$ as the market maker's information structure. Under the new notation, $V(t) = E[v|\mathcal{F}(t)]$ where the conditional expectation is taken after the disclosure at time t. We define $\Sigma(t)$ as[8]

$$\frac{1}{\Sigma(t)} \equiv \int_0^t \beta(u)^2\, du + \frac{1}{\Sigma(0)}. \tag{3.8}$$

Lemma 3.1 *Assume each trader i follows a linear strategy as in Equation 3.3. Then* $\Sigma(t) = var[v|\mathcal{F}(t)]$*, where the variance is calculated after disclosure at time t. Define*

$$W(t) \equiv \sum_{1\leq i\leq 2} z^i(t) + \int_0^t \beta(u)\left\{v - V(u)\right\}\, du. \tag{3.9}$$

The process W is a Wiener process on the market maker's information structure $\mathbf{F}$*. Furthermore,*

$$V(t) = \int_0^t \beta(u)\Sigma(u) dW(u). \tag{3.10}$$

The process W is called the "innovation" process for the market maker's estimation problem. The differential

$$dW(t) = \sum_{1\leq i\leq 2} dz^i + \beta(t)\left\{v - V(t)\right\} dt$$

is the unpredictable part of the order flow from informed traders (recall that from the market maker's viewpoint, the expected order from informed traders is 0). The lemma shows that the market's estimate of v is revised according to $dV = \beta\Sigma\, dW$. Moreover, having the changes of both value estimates and prices proportional to orders as in Equations 3.1 and 3.2 implies that these changes are unpredictable, as they must be when the market maker is risk neutral and competitive.

Consider an arbitrary informed investor j $(1 \leq j \leq 2)$. Assume that the other informed investor i $(i \neq j)$ follows a linear strategy as in Equation 3.3, and assume that j follows an arbitrary strategy, which may or may not follow Eqaution 3.3. Let $\mathbf{F}^j \equiv \{\mathcal{F}^j(t)|0 \leq t < 1\}$ denote the filtration generated

[8] In the discrete-time model, we define $\Sigma(t)$ as the variance of the asset value conditional on the market maker's information. Here, we choose to define $\Sigma(t)$ by a mathematical equation and then prove that it is equal to the same conditional variance under certain conditions (see Lemma 3.1). Alternatively, we could define $\Sigma(t)$ as the desired conditional variance and then prove Equality 3.8 in Lemma 3.1. But such an alternative approach does not offer us a easy-to- use mathematical formula for $\Sigma(t)$ when conditions in Lemma 3.1 do not hold. Finally, we remark that it can be verified (see the proof of Theorem 3.1) that the function $\Sigma(t)$ defined here is the same as that used in the statement of Theorem 3.1.

by s^j and the order flow of all traders i ($i \neq j$). This is informed trader i's information structure. We want to describe the conditional expectation and conditional variance of v, given his information. In particular, we define

$$\begin{aligned} U^j &\equiv E[v - s^j | \mathcal{F}^j(t)], \quad \text{and} \\ V^j &\equiv s^j + U^j \end{aligned}$$

where the expectation is taken at time t after the informed traders' disclosure. We also define[9]

$$\frac{1}{\Omega(t)} \equiv 2 \int_0^t \beta(u)^2 \, du + \frac{1}{\sigma_v^2} + \frac{1}{\sigma_\epsilon^2} \tag{3.11}$$

Lemma 3.2 *Consider an arbitrary informed trader j ($1 \leq j \leq 2$). Assume each trader $i \neq j$ follows a linear strategy as in Equation 3.3. Then, $\Omega(t) = var[v|\mathcal{F}^j(t)]$, where the variance is calculated after disclosure at time t. Define*

$$W^j(t) \equiv \sqrt{2} \left[z^i(t) + \int_0^t \beta(u)\{v - V^j(u)\} \, du \right]. \tag{3.12}$$

The process W^j is a Weiner process on the information structure $\mathbf{F}^j$, and

$$U^j(t) = \rho s^j + \int_0^t \sqrt{2}\beta(u)\Omega(u) dW^j(u). \tag{3.13}$$

The differential of the innovation process W^j is again the difference between the actual order and the expected order, but now we are computing the expected order using trader j's information.The lemma shows that his estimate of the asset value v is revised as $dV^j = \sqrt{2}\beta\Omega \, dW^j$.

For ease of notation, we define

$$\delta(t) \equiv \frac{\Sigma(t) - \Omega(t)}{\Sigma(t)} = \frac{\Omega^{-1}(t) - \Sigma^{-1}(t)}{\Omega^{-1}(t)}. \tag{3.14}$$

Lemma 3.3 *Assume (1) each informed trader believes that all other informed traders follow Strategy 3.3, and (2) the market maker believes that all informed traders follow Strategy 3.3. Then,*

$$\sum_{1 \leq i \leq 2} \left(V^i(t) - V(t) \right) = 2\delta(t)(v - V(t)). \tag{3.15}$$

The next lemma gives explicitly formula for each informed trader's trading strategy in equilibrium. It may be worth noting that, somewhat surprisingly, the deterministic part of an informed trader's order flow is identical to that in the no-disclosure case (see Equations 1.6 and 3.11 in Back, Cao, and Willard (2000)).

[9] Here we choose to define $\Omega(t)$ by a mathematical equation, rather than by defining it to be the conditional variance of the asset value as in the discrete-time model. The reason is the same as that given in Footnote 8 for $\Sigma(t)$.

Lemma 3.4 *Assume that each informed trader believes that all other informed traders follow Strategy 3.3. The following is the only trading strategy such that (1) it satisfies Equation 3.3 and (2) Equation 3.1 is a rational pricing rules for the market maker:*

$$dx^i(t) = \frac{\beta(t)}{2\delta(t)}\left(V^i(t) - V(t)\right)dt + \sigma^i dW^i(t), \quad 1 \leq i \leq 2. \tag{3.16}$$

Moreover,

$$\bar{\lambda}(t) \;=\; \beta(t)\Sigma(t), \tag{3.17}$$

and the trading strategy supports pricing rule given in Equation 3.2 with

$$\lambda(t) \;=\; \beta(t)\Sigma(t)\frac{\sum_{1\leq i\leq 2}(\sigma^i)^2}{1+\sum_{1\leq i\leq 2}(\sigma^i)^2}. \tag{3.18}$$

Given Equation 3.17, the entire equilibrium is determined by $\bar{\lambda}(t)$. To see this, note that

$$\bar{\lambda}(t)^2 = \beta(t)^2\Sigma(t)^2 = -\Sigma'(t),$$

where the second equation follows from Equation 3.8. Therefore, the function $\Sigma(t)$ is determined by $\bar{\lambda}(t)$. The condition $\bar{\lambda}(t) = \beta(t)\Sigma(t)$ then determines $\beta(t)$.

To determine $\bar{\lambda}(t)$ or, equivalently $1/\bar{\lambda}(t)$, which Kyle (1985) calls "the depth of the market," we turn to the equilibrium condition that has not yet been exploited, namely, the requirement that each informed trader's trading strategy be optimal.

3.4 Optimal Trading and Market Depth

In this subsection, we derive the optimality condition for an informed trader's trading rules. Such a condition turns out to be a restriction on market depth.

Throughout the subsection, we focus on an arbitrarily chosen trader, say trader j. Assume that each trader $i \neq j$ follows Strategy 3.16. By Lemma 3.4, trader j's trading strategy can be written as $x^j(s^j, t, P^{x^j})$, where we use P^{x^j} to emphasize that trader j's strategy affects the price process. We define a trading strategy x^j to be feasible for trader j if there exists a unique solution P^{x^j} to Equation 3.1 (with boundary condition $P^{x^j}(0) = 0$) for the given $\bar{\lambda}$ and for the given β that characterizes the other traders' strategies and if

$$\lim_{t\to 1} P(t) \text{ exists and is finite a.s.,} \tag{3.19}$$

$$\int_0^1 dx^j\left(s^j, u, P^{x^j}\right) \text{ exists and is finite a.s., and} \tag{3.20}$$

$$E\int_0^1 P^{x^j}(t)^2\,dt < \infty. \tag{3.21}$$

Note that the integral in Expression 3.21 is the limit of the integral over $[0, t]$ as $t \to 1$. The limits in Expressions 3.19 and 3.20 define, respectively, the price and number of shares held by trader j just before the announcement. Condition 3.21 is the "no doubling strategies" condition introduced in Back (1992). Given the existence of the limits, the integral

$$\int_0^1 \left(v - P^{x^j}(t+dt)\right) dx\left(s^j, t, P^{x^j}\right), \tag{3.22}$$

exists and equals to the profit of trader j. The formula is is derived from the Merton-type wealth equation, and the existence of the integral can be verified by integrating by parts as in Back (1992).

Lemma 3.5 *Assume each trader $i \neq j$ plays a linear strategy as in Equation 3.16. The conditions*

$$\frac{d}{dt}\left(\frac{1}{\bar{\lambda}(t)}\right) = \beta(t)\left(2 - \frac{1}{\delta(t)}\right), \tag{3.23}$$

$$\lambda(t) = \frac{\bar{\lambda(t)}}{2}, \tag{3.24}$$

and

$$\lim_{t\to1} \Sigma(t) = 0 \ \ \textit{or} \ \ \lim_{t\to1} \bar{\lambda}(t) = +\infty \tag{3.25}$$

are necessary and sufficient for Strategy 3.16 to create an optimal expected profit for trader j, which is equal to

$$\frac{2(\sigma_v^2 s^j)^2}{(\sigma_v^2+\sigma_\epsilon^2)^2\bar{\lambda}(0)} + \frac{1}{4}\int_0^1 \frac{1}{\bar{\lambda}(u)}\left(\bar{\lambda}(u) - 2\beta(u)\Omega(u)\right)^2 du. \tag{3.26}$$

If $\sigma_\epsilon^2 = \sigma_v^2$, then the right-hand side of Equation 3.23 is zero. Therefore, market depth (which is $1/\lambda = 2/\bar{\lambda}$) must be constant. If $\sigma_\epsilon^2 > \sigma_v^2$, then the right-hand side of Equation 3.23 is negative. This implies that in such a case market depth $1/\lambda$ must be declining over time. If $\sigma_\epsilon^2 < \sigma_v^2$, then the right-hand side of Equation 3.23 is always positive. This implies that in such a case market depth $1/\lambda$ must be rising over time, in contrast to the results in the setting without disclosure obtained by Back, Cao, and Willard (2000), in which market depth first rises to its maximum and then fall to 0. The difference occurs because the conditional correlation in our model is positive when $\sigma_\epsilon^2 < \sigma_v^2$ and never changes sign but in Back, Cao and Willard, the conditional correlation will converges to -1 even when it was positive at time zero.

Condition 3.23 is a local condition for optimality at each $t < 1$, which we will discuss below. Condition 3.25 means there is no money "left on the table" an instant before the announcement. If the first condition of 3.25 holds, then the market's information about v is precise by the announcement date, and the asset will be correctly priced. If the second condition of 3.25 holds, then the market is completely illiquid just before the announcement, so, even if the asset were mis-priced, there would be no profitable trades available. These conditions are not mutually exclusive. In fact, only the first condition holds in our case, which is contrasting with both conditions hold in Back, Cao, and Willard (2000).

4 Comparative Dynamics

In this section, we use the closed-form equilibrium formula derived in the previous section to study the comparative dynamics of the equilibrium and compare the equilibrium against that obtained by Back, Cao and Willard (2000) in the case of no disclosure. For comparison, we use $\hat{\Sigma}$ to denote the conditional variance in the BCW model and the same holds for other parameters.

Theorem 4.1 *In the continuous time trading model without public disclosure, there exists a unique symmetric linear equilibrium. In this equilibrium, the informed investors submit a market order of*

$$dx^i(t) = \hat{\beta}(t)\left(s^i - \frac{V(t)}{2}\right)dt = \frac{\hat{\beta}(t)}{2\hat{\delta}(t)}(V^i(t) - V(t))dt, \tag{4.27}$$

and the market maker set the price according to

$$dP(t) = \hat{\lambda}\left(\sum_{i=1}^{2} dx^i(t) + dz^0(t)\right). \tag{4.28}$$

and

$$\hat{\Sigma}(t) = \frac{\sigma_v^2\sigma_\epsilon^2}{\sigma_\epsilon^2 - \sigma_v^2\ln(1-t)}. \tag{4.29}$$

$$\hat{\beta}(t) = \frac{1}{\sigma_\epsilon\sqrt{1-t}}, \tag{4.30}$$

$$\hat{\lambda}(t) = \hat{\beta}(t)\hat{\Sigma}(t). \tag{4.31}$$

Notice that $\hat{\Sigma}(t), \hat{\beta}(t), \hat{\lambda}(t), \hat{\delta}(t)$ in the economy without disclosure corresponds to the same parameters without the hat in the economy with disclosure.

While in most strategic trading models, the trading volume coming from the informed traders is negligible compared to the noise traders. However, when disclosure is required, informed investors' trades contains a component of positive quadratic variation that is comparable to that of the liquidity traders:

Corollary 4.1 *Informed investors contribute half of the trading volume in the market with disclosure.*

In each period, the informativeness of informed investors' trade is measured by $\beta(t)$ because the total information based trade in period t to $t+dt$ is proportional to $\beta(t)(v - V(t))dt$. The variance of the aggregate randomization noise is dt and the increase in market maker's precision is $\beta^2(t)dt$. The derivative of market maker's conditional precision is $(1/\Sigma(t))' = \beta^2(t)$. The following describes how disclosure affects $\beta(t), \Sigma(t)$.

Corollary 4.2 *Informed investors' trade information based trades are more aggressive and the market is more efficient, that is*

$$\frac{\beta(t)}{\hat{\beta}(t)} = \frac{1}{\sqrt{1-t}} > 1,$$

$$\frac{\Sigma(t)}{\hat{\Sigma}(t)} = \frac{\sigma_\epsilon^2 - \sigma_v^2 \ln(1-t)}{\sigma_\epsilon^2 + \sigma_v^2 t/(1-t)} < 1.$$

Moreover as time approaches 1, we have,

$$\lim_{t \to 1} \frac{\beta(t)}{\hat{\beta}(t)} = \infty, \ \lim_{t \to 1} \frac{\Sigma(t)}{\hat{\Sigma}(t)} = 0,$$

Disclosure makes the market more efficient. Since informed investors' information based trade is mixed with more noise trades, they trade more aggressively with respect to their signal. This effect is most profound near the end of trade as the ratio of Σ with and without disclosure goes to zero. Figure 1A shows the intensity of informed investors' trading in relation to that of informed trading without disclosure. The intensity is greater when disclosure is required. Figure 1B shows the ratio of trading intensity with and without disclosure. It is always larger than 1 and goes to infinity near the end of trade.

As a result of more aggressive trading by informed investors and the fact that the random order from all informed investors collectively equals, in distribution, to that of the liquidity traders, market becomes more efficient under the disclosure rules. This is clearly demonstrated in Figure 2.

We next examine the comparative statics of $\beta(t), \Sigma(t), \lambda(t)$ with respect to time and the degree of noise of informed investors' signals, as measured by σ_ϵ.

Corollary 4.3 *The variables $\beta(t)$, $\Sigma(t)^{-1}$ all increase with t and decreases with σ_ϵ. The variable $\lambda(0)$ decreases with σ_ϵ and $\lambda(1)$ increases with σ_ϵ. The variable $\lambda(t)$ decreases over time when $\sigma_\epsilon \geq \sigma_v$ while $\lambda(t)$ increases over time when $\sigma_\epsilon < \sigma_v$.*

When σ_ϵ is small, informed investors trade very aggressively with each other and thus $\beta(t)$ is high and $\Sigma(t)$ is low. As more information is revealed through trading and disclosure, clearly $\Sigma(t)$ will increase over time. Similarly, as investors learn more from trading and disclosure and market becoming more efficient, the trading intensity increases over time as well. The comparative statics on $\lambda(t)$ is more complicated. When σ_ϵ is small, each investor is very well informed and they trade very aggressively in the beginning. Thus $\lambda(0)$ decreases with σ_ϵ. Similarly, with very aggressive trading in the beginning, the market becomes more efficient later and thus $\lambda(1)$ is low with small σ_ϵ. Moreover, with small σ_ϵ, higher market efficiency due to aggressive trading also means that market liquidity will increase over time. On the contrary when σ_ϵ is low, investors will trade very cautiously initially and only increase their trades aggressively later on. This means that the market liquidity will decrease over time.

Disclosure not only increases market efficiency, it also affects how informed investors compete with each other. It is interesting to compare the trading strategy of informed investors in the aggregate to that of a monopolist. We have the following results.

Corollary 4.4 *When $\sigma_\epsilon^2 = \sigma_v^2$, informed investors trade cooperatively like a monopolistic investor in the aggregate and their profits are maximized. Conditional correlation of investors' private valuation remains zero throughout the trading period.*

When informed investors' signals are uncorrelated initially, each informed investor's conditional precision is twice of that of the market maker. As trading goes on, since each informed trader knows his own randomizing trade, the noise in the other informed investor's trades is also half of the variance of the noise in the market maker's observation. As a result, the conditional precision of each informed trader's expectation about the asset value is remains twice of that of the market maker. As a result, informed investors conditional correlation remains zero. Disclosure makes informed investors cooperate with each other.

Corollary 4.5 *When $\sigma_\epsilon^2 \neq \sigma_v^2$, as $t \to 1$, $2\delta \to 1$ and informed traders' private valuations become uncorrelated and they all behave in the aggregate like a monopolistic informed trader with all the information in the economy. We have*

$$\frac{\beta(t)}{1/(\sigma_\epsilon(1-t))} = 1, \ \lim_{t\to 1} \frac{\Sigma(t)}{\sigma_\epsilon^2(1-t)} = 1, \ \lim_{t\to 1} \frac{\lambda(t)}{\sigma_\epsilon/2} = 1.$$

Even with correlated signals, informed investors learns to become cooperative. As discussed earlier, the increase in conditional precision for the informed investor is twice of that of the market maker. As learning accumulates, the ratio of the conditional precision of the informed investor and the market maker about the asset value converges to two. As a result, the conditional correlation among informed investors converges to zero. This is drastically different from the case without disclosure. In Back, Cao and Willard (2000)'s model without disclosure, near the end of trading, the ratio of the conditional precision of the informed investors and that of the market maker about the asset value converges to 1 as the increase in conditional precision goes to infinity. This holds because the noise in the price comes from the noise traders and no one has any extra information about the noise trades. Therefore the increase in conditional precision is the same for the market maker and the informed traders. As time goes to 1, the increase in conditional precision goes to infinity and the ratio of conditional precision between the informed investor and market maker goes to 1. Informed investors has little informational advantage over the market maker, the asset value the conditional correlation of investors' private valuation goes to -1 and $\lambda(t)$ goes to infinity. On the contrary, in continuous time trading with disclosure, investors learn to become cooperative. The conditional correlation of investors' private valuation goes to zero and $\lambda(t)$ goes to a constant.

In Figures 6-9, we examine how $\beta(t), \Sigma(t), 1/\lambda(t), \pi(0)$ changes with σ_ϵ. In Figure 6, it is clear that $\beta(t)$ decreases with σ_ϵ. Coarser information makes investors compete with each other less intensively. As shown in Figure 7, market also becomes less efficient as σ_ϵ increases. Near the end of trading, conditional variance decreases almost as a straight line, like the monopolistic setting. Figure 8 plots market depth with disclosure. With low σ_ϵ, market depth increases over time as informed traders trade very aggressively to start with and market is less liquid in the beginning but as more information is revealed, the market becomes more liquid. With high σ_ϵ, investors trade cautiously in the beginning and start to trade more aggressively later as they learn more from each other through disclosure. As a result, market liquidity drops over time. With uncorrelated signals, market efficiency and market liquidity are the same as if there exists a monopolistic informed investor with all the signals in the market. It follows that informed investors' profits is maximized with when $\sigma_\epsilon = \sigma_v$ as shown in Figure 9. Notice that in the setting without disclosure, informed investors' profits are maximized when σ_ϵ is slightly larger than σ_v.

Next we examine market depth, $1/\lambda(t)$ defined by Kyle (1985). The expected profits π_D is related to market depth as described in Theorem

Corollary 4.6 *As time approaches 1, we have*

$$\lim_{t\to 1} \frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \infty,$$

Moreover, when $\sigma_\epsilon \leq \sigma_v$, then $1/\lambda(t) > 1/\hat{\lambda}(t)$. In addition, $\pi(0) < \hat{\pi}(0)$.

There are three factors that affect market liquidity. The first effect is the randomization effect which will increase market liquidity under disclosure. Other things being equal, this effect will double market liquidity. The second effect is the trading intensity effect due to private information which decrease market liquidity under disclosure. While both the informed investor and the market maker learns from public disclosure. The noise in the publicly disclosed trades for the market maker is $\sum_{i=1}^{2} \sigma(t)dW^i(t)$ but the noise for each investor i is $\sigma(t)dW^j(t), j \neq i$. Therefore informed investor i learns more from the public disclosure than the market maker and this effect will decrease market liquidity. The third effect is the market efficiency effect which increase market liquidity under disclosure because of a lower residual uncertainty.

Figure 3 plots the market depth with positively correlated signals. As shown in Figure 3, when $\sigma_v^2 \geq \sigma_\epsilon^2$ the last two effects roughly offset each other except near the end of trade. Therefore, the first effect is dominant in early part of the trading period and market liquidity roughly doubles. However in the latter part of the trading period, disclosure makes the market much more efficient and the third effect is dominant and market liquidity is much higher. Therefore, market is always more liquid with disclosure.

In brief, when the noise in informed investors signals is small, informed investors don't learn from each other as much. As they trade more aggressively on their perceived difference from market expectation under disclosure, market depth is higher with disclosure due to randomization and higher market efficiency, a component in informed trader's trade which makes the proportion of informed trade less significant. It is interesting to observe that market depth changes over time in a pattern that is different from no-disclosure case. In the case of multiple informed traders of positively correlated signals, market first rises and then declines to 0 when this is no disclosure requirement; but market depth always rises in when there is disclosure requirement.

Corollary 4.7 *For $t > 3/4$, there exists $\sigma_\epsilon^* > \sigma_v$, such that for $\sigma_\epsilon > \sigma_\epsilon^*$, that $1/\lambda(t) < 1/\hat{\lambda}(t)$. In addition, there exists $\sigma_\epsilon^{**} > \sigma_v$, such that for $\sigma_\epsilon > \sigma_\epsilon^{**}$, $\pi(0) > \hat{\pi}(0)$.*

This is a rather surprising result. Intuitively, one would have expected that disclosure should always increase market liquidity. As we discussed earlier, the effects of trade disclosure on market liquidity can be decomposed to three components: randomization effect, trade intensity effect and the market efficiency effect. When σ_ϵ is very small, each informed investor on his own knows very little about the value of the liquidation value of the risky asset. Therefore they learn a lot from disclosure of informed investors' trades. Since the variance of noise in disclosed trades is $2\sigma^2 dt$ for the market maker and $\sigma^2 dt$ for each informed trader, informed investors learns faster from disclosed trades than the market maker. When σ_ϵ is very large, the learning from public disclosure becomes very significant and this effect dominates the other two effects which causes the market liquidity to be higher for some t. Moreover, the reduction in market liquidity can result in higher profits for very large σ_ϵ.

The effect of disclosure on informed investors' profits is ambiguous. Other things being equal, disclosure causes the informed investors to lose half of their information based trading profits due to randomization. This results in a reduction of informed investors' profits when σ_ϵ is small. With large σ_ϵ, the results can be reversed. In the latter case, informed investors learn a lot from the disclosed trades about the asset value as they each have very imprecise signals in the beginning. In addition, the informed investors learns more from the disclosed trades than the market maker. The increase of precision is 4 times of that of the market maker. Consequently, the increase of learning by informed investors could more than offset the loss due to randomization and make them earn more profits than what they would receive in a setting without disclosure. Alternatively, we can view disclosure as an apparatus for coordination. Notice that informed investors' profits would be maximized if they could coordinate and trade at the same intensity as a monopolist with the same information. When each informed investor has very imprecise signals, they trade very cautiously, far from the level of a monopolist. Disclosure of trades releases information and make them trade more aggressively toward the level of a monopolist. The increase of trading intensity effectively coordinates their trading activity toward higher profits, and can offset the losses due to randomization when σ_ϵ is low.

Figure 5 plots the ratio of informed traders' profits as a function of σ_ϵ. Notice that the informed investors' total expected profits could be larger under trade disclosure for large σ_ϵ.

Disclosure makes informed investors learn to cooperate. Thus it is interesting to determine how disclosure affects an informed investor's profit with and without competition. Will an informed investor facing competition be better off? As shown in BCW (2000), this can never happen in a setting without trade disclosure. However in our setting with disclosure of trades, it is possible. Let π_D denote the expected profits of a single informed investor in a duopolistic setting and π_M denote the expected profits of a single informed investor in a monopolistic setting and $\hat{\pi}_M$ that of an informed investor in a monopolistic setting without disclosure. We have

Corollary 4.8 *There exists $\hat{\sigma}_\epsilon$ such that for $\sigma_\epsilon > \hat{\sigma}_\epsilon$, $\pi_D > \hat{\pi}_M > \pi_M$.*

This holds because, with very large σ_ϵ, investors each has very noisy signals and are eager to learn about from each other. Disclosure of trades let investors to learn from each other about the market value at a speed (as measured by the increase in conditional precision) four times as fast as that of the market maker. Notice this cannot happen in the setting without disclosure as informed investors learn at the same speed as the market maker. With very large σ_ϵ, the benefit of learning can offset the loss due to competition and informed investors are better off with competition. Interestingly, learning from each other is so beneficial that an informed investor with disclosure and competition is better off than what he expects to receive with neither disclosure nor competition. disclosure

Our analysis indicates that learning can create synergies in the presence of disclosure. Suppose that each informed investor has to spend c to collect differential signals as described before and the act to collect information is observable by market participants, then we have the following herding result regarding information acquisition:

Proposition 1 *When $\sigma_\epsilon > \hat{\sigma}_\epsilon$ and $\pi_D > c > \pi_M$, there exists two information acquisition equilibria: (i) in the first equilibria, no one would acquire any private signals; (ii) in the second equilibria, both informed investors will acquire private signals.*

5 Extension

Our model can be extended to arbitrary number of informed investors with the following modification. Assuming that each informed investor $i = 1, ..., N$ receives a signal in the form of

$$s_i = \frac{v + \epsilon_i}{N} \tag{5.32}$$

in addition we have

$$\epsilon_i = \eta_i - \frac{\sum_{j=1}^N \eta_j}{N} \tag{5.33}$$

and that $v, \{\eta_i, i = 1, \ldots, N\}$ are multi-variate normally distributed and independent with mean zero. Moreover, η_i has variance σ_η^2 for all i. Let σ_ϵ^2 denote the variance of ϵ, it follows that

$$\sigma_\epsilon^2 = \frac{N-1}{N}\sigma_\eta^2. \tag{5.34}$$

When $N = 1$, the informed investor knows v and our model reduces to HHL (2001). Let ρ denote the correlation of investor's private signals, it is easy to verify that

$$\rho = \frac{\sigma_v^2 - \sigma_\epsilon^2/(N-1)}{\sigma_v^2 + \sigma_\epsilon^2}. \tag{5.35}$$

Given these notations, we present the discrete time model and continuous time model below:

Theorem 5.1 *The necessary and sufficient conditions for a recursive linear symmetric equilibrium to exist are described below. For all $m = 1, \cdots, M-1$ and for all informed traders $i = 1, \cdots, N$,*

$$x_m^i = \frac{\beta_m \Delta t}{N\delta_{m-1}}(V_{m-1}^i - V_{m-1}) + z_m^i \tag{5.36}$$

$$P_m = V_{m-1} + \lambda_m \left(z_m^0 + \sum_{i=1}^N x_m^i \right) \tag{5.37}$$

$$V_m = V_{m-1} + \bar{\lambda}_m \sum_{i=1}^N x_m^i \tag{5.38}$$

$$\bar{\lambda}_m = \beta_m \Sigma_m/(N\sigma_m^2) \tag{5.39}$$

$$\lambda_m = \beta_m \Sigma_{m-1}/(\beta_m^2 \Delta t \Sigma_{m-1} + 1 + N\sigma_m^2) \tag{5.40}$$

$$V_m^i - V_{m-1}^i = \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}} \left(v - V_{m-1}^i + \sum_{j \neq i} \frac{z_m^j}{\beta_m \Delta t} \right) \tag{5.41}$$

$$V_m - V_{m-1} = \frac{\Sigma_{m-1} - \Sigma_m}{\Sigma_{m-1}} \left(v - V_{m-1} + \sum_{1 \leq j \leq N} \frac{z_m^j}{\beta_m \Delta t} \right) \tag{5.42}$$

$$\Omega_m^{-1} = \Omega_{m-1}^{-1} + \beta_m^2 \Delta t/((N-1)\sigma_m^2) \tag{5.43}$$

$$\Sigma_m^{-1} = \Sigma_{m-1}^{-1} + \beta_m^2 \Delta t/(N\sigma_m^2) \tag{5.44}$$

$$E[\pi_m^i | F_{m-1}^i] = \alpha_{m-1}(V_{m-1}^i - V_{m-1})^2 + \zeta_{m-1} \tag{5.45}$$

$$\lambda_m = \alpha_m \bar{\lambda}_m^2 \tag{5.46}$$

$$\lambda_m = \frac{\bar{\lambda}_m}{2 - \bar{\lambda}_m \beta_m \Delta t(1 - 1/(N\delta_{m-1}))} \tag{5.47}$$

$$\alpha_{m-1} = \alpha_m \left(1 - \frac{\beta_m^2 \Delta t \Sigma_m}{N\sigma_m^2} \left(1 - \frac{1}{N\delta_{m-1}} \right) \right)^2 \tag{5.48}$$

$$\zeta_{m-1} = \zeta_m + \alpha_m \beta_m^2 \Delta t \left(\frac{\Omega_m}{(N-1)\sigma_m^2} - \frac{\Sigma_m}{N\sigma_m^2} \right)^2 \left(\Omega_{m-1}\beta_m^2 \Delta t + (N-1)\sigma_m^2 \right) \tag{5.49}$$

subjecting to the boundary conditions

$$\beta_M = \sqrt{\frac{N\delta_{M-1}}{\Sigma_{M-1}\Delta t}}, \tag{5.50}$$

$$\lambda_M = \frac{\sqrt{N\delta_{M-1}\Sigma_{M-1}/\Delta t}}{1 + N\delta_{M-1}}, \tag{5.51}$$

$$\alpha_{M-1} = \frac{1}{\lambda_M(1 + N\delta_{M-1})^2}, \tag{5.52}$$

$$\zeta_{M-1} = 0, \tag{5.53}$$

and the second order condition

$$\lambda_M > 0. \tag{5.54}$$

Similar to the case of two informed traders, the equilibrium can be solved recursively. When Δ_t goes to zero, the system converges to a set a differential equations derived in the continuous time trading model presented below

Theorem 5.2 *In continuous time trading, there is a unique symmetric linear equilibrium specified as follows*

$$\beta(t) = \frac{\sqrt{-\Sigma(t)'}}{\Sigma(t)}, \lambda(t) = \frac{\sqrt{-\Sigma'(t)}}{2}, \bar{\lambda}(t) = \sqrt{-\Sigma'(t)},$$

where

$$\Sigma(t) = \Sigma(0)(1-t) \;\; \textit{for } \sigma_\epsilon^2 = (N-1)\sigma_v^2 \textit{ or } N = 1$$

$$
\begin{aligned}
\Sigma(t) &= \frac{\sigma_\epsilon^2 \Sigma(0)}{(N-1)\sigma_v^2 - \sigma_\epsilon^2}\left[\left((1-B)\,t + B\right)^{\frac{N}{4-3N}} - 1\right] \; \textit{otherwise} \\
\textit{where } B &= \left(\frac{\sigma_\epsilon^2}{(N-1)\sigma_v^2}\right)^{3-\frac{4}{N}}, \Sigma(0) = \sigma_v^2.
\end{aligned}
$$

In equilibrium, the expected profit of each informed investor is

$$
\frac{1}{N}\int_0^1 \lambda(t)dt = \begin{cases} \frac{1}{2N}\sqrt{\Sigma(0)} & \textit{for } \sigma_\epsilon^2 = (N-1)\sigma_v \textit{ or } N = 1, \\ \sqrt{\frac{\Sigma(0)(3N-4)\sigma_\epsilon^2}{N(1-B)((N-1)\sigma_v^2 - \sigma_\epsilon^2)}}\;\frac{\left|1-B^{\frac{N-2}{3N-4}}\right|}{2|N-2|} & \textit{otherwise.} \end{cases} \tag{5.55}
$$

For the purpose of comparison, we restate the Back, Cao, and Willard (2000) result of continuous trading equilibrium without disclosure in the next theorem.

Theorem 5.3 *If there is more than one informed trader ($N > 1$) and their signals are perfectly correlated ($\rho = 1$), then there is no symmetric linear equilibrium. Otherwise, there is a unique symmetric linear equilibrium. Set $\hat{\Sigma}(0) = var(v)$, and consider the constant*

$$
k = \int_1^\infty x^{2(N-2)/N} e^{-2x(1-\phi)/(N\phi)}\, dx. \tag{5.56}
$$

For each $t < 1$, define $\hat{\Sigma}(t)$ by

$$
\int_1^{\hat{\Sigma}(0)/\hat{\Sigma}(t)} x^{2(N-2)/N} e^{-2x(1-\phi)/(N\phi)}\, dx = kt. \tag{5.57}
$$

The equilibrium is

$$
\hat{\beta}(t) = \left(\frac{k}{\hat{\Sigma}(0)}\right)^{1/2} \left(\frac{\hat{\Sigma}(t)}{\hat{\Sigma}(0)}\right)^{(N-2)/N} \exp\left\{\frac{1}{N}\left(\frac{1-\phi}{\phi}\right)\frac{\hat{\Sigma}(0)}{\hat{\Sigma}(t)}\right\}, \tag{5.58}
$$

$$
\hat{\lambda}(t) = \hat{\beta}(t)\hat{\Sigma}(t). \tag{5.59}
$$

With respect to the comparative statics of the case with more informed investors, we have the following results:

Corollary 5.1 *(i)Informed investors contribute half of the trading volume in the market with disclosure; (ii) For $N = 1$, we have $\beta(t) = \hat{\beta}(t), \Sigma(t) = \hat{\Sigma}(t), \lambda(t) = \hat{\lambda}(t)/2 = 1/(2\sigma_v)$; For $N > 1$, we have the following results: (iii)*

$$
\lim_{t\to 1}\frac{\beta(t)}{\hat{\beta}(t)} = \infty, \; \lim_{t\to 1}\frac{\Sigma(t)}{\hat{\Sigma}(t)} = 0,
$$

(iv)

$$
\lim_{t\to 1}\frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \infty,
$$

(v)The conditional variance of the asset value Σ decreases with t and increases with σ_ϵ. The initial market depth $1/\lambda(0)$ increases with σ_ϵ and the market depth in the end of trading, $1/\lambda(1)$ decreases with σ_ϵ. The variable $\lambda(t)$ decreases over time when $\sigma_\epsilon^2 < (N-1)\sigma_v^2$ while $\lambda(t)$ increases over time when $\sigma_\epsilon^2 \geq (N-1)\sigma_v^2$; (vi)When $\sigma_\epsilon^2 = (N-1)\sigma_v^2$, informed investors trade in aggregate like a monopolistic investor and informed investors' profits are maximized. Therefore, market efficiency and market liquidity are the same as if there exists a monopolistic informed investor with all the signals in the market. Conditional correlation of investors' private valuation remains uncorrelated throughout the trading period. When $\sigma_\epsilon^2 \neq (N-1)\sigma_v^2$, as $t \to 1$, $N\delta \to 1$ and informed traders' private valuations become uncorrelated and they all behave in aggregate like a monopolistic informed trader with all the information in the economy. We have

$$\lim_{t\to 1} \frac{\beta(t)}{1/(\sqrt{S_0}(1-t))} = 1, \ \lim_{t\to 1} \frac{\Sigma(t)}{S_0(1-t)} = 1, \ \lim_{t\to 1} \frac{\lambda(t)}{\sqrt{S_0}/2} = 1.$$

Here, $S_0 = \frac{(1-\rho)(1-B)\Sigma_0}{\rho(3N-4)}$, B is defined in theorem 5.2.

Notice that our results on comparative statics obtained with two informed investors broadly hold for larger N with some notable exceptions. Although we can't prove that $\beta(t) > \hat{\beta}(t)$, and $\Sigma > \hat{\Sigma}$ for all t, we prove it for t close to 1 and our numerical analysis shows that disclosure increases the intensity of informed trading and improves market efficiency. The increase in market efficiency due to disclosure also makes the market depth higher near the end of trading. The conditional variance increases as investors receive noisier signals. Initial market depth is higher with noisier signals as investors trade cautiously initially. However, market depth in the end of trading will be lower with noisier signals as there will be more residual asymmetric information near the end. As a result, market depth will be decreasing with noisy signals and increasing with precise signals. Informed investors' profits will be maximized if they have uncorrelated signals in which case they coordinate and trade like a monopolist. Moreover, the conditional correlation goes to zero near the end of trading even when investors initially have correlated signals. Informed investors learn to be cooperative.

Next we consider whether informed investors is better off or worse off with more informed investors in the market. Let π_N denote what an informed investor would expected to receive in a setting with multiple informed traders. Let $\pi_{N\to N-1}$ denote the profits each informed investor would obtain if one informed investor leaves the market and the other $N-1$ informed investors will stay and trade in this market and π_m denote what he would receive if he is the only informed investors in the market.

Corollary 5.2 *Then for any $N > 1$, there exists $\bar{\sigma}_\epsilon$ such that $\pi_N > \pi_{N\to N-1}$ for all $\sigma_\epsilon > \bar{\sigma}_\epsilon$. In addition, for $1 < N < 5$, there exists $\hat{\sigma}_\epsilon$ such that for $\sigma_\epsilon > \hat{\sigma}_\epsilon$, $\pi_N > \pi_M$.*

Just like the case with two informed investors, $N-1$ informed investors can benefit from the participation of one more informed investor, if they collectively learn a lot from the new participant through trading. Indeed, learning can be so beneficial that a monopolist will be better off if $N-1$ informed investors all participate when $N < 5$. However, as N goes to infinity, each informed investor's profits goes to zero. In Figure 10, we show numerically that for $N = 5$, a monopolist would prefer the other four informed investors not to participate in the market.[10]

With two informed investors, it is possible that disclosure increase the aggregate profits of informed investors. We show numerically in Figure 5 that this is impossible when $N > 2$. With larger N, each informed investor will learn at the speed $N^2/(N-1)^2$ times that of the market maker. However $N^2/(N-1)^2$ is decreasing in N, therefore, for larger N the benefit of learning from each other and coordinating with each other is not big enough to offset the loss due to randomization.

6 Conclusion

How would disclosure of informed investors' trades affect market efficiency, market liquidity and expected profits of informed investors? In a setting with two informed investors, we show that informed investors will randomize their trades to hide their private information and to manipulate market maker's and others' beliefs. As a result, they sometimes trade against their own valuation. The instantaneous variance of informed traders' trade is the same as that of the liquidity traders. Similar to the single informed investor model of Huddart, Hughes and Levine (2001), the market is more efficient with trade disclosure.

However with more than one informed investor in the market, informed investors also learn from each other. Contrary to the model of Back, Cao and Willard (2000) in which informed investors learns at the same speed (measured by the increase of conditional precision) as the market maker, in our model informed investors learns more than market maker as they know of the manipulating component in their trades. Consequently, with noisy initial signals, the learning effect by informed investors dominates and they make more expected profits than what they would obtain in a setting without disclosure. In addition, an informed investor learn much from disclosure such that he make more profits than he would make if he is the only informed investor in the market. Synergy in the gains from informed trading also implies that when there is cost in information collection, there could exit multiple information acquisition equilibria. In one equilibrium, no one would acquire information but in the other both investors would acquire information.

In the extension to three or more informed investors, each informed investor still learns more than the market maker, albeit at a lower speed advantage (at the speed of $N^2/(N-1)^2$ of that of the market maker). The reduction in the relative speed of learning cause the gains informed investors receive from learning to be lower than that of the loss due to randomization. As a result disclosure always reduce informed investors' profits. However, even in the case of three or more informed investors, for very noisy signals, each informed investor can still benefit from the presence of more informed investors as informed investors can learn more from each other.

Disclosure also changes the inter-temporal patterns of the market liquidity. In models without disclosure, Back, Cao and Willard show that informed investors will eventually be on the other side of the market and market liquidity goes to zero as they cluster their trades near the end of trading.

[10] This holds also for $N > 5$ numerically although we cannot provide an analytical proof for this result.

When the noise in informed investors signals is small, market liquidity will first increase and then decrease. For large noises, market liquidity always decreases over time. On the contrary, in our model, market liquidity is always finite. When informed investors have very noisy signals they will trade more cautiously in the beginning and thus initial market liquidity $1/\lambda(0)$ increases with σ_ϵ. As time goes on, investors learn more and trade more aggressively, and market liquidity will also decrease over time. Indeed, at the end of period market liquidity decreases with σ_ϵ. With small noises in informed investors' private signals, informed investors will trade aggressively initially which results in a lower market liquidity that increases over time.

We considered only the case in which the signals have a symmetric structure that is they all have the same correlation with each other and the same variance. In the future, it would be interesting to relax this restriction and it is possible that some informed investors benefit from disclosure while others would be worse off. Similarly, with asymmetric information structure, it is also possible that some informed investors may prefer more informed investors to learn from each other while others would be better off with less competition.

Our model provides the first example in which informed investors are better with with more public information. It is worthwhile to examine if this also holds in cases of information disclosure of signals about asset value, not in terms of trade disclosure. We leave that for future research.

A Proofs for Section 2

Proof of Theorem 2.1 We are here proving Theorem 5.1 the general case with $N \geq 1$ and Theorem 2.1 is included as a special case $N = 2$. We focus on proving the necessity of the claimed equations. The sufficiency of these equations can be established by reversing the necessity arguments (see the end of this proof for more details). So in the rest of this proof except in the last paragraph, we assume that a symmetric linear equilibrium exists, and we prove the claimed equations.

We first prove Equations 5.41, 5.42, 5.43, and 5.44 simply by assuming that each informed trader follows Strategy 2.1. These equations will be used in the inductive proofs for other equations.

First, we can easily check the correctness of Equations 5.41 and 5.42 by the fact that the expectation of a normal variable is precision-weighted average of all received signals. Moreover, the updating rules of normally distributed variables states that posterior precision equals prior precision plus the precision of the noise of the signals. Hence, we immediately establish the correctness of Equations 5.43 and 5.44.

Before proving the rest of the desired equations, we first establish the following useful lemma.

Lemma A.1 *Assume (1) each informed trader believes that all other informed traders follow Strategy 2.1, and (2) the market maker believes that all informed traders follow Strategy 2.1. Then,*

$$\sum_{1 \leq i \leq N} (V_m^i - V_m) = N\delta_m(v - V_m).$$

Proof First, it is easy to check the correctness of the following mathematical identidy by properties of normal variables

$$\Omega_0 = \frac{N-1}{N}(1-\rho)\Sigma_0. \tag{A.1}$$

Using this relation and Equations 5.43 and 5.44, we can easily check

$$\frac{\Omega_m}{\Omega_0}(N-1)\rho + 1 = N\delta_m. \tag{A.2}$$

In what follows, define

$$U^i_m \equiv E[v - s^i | F^i_m]$$

where the expectation is computed after trade disclosures in period m. Equivaently, we could have defined $U^i_m \equiv V^i_m - s^i_m$.

Since the expected value of a normal variable is equal to the precision weighted average of all received signals, we have

$$\begin{aligned} U^j_m &= \frac{\Omega_m}{\Omega_0}U^j_0 + \Omega_m \sum_{1\le k\le m}\left[\left(\frac{1}{\Omega_k} - \frac{1}{\Omega_{k-1}}\right)\sum_{i\neq j}\left(s^i_k + \frac{z^i_k}{\beta_k \Delta t}\right)\right] \\ &= \frac{\Omega_m}{\Omega_0}U^j_0 + \Omega_m \sum_{1\le k\le m}\left[\frac{\beta_k^2 \Delta t}{(N-1)\sigma^2_m}\sum_{i\neq j}\left(s^i_k + \frac{z^i_k}{\beta_k \Delta t}\right)\right], \end{aligned} \tag{A.3}$$

where the second equation follows from Equation 5.43. (It is easy to verify that Equation 5.43 holds when each informed trader merely *belives* all other informed traders follow Strategy 2.1.) Similarly,

$$V_m = 0 + (\Sigma_m) \cdot \sum_{1\le k\le m}\left[\frac{\beta_k^2 \Delta t}{N\sigma^2_m}\sum_{1\le i\le N}\left(s^i_k + \frac{z^i_k}{\beta_k \Delta t}\right)\right]. \tag{A.4}$$

Summing up Equation A.3 over $j = 1, 2, \ldots, N$, we have

$$\begin{aligned} \sum_{1\le j\le N} U^j_m &= \frac{\Omega_m}{\Omega_0}(N-1)\rho \sum_{1\le j\le N} s^j + \Omega_m \sum_{1\le k\le m}\left[\frac{\beta_k^2 \Delta t}{\sigma^2_m}\sum_{1\le i\le N}\left(s^i_k + \frac{z^i_k}{\beta_k \Delta t}\right)\right] \\ &= \frac{\Omega_m}{\Omega_0}(N-1)\rho v + N\frac{\Omega_m}{\Sigma_m}V_m \quad \text{(by Equation A.4)} \\ &= (N\delta_m - 1)v + N\frac{\Omega_m}{\Sigma_m}V_m \quad \text{(by Equation A.2).} \end{aligned}$$

The last equation is only a slight variation of the equatlity claimed in the lemma.

We have thus completed the proof of Lemma A.1. Using the results established in proving the lemma, we next prove that Strategy 5.36 satisfies Equation 2.1. In Equation 5.36, x^i_m consists of a random component (z^i_m), a component based on public information ($\frac{\beta_m \Delta t}{N\delta_{m-1}}V_{m-1}$), and a private-information-related component ($\frac{\beta_m \Delta t}{N\delta_{m-1}}V^i_{m-1}$). By Equation A.3, the only private component in $\frac{\beta_m \Delta t}{N\delta_{m-1}}V^i_{m-1}$ is equal to

$$\frac{\beta_m \Delta t}{N\delta_{m-1}}\left(s^i + \frac{\Omega_{m-1}}{\Omega_0}(N-1)\rho s^i\right) = \beta_m \Delta t s^i \quad \text{(by Equation A.2).}$$

This proves that Strategy 5.36 satisfies Equation 2.1. Moreover, our arguments also imply that to support a symmetric linear equilibrium, x_m^i must have the following form:

$$x_m^i - z_m^i = \frac{\beta_m \Delta t}{N\delta_{m-1}} V_{m-1}^i + \text{ a public-information-based component.} \tag{A.5}$$

Using Lemma A.1 and Equation 5.36, we have

$$\sum_{1\le i\le N} x_m^i = \beta_m \Delta t \left(v - V_{m-1} + \sum_{1\le i\le N} \frac{z_m^i}{\beta_m \Delta t} \right). \tag{A.6}$$

Therefore, using Equation 5.42 we immediately obtain Equation 5.38 and Equation 5.39 (the derivation of Equation 5.39 also needs Equation 5.44). Note that in a symmetric linear equilibrium, the value updating rules must be of the form specified in Equation 5.38. Our arguments in this paragraph together with Equation A.5 also show that to support a symmetric linear equilibrium, Equation 5.36 must hold.

Using Equation A.6 and the rules of conditional expectation of normally distributed variables, we immediately obtain Equation 5.37 with

$$\begin{aligned} \lambda_m &= \frac{\operatorname{cov}\left(v, \sum_{1\le j\le N} x_m^j + z_m^0\right)}{\operatorname{var}(z_m^0 + \sum_{1\le j\le N} x_m^j)} \\ &= \beta_m \Sigma_{m-1}/(\beta_m^2 \Delta t \Sigma_{m-1} + 1 + N\sigma_m^2). \end{aligned}$$

The last equation is exactly Equation 5.40.

We next proceed to prove Equations 5.45 to 5.54 by backward induction on m, starting with the last period $m = M$. As there is no more trading opportunities after the last period, the maximization problem for each informed trader i is the same as the case without disclosure which has been derived in Foster and Viswanathan (1996) and Cao (1995). In particular, applying Thereom **??**, we know that the expected profit function of informed trader i has the form described in Equation 5.45 with the boundary conditions specified in Equations 5.50 to 5.54.

Thus, we have completed the base step. Next, we assume Equations 5.45 to 5.49 are correct for period $m+1$ and prove them for period m. By the induction hypothesis, immediately after the mth period disclosure, the expected profits for future trades (i.e., trades from period $m+1$ onwards) can be written as,

$$E_m^i[\pi_{(m+1)}^i] \equiv E[\pi_{(m+1)}^i | F_m^i] = \alpha_m (V_m^i - V_m)^2 + \zeta_m. \tag{A.7}$$

Hence, the maximization problem of informed trader i immediately after the $(m-1)$th period trade disclosure is:

$$\max_{x_m^i} E_{m-1}^i \left[x_m^i \left(v - V_{m-1} - \lambda_m \left(z_m^0 + \sum_{1\le j\le N} x_m^j \right) \right) + \alpha_m (V_m^i - V_m)^2 \right] + \zeta_m, \tag{A.8}$$

where the two terms inside the squared brackets represent the profit of the mth trade and the total profit of all future trades.

For informed trader i to follow a random strategy, he must be indifferent between different values of x^i_m. Thus, the coefficients of $(x^i_m)^2$ and x^i_m in Expression A.8 must be zero. These two restrictions respectively imply

$$\lambda_m = \alpha_m \bar{\lambda}_m^2, \quad \text{and} \tag{A.9}$$

$$E^i_{m-1}\left[v - V_{m-1} - \lambda_m \sum_{j\neq i} x^j_m\right] = 2\alpha_m \bar{\lambda}_m E^i_{m-1}\left[V^i_m - V_{m-1} - \bar{\lambda}_m \sum_{j\neq i} x^j_m\right]. \tag{A.10}$$

Note that Equation A.9 is the same as Equation 5.46. In what follows, we show that Equations A.9 and A.10 together imply Equation 5.47. On the other hand, by Lemma A.1,

$$\sum_{j\neq i} x^j_m = \beta_m \Delta t \left(v - V_{m-1} - \frac{1}{N\delta_{m-1}}(V^i_{m-1} - V_{m-1})\right) + \sum_{j\neq i} z^j_m. \tag{A.11}$$

Hence,

$$E^i_{m-1}\left[\sum_{j\neq i} x^j_m\right] = \beta_m \Delta t \left(1 - \frac{1}{N\delta_{m-1}}\right)\left(V^i_{m-1} - V_{m-1}\right)$$

Applying this relation to Equation A.10, we obtain

$$\frac{1 - \lambda_m \beta_m \Delta t + \lambda_m \beta_m \Delta t/(N\delta_{m-1})}{1 - \bar{\lambda}_m \beta_m \Delta t + \bar{\lambda}_m \beta_m \Delta t/(N\delta_{m-1})} = 2\alpha_m \bar{\lambda}_m.$$

Now we multiply both sides of the preceding equation with the denominator of the left-hand side of the equation, and then we use Equation A.9 to substitute all the $\alpha_m \bar{\lambda}^2$ terms by λ_m. This leads to

$$2\alpha_m \bar{\lambda}_m = 1 + \lambda_m \left(\beta_m \Delta t - \frac{\beta_m \Delta t}{N\delta_{m-1}}\right).$$

Next, multiplyuing both sides of the above equation with $\bar{\lambda}_m$ and using Equation A.9 to substitute $\alpha_m \bar{\lambda}^2$ by λ_m, we immediately obtain Equation 5.47.

Since we have established that informed trader i is indifferent to x^i_m, Expression A.8 can be simplified by setting $x^i_m = 0$. Thus,

$$\begin{aligned} E^i_{m-1}[\pi_m] &= \alpha_m E^i_{m-1}\left[\left(V^i_m - V_m\right)^2\right] + \zeta_m \\ &= \alpha_m \left(E^i_{m-1}\left[V^i_m - V_m\right]\right)^2 + \alpha_m \mathrm{var}^i_{m-1}(V^i_m - V_m) + \zeta_m \end{aligned} \tag{A.12}$$

On the other hand, since we have assumed $x^i_m = 0$ in the profit calculation, using the updating rule for normal variables we have

$$\begin{aligned} V^i_m &= \frac{\Omega_m}{\Omega_{m-1}} V^i_{m-1} + \frac{\Omega_{m-1} - \Omega_m}{\Omega_{m-1}}\left(v + \sum_{j\neq i} \frac{z^j_m}{\beta_m \Delta t}\right) \\ &= \frac{\Omega_m}{\Omega_{m-1}} V^i_{m-1} + \frac{\Omega_m \beta_m^2 \Delta t}{(N-1)\sigma_m^2}\left(v + \sum_{j\neq i} \frac{z^j_m}{\beta_m \Delta t}\right), \end{aligned} \tag{A.13}$$

where the second equation follows from Equation 5.43. Moreover, using the pricing rules by market maker and applying Equation A.11, we have

$$\begin{aligned} V_m &= V_{m-1} + \bar{\lambda}_m \beta_m \Delta t \left(v - V_{m-1} - \frac{V^i_{m-1} - V_{m-1}}{N\delta_{m-1}} \right) + \bar{\lambda}_m \sum_{j \neq i} z^j_m \\ &= V_{m-1} + \frac{\beta_m^2 \Sigma_m \Delta t}{N\sigma_m^2} \left(v - V_{m-1} - \frac{V^i_{m-1} - V_{m-1}}{N\delta_{m-1}} + \sum_{j \neq i} \frac{z^j_m}{\beta_m \Delta t} \right), \end{aligned} \tag{A.14}$$

where the second equation follows from Equation 5.39.

Now, using Equations A.14 and A.13 and the fact that v is independent of $\sum_{j\neq i} z^j_m$, we have

$$\mathrm{var}^i_{m-1}(V^i_m - V_m) = \left(\frac{\Omega_m \beta_m^2 \Delta t}{(N-1)\sigma_m^2} - \frac{\beta_m^2 \Sigma_m \Delta t}{N\sigma_m^2} \right)^2 \left(\Omega_{m-1} + \frac{(N-1)\sigma_m^2 \Delta t}{(\beta_m \Delta t)^2} \right) \tag{A.15}$$

Moreover,

$$\begin{aligned} E^i_{m-1}[V^i_m - V_m] &= V^i_{m-1} - E^i_{m-1}[V_m] \\ &= \left(1 - \frac{\beta_m^2 \Sigma_m \Delta t}{N\sigma_m^2} \left(1 - \frac{1}{N\delta_{m-1}} \right) \right) (V^i_{m-1} - V_{m-1}), \end{aligned} \tag{A.16}$$

where the last equation follows from Equation A.14. Substituting Equations A.15 and A.16 into Equation A.12, we immediately see that Equation 5.45 is correct for m with α and ζ satisfying Equations 5.48 and 5.49. This completes our inductive step.

So far, we have proved all the desired equations as necessary conditions to support a symmetric linear equilibrium. In proving these equations, we have used (1) the rationality of the market maker's pricing rules and value updating rules, and (2) the optimality of all informed traders' trading strategies. Moreover, by reversing these arguments, we can easily check that when these equations indeed hold, (1) the pricing rules and value updating rules are indeed rational for the market maker, and (2) the trading strategies of all informed traders are indeed optimal. Therefore, all these equations collectively form a set of sufficient conditions to support a symmetric linear equilibrium. □

Discussion on Solving the System of Equations in Theorem 5.1

The whole recursive system of $\alpha_m, \beta_m, \lambda_m, \bar{\lambda}_m, \Sigma_m, \Omega_m, \zeta_m$ can be numerically solved by first conjecturing a value of Ω_{M-1} and then solving recursively for $\Omega_{M-2}, \ldots, \Omega_0$. Given the conjectured Ω_{M-1}, we can compute δ_{M-1}, since the definition of δ_M and Equations 5.43 and 5.44 imply

$$N\delta_{M-1} = 1 + \frac{\Omega_{M-1}}{\Omega_0}(N\delta_0 - 1).$$

From Ω_{M-1} and δ_{M-1}, we can now derive Σ_{M-1}. From the boundary condition in Equation 5.52, we can determine α_{M-1}. Now again we conjecture a value for Ω_{M-2}, which allows us to derive δ_{M-2} and Σ_{M-2} as before. From Equations 5.44 and 5.39,

$$\Sigma^{-1}_{M-1} = \Sigma^{-1}_{M-2} + \bar{\lambda}_{M-1}\beta_{M-1}\Delta t / \Sigma_{M-1}.$$

Consequently, we obtain $\beta_{M-1}\bar{\lambda}_{M-1}$. Comparing Equations 5.40 and 5.46, we arrive at

$$\beta_{M-1}\Sigma_{M-2}/(\beta_{M-1}^2\Delta t\Sigma_{M-2} + 1 + N\sigma_{M-1}^2) = \bar{\lambda}_{M-1}^2\alpha_{M-1}.$$

In the preceding equation, we can use the derived expression for $\beta_{M-1}\bar{\lambda}_{M-1}$ to substitute $\bar{\lambda}_{M-1}$ for β_{M-1}, and we can use Equation 5.39 to substitute $\bar{\lambda}_{M-1}$ for σ_{M-1}^2. Doing so results in an equation with $\bar{\lambda}$ being the only unknown. Solving the resulting equation gives a formula for $\bar{\lambda}_{M-1}$. Next we can derive β_{M-1} from $(\beta_{M-1}\bar{\lambda}_{M-1})/\bar{\lambda}_{M-1}$, λ_{M-1} from Equation 5.46, and σ_{M-1}^2 from Equation 5.39. Given the expressions for $\lambda_{M-1}, \bar{\lambda}_{M-1}, \beta_{M-1}$ and σ_{M-1}^2, we can now check whether Equation 5.47 holds or not. If it doesn't, we modify our initial value of Ω_{M-2} until it holds. We repeat the procedure to derive $\Omega_{M-3}, ..., \Omega_0$. If the derived Ω_0 is different from the initial given value, we adjust Ω_{M-1} and repeat the whole procedure until the derived Ω_0 equals to the initial given value.

B Proofs for Section 3

Here, we prove the general case $N \geq 1$ for all corollaries and theorem rather than the special case $N = 2$, so the proof of Theorem 5.2 is covered.

Proof of Lemma 3.1 Recall that each dz^i $(1 \leq i \leq N)$ is a non-standard Brownian motion and that dz^i and dz^j are independent for $i \neq j$. Hence, by Equation 3.4, $\sum_{1\leq i\leq N} dz^i$ is a standard Brownian motion with instantaneous variance dt. The correctness of the lemma then follows from the Kalman-Bucy filter (see, e.g., Kallianpur (1980)). □

Proof of Lemma 3.2 Note that $U^j(0) = (N-1)\rho s^j$ and that $\sum_{i\neq j} z^i(t)$ is a Brownian motion with instantaneous variance equal to $\frac{N-1}{N}dt$. On the other hand, Equation A.1 implies $\Omega(0) = \text{var}(v|\mathcal{F}^j(0))$ (since there is no trade at time 0, it makes no difference whether the variance is taken before or after disclosure at time 0). Now, the correctness of the lemma follows from the Kalman-Bucy filter (see, e.g., Kallianpur (1980)). □

Proof of Lemma 3.3 Consider an arbitrary informed trader j $(1 \leq j \leq N)$. If indeed all other traders follow Strategy 3.3, then from Equations 3.12 and 3.13,

$$dU^j + \left(\frac{N}{N-1}\beta^2\Omega\right)U^j dt = \frac{N}{N-1}\beta\Omega\left(\sum_{i\neq j} dz^i + \beta\sum_{i\neq j} s^i dt\right).$$

Together with Equation 3.11, this immediately implies

$$\frac{d}{dt}\left(\frac{1}{\Omega}U^j\right) = \frac{N}{N-1}\beta\left(\sum_{i\neq j} dz^i + \beta\sum_{i\neq j} s^i dt\right).$$

Since trader j believes all other traders follow Strategy 3.3, he expects $dz^i + \beta s^i dt = dx^i - f(t)dt$. Hence, he uses the following rule to update his U^j,

$$\frac{d}{dt}\left(\frac{1}{\Omega}U^j\right) = \frac{N}{N-1}\beta\sum_{i\neq j}(dx^i - f(t)dt).$$

Therefore,

$$\begin{aligned} & U^j(t) \\ = & \frac{\Omega(t)}{\Omega(0)} U^j(0) + \Omega(t) \frac{N}{N-1} \int_0^t \beta(t) \sum_{i \neq j} (dx^i - f(t)dt) \qquad \text{(B.1)} \\ = & \frac{\Omega(t)}{\Omega(0)} (N-1)\rho s^j + \Omega(t) \frac{N}{N-1} \int_0^t \beta(t) \sum_{i \neq j} (dx^i - f(t)dt). \qquad \text{(B.2)} \end{aligned}$$

Note that Equation B.1 can also be directly derived by the fact that (under our normality assumption) the conditional expectation of the asset value is the precision-weighted average of all observable signals.

By exactly the same reasoning, the market maker, who believes that all informed traders follow Strategy 3.3, has his estimate of asset value as

$$V(t) = 0 + \Sigma(t) \int_0^t \beta(t) \sum_{1 \leq i \leq N} (dx^i - f(t)dt). \qquad \text{(B.3)}$$

Summing up Equation B.2 over all j, we obtain

$$\begin{aligned} \sum_{1 \leq j \leq N} U^j(t) &= \frac{\Omega(t)}{\Omega(0)} (N-1)\rho v + \Omega(t) N \int_0^t \beta(t) \sum_{1 \leq i \leq N} (dx^i - f(t)dt) \\ &= (N-1) \frac{\Omega(t)}{\Omega(0)} \rho v + N \frac{\Omega(t)}{\Sigma(t)} V(t) \quad \text{(by Equation B.3)}. \qquad \text{(B.4)} \end{aligned}$$

Now the correctness of Equation 3.15 reduces to the following

$$\frac{\Omega(t)}{\Omega(0)} (N-1)\rho + 1 = N \frac{\Sigma(t) - \Omega(t)}{\Sigma(t)}. \qquad \text{(B.5)}$$

This equality can be directly verified by Equations 3.8 and 3.11.

Proof of Lemma 3.4 By Eqaution B.2, V^j consists of two components.[11] The first component is based on private-information (i.e., it depends on s^j) and is equal to

$$\left(1 + \frac{\Omega(t)}{\Omega(0)} (N-1)\rho\right) s^j.$$

The second component is purely based on public information. Hence, the private-information component in dx^j (i.e., the component dependent on s^j) is equal to

$$\frac{\beta}{N\delta} \left(1 + \frac{\Omega(t)}{\Omega(0)} (N-1)\rho\right) s^j = \beta s^j,$$

where the equality follows from Equation B.5 (which is purely a mathematical identity). This proves that Stratergy 3.16 satisfies Equation 3.3. The above arguments also show that if a strategy satisfies Equation 3.3 and its deterministic part can be decomposed into a public- information component

[11] We can use this equation here since it is derived from merely assuming that each informed trader believes all other informed traders follow Strategy 3.3.

and another component involving V^j, then the latter component must be of the form specified in Equation 3.16.

On the other hand, if all informed traders indeed follow Strategy 3.16, then by Lemma 3.1,

$$dV(t) = \beta(t)\Sigma(t)\left[\sum_{1\le i\le N} dz^i + \beta(v - V(t))dt\right]. \tag{B.6}$$

However, the market maker does not observe v directly. Hence, believing that all informed traders follow Strategy 3.16, he can use Equation 3.15 to substitute the $v - V(t)$ term in the above equation and obtain

$$dV(t) = \beta(t)\Sigma(t)\left[\sum_{1\le i\le N} dz^i + \frac{\beta}{N\delta}\sum_{1\le i\le N}\left(V^i(t) - V(t)\right)dt\right]. \tag{B.7}$$

Since we have already proved that each informed trader i's information-based component has the form of $\frac{\beta(t)}{N\delta(t)}V^i(t)$, the above equation is consistent with Equation 3.1 if and only if the public- information component of each informed trader i's deterministic trade is equal to $\frac{\beta(t)}{N\delta(t)}V(t)$. Hence, we conclude that Strategy 3.16 is the unique trading strategy with the claimed properties.

Now, comparing Equations 3.1 and B.7, we immediately obtain Equation 3.17. Finally, given that Strategy 3.16 supports the pricing rule in Equation 3.1 with $\bar{\lambda}$ specified in Equation 3.17, a direct application of the Kalman-Bucy filter (see, e.g., Kallianpur (1980)) proves that Strategy 3.16 also supports the pricing rule in Equation 3.2 with λ specified in Equation 3.18. □

Proof of Lemma 3.5 Since we will focus on an arbitrary informed trader j $(1 \le j \le N)$ throughout this proof, we use $dx(t)$ as a shorthand for $dx^j(s^j, V^j, V^{x^j})$. Also, we rewrite $V(t)$ and $P(t)$ as $V^x(t)$ and $P^x(t)$, respectively, to emphasize that trading strategy x affects the processes V and P. Using Expression 3.22 and the law of iterated expectations, we know that the objective of trader j is to maximize

$$E\int_0^1 \left(V^j(t) - P^x(t+dt)\right)dx(t)$$

under the dynamics of the state variables V^j, V^x, and P^x.

From Equation 3.13, V^j follows the following dynamics

$$dV^j(t) = \sqrt{\frac{N}{N-1}}\beta(t)\Omega(t)dW^j(t). \tag{B.8}$$

On the other hand, the instantaneous order submitted by all traders $i \ne j$ sum to

$$\begin{aligned} & \sum_{i\ne j}\frac{\beta}{N\delta}\left(V^i - V^x\right)dt + \sum_{i\ne j} dz^i \\ = & \left[\beta(v - V^x) - \frac{\beta}{N\delta}\left(V^j - V^x\right)\right]dt + \sum_{i\ne j} dz^i \quad \text{(by Equation 3.15)} \\ = & \sqrt{\frac{N-1}{N}}dW^j + \beta\left(1 - \frac{1}{N\delta}\right)(V^j - V^x)dt \quad \text{(by Lemma 3.2)} \end{aligned}$$

Hence, by the pricing rule in Equation 3.1,

$$
\begin{aligned}
dV^x(t) &= \bar{\lambda}(t)dx(t) + \bar{\lambda}(t)\sqrt{\frac{N-1}{N}}dW^j(t) \\
&\quad + \bar{\lambda}(t)\beta(t)\left(1 - \frac{1}{N\delta(t)}\right)\left(V^j(t) - V^x(t)\right)dt.
\end{aligned} \tag{B.9}
$$

The optimization problem is a Markovian stochastic control problem with state variables $(V^j(t), V^x(t), P^x(t))$. Let $J(t, s, V^j, V^x)$ denote a candidate for the following value function

$$
\begin{aligned}
&\sup_x E\int_t^1 \left(V^j(u) - P^x(u+du)\right)dx(u) \\
= &\sup_x E\int_t^1 \left(V^j(u) - V^x(u) - \lambda(u)dx(u)\right)dx(u),
\end{aligned}
$$

where the expectation is conditioned on $F^j(t)$ and the equality follows from Equation 3.2. Note that we have dropped "$-\lambda(dz^0(u) + \sum_{i\neq j} dx^i)$" in the parentheses on the right-hand side of the above equation. All of the droped terms there are either a random variable uncorrelated with dx^i or a deterministic term of an order at least dt, and therefore they do not contribute to the expectation.

The Bellman equation for this control problem is

$$
\begin{aligned}
0 &= \max_x E_t^j\Big[\left(V^j - V^x - \lambda dx\right)dx + J_t dt + J_{V^x}dV^x + J_{V^j}dV^j + \\
&\quad + \frac{1}{2}J_{V^xV^x}(dV^x)^2 + J_{V^xV^j}dVdV^j + \frac{1}{2}J_{V^jV^j}(dV^j)^2.\Big]
\end{aligned} \tag{B.10}
$$

Here, J_x, J_{xy} are the first- and second-order partial derivatives of J with respect to x and x, y. Intuitively, the Bellman equation states that the over x of the drift of J plus the instantaneous profit $(V^j - P^x)x$ equals zero; i.e., the expected decline in future profit should be exactly offset by the realized current profit.

Note that the right-hand side of the above Bellman equation depends on x through a quadratic function of dx. In particular, since dx only appears in the dynamics of dV^x but not in the dynamics of dV^j, the only terms involving dx (except those of higher orders) on the right-hand side of the Bellman equation are: $(-\lambda + \frac{1}{2}J_{V^xV^x}\bar{\lambda})(dx)^2$ and $(\bar{\lambda}J_{V^x} + V^j - V^x)dx$. For trader j to follow a random strategy, he must be indifferent across the various possible orders induced by the random strategy; i.e., the coefficient of dx and $(dx)^2$ must be zero. Reasoning from the linear term, we have

$$
J_{V^x} = \frac{V^x - V^j}{\bar{\lambda}}. \tag{B.11}
$$

Reasoning from the quadratic term, we have $\frac{1}{2}J_{V^xV^x}\bar{\lambda}^2 = \lambda$. Then, applying Equation B.11, we obtain

$$
\lambda = \frac{\bar{\lambda}}{2} \tag{B.12}
$$

which establishes Equation 3.24. Recall that our postulated equilibrium strategy in Equation 3.16 includes a stochastic term in trader j's order flow. For trader j to follow such a random strategy, he must be indifferent across the various possible orders induced by the random strategy. The above two equations serves to ensure that trader j will be indeed indifferent.

From Equations B.12, 3.18, and 3.17, we immediately know that $\text{var}(dz^0(t)) + \sum_{1\le i\le N} \text{var}(dz^i(t)) = 2\sum_{1\le i\le N} \text{var}(dz^i(t))$. This confirms our earlier claim (see Equation 3.5) that Equation 3.4 leads to

$$\text{var}(dz^0(t)) = dt. \tag{B.13}$$

Using Equations B.8, B.9, B.11, and B.12, we can simplify Equation B.10 to

$$\begin{aligned} 0 &= J_t + J_{V^x}\bar{\lambda}\beta\left(1 - \frac{\Sigma}{N(\Sigma-\Omega)}\right)(V^j - V^x) \\ &+ \frac{1}{2}J_{V^xV^x}\bar{\lambda}^2\frac{N-1}{N} + J_{V^xV^j}\bar{\lambda}\beta\Omega + \frac{1}{2}J_{V^jV^j}\beta^2\Omega^2\frac{N}{N-1}. \end{aligned} \tag{B.14}$$

By taking the derivatives of Equation B.14 with respect to V^x and using Equations B.11 and B.12 to simplify terms, we arrive at

$$0 = \frac{d}{dt}\left(\frac{V^x - V^j}{\bar{\lambda}}\right) + \frac{d}{dV^x}\left(\beta\left(1 - \frac{\Sigma}{N(\Sigma-\Omega)}\right)(-1)(V^j - V^x)^2\right).$$

It is straightforward to show that this is equivalent to Equation 3.23. Since Bellman equation is a necessary condition for the optimality of the trading strategy for trader i, the above arguments prove the necessity of Equation 3.23.

To prove the necessary and sufficient conditions for the optimality of the trading strategy, we can assume in the rest of the proof that Equations 3.23 and 3.24 hold, and we only need to show that Equation 3.25 is necessary and sufficient for the optimality of trader j's strategy.

First, straightforward calculations show that the following function J does satisfy the Bellman equation as specified in Equations B.11, B.12, and B.14.

$$J(t, s, V^j, V^x) = \frac{1}{2\bar{\lambda}(t)}(V^x - V^j)^2 + \frac{N-1}{2N}\int_t^1 \frac{1}{\bar{\lambda}(u)}\left(\bar{\lambda}(u) - \frac{N\beta(u)\Omega(u)}{N-1}\right)^2 du. \tag{B.15}$$

Reasoning with the above J as in Back (1992), we can show that an optimal strategy should not include discrete orders (this is due to the convexity of J as a function of V^j and V^x). Given any trading strategy x with continuous orders, we can apply Ito's lemma to obtain

$$\begin{aligned} & J(1, s, V^j(1-), V^x(1-)) - J(0, s, V^j(0), V^x(0)) \\ =& \int_0^1 \left(J_t dt + J_{V^x}dV^x + J_{V^j}dV^j + \frac{1}{2}J_{V^xV^x}(dV^x)^2 + J_{V^xV^j}dV^x dV^j + \frac{1}{2}J_{V^jV^j}(dV^j)^2\right) \\ =& J(0, s, V^j(0), V^x(0)) + \int_0^1 g(t)dW^j(t) - \int_0^1 (V^j - V^x - \lambda dx)dx \end{aligned}$$

for some function g that depends on time t only and it's easy to verify

$$E\left[\int_0^1 g(t)^2 dt\right] < \infty,$$

where the last equality holds since J satisfies Equations B.11, B.12, and B.14. Thus,

$$E\left(\int_0^1 (V^j - V^x - \lambda dx)dx\right) = J(0, s, V^j(0), V^x(0)) - E(J(1, s, V^j(1-), V^x(1-))).$$

By the definition of J, $-E(J(1,s,V^j(1-),V^x(1-))) \leq 0$. Thus from the preceding equality, we see that the proposed trading strategy is optimal if and only if $J(1,s,V^j(1-),V^x(1-)) = 0$, *a.s.*, which is equivalent to

$$\lim_{t\to 1} V^x(t) - V^j(t) = 0 \ a.s. \ \text{or} \ \lim_{t\to 1} \bar{\lambda}(t) = +\infty. \tag{B.16}$$

To complete the correctness proof that Equations 3.23 and 3.25 are indeed necessary and sufficient. We are left to prove that Equations B.16 and 3.25 are equivalent. First, if $\lim_{t\to 1} \Sigma(t) = 0$, then $\lim_{t\to 1} V^x(t)$ is a precise estimate of v and so $V^j(t)$ should also approach to v. On the other hand, by Lemma 3.3, we know that $\lim_{t\to 1} V^x(t) - V^j(t) = 0$ *a.s.* imply:

$$\begin{aligned} \lim_{t\to 1} \sum_j [V^x(t) - V^j(t)] &= \lim_{t\to 1} N\delta(t)(v - V^x) \\ &= 0 \end{aligned}$$

This must imply $\lim_{t\to 1} \Sigma(t) = 0$, otherwise we have $\lim_{t\to 1} N\delta(t) \neq 0$ (by Equation (B.5)), $\lim_{t\to 1} v - V^x \neq 0$, a contradiction.

Finally, to complete the proof, note that the above argument implies that the expected trading profit for Strategy 3.16

$$\begin{aligned} & E(J(0,V^j(0),V^x(0))) \\ = \ & \frac{1}{2\bar{\lambda}(0)}(V^j(0) - 0)^2 + \frac{N-1}{2N}\int_0^1 \frac{1}{\bar{\lambda}(u)}\left(\bar{\lambda}(u) - \frac{N\beta(u)\Omega(u)}{N-1}\right)^2 du. \end{aligned}$$

as claimed. □

Proof of Theorem 3.1 This proof consists of two parts: (1) assuming Σ as given, we first prove the formulae for all other quantities; (2) then we prove that Σ exists if and only if $\rho < 1$ or $N = 1$ and that when σ exists it is uniquely determined by the formula given in the lemma.

First, we prove the formulae for all the other formulae assuming the correctness of the formula for Σ. The formula for $\beta(t)$ as a function of $\Sigma(t)$ follows directly from Equation 3.8. The formula for $\bar{\lambda}(t)$ follows from the fact $\bar{\lambda}(t) = \beta(t)\Sigma(t)$ (see Lemma 3.4), and hence the formula for $\lambda(t)$ follows from the fact that $\lambda(t) = \frac{1}{2}\bar{\lambda}(t)$.

Since the market maker makes no profit, the expected profit of all the informed traders is equal to the loss of liquidity traders, which is equal to

$$\int_0^1 \lambda(t) dz^0(t) dz^0(t) = \int_0^1 \lambda(t) dt.$$

By symmetry, each informed trader's profit is $\frac{1}{N}$ of the total expected profits of all informed traders, and hence it is equal to $\frac{1}{N}\int_0^1 \lambda(t)dt$, as claimed. To prove the correctness of Expression 5.55, note that

$$\begin{aligned} \lambda &= \frac{1}{2}\bar{\lambda} \\ &= \frac{1}{2}\beta\Sigma \\ &= \frac{1}{2}\sqrt{\left(\frac{1}{\Sigma}\right)'}\Sigma \end{aligned}$$

$$= \frac{1}{2}\sqrt{\Sigma(0)\left(\frac{1-\rho}{\rho}\right)\left(\frac{1-B}{3N-4}\right)}((1-B)t+B)^{-\frac{2N-2}{3N-4}},$$

where the last equality follows from the formula for Σ in Theorem 3.1. Algebraic calculations then show that the integral of the last expression with respect to t from 0 to 1 is equal to Expression 5.55 times N, as desired.[12]

We have thus proved the correctness of all the formulae except the one for Σ. Moreover, this means that the existence, non-existence, or uniqueness of the equilibrium is equivalent to the existence, non-existence, or uniqueness of $\Sigma(t)$, respectively. So in what follows, we only need to derive the formulae for $\Sigma(t)$ or prove its non-existence. We will do so by solving the differential Equation 3.23 with boundary condition 3.25.

From Equation 3.23, we have

$$\frac{d}{dt}\left(\frac{1}{\bar{\lambda}}\right) = \frac{d}{dt}\left(\frac{1}{\beta\Sigma}\right) = -\frac{\beta'}{\beta^2\Sigma} + \beta,$$

where we have used Equation 3.8 to derive the last equality. Thus, Equation 3.23 implies

$$-\frac{\beta'}{\beta^3\Sigma} = 1 - \frac{2\Sigma}{N(\Sigma-\Omega)}. \tag{B.17}$$

On the other hand, the definitions of $\Sigma(t)$ and $\Omega(t)$ (Equations 3.8 and 3.11) implies

$$\frac{N-1}{N}\frac{1}{\Omega(t)} - \frac{1}{\Sigma(t)} = \frac{1}{(1-\rho)\Sigma(0)} - \frac{1}{\Sigma(0)} = \frac{A}{N},$$

where

$$A = \frac{\rho N}{(1-\rho)\Sigma(0)}$$

is a constant. Substituting $\Sigma(t)$ for $\Omega(t)$ in Equation B.17, we get

$$-\frac{\beta'}{\beta^3} = \left(1-\frac{2}{N}\right)\Sigma + \frac{2(N-1)}{(-A-\frac{1}{\Sigma})N}$$

In what follows, we let $\Gamma = \frac{1}{\Sigma}$. Using the fact $\frac{d}{dt}(\frac{1}{\Sigma}) = \beta^2$, we can rewrite the preceding differential equation as

$$0 = \frac{\Gamma''}{\Gamma'} + \left(2-\frac{4}{N}\right)\frac{\Gamma'}{\Gamma} + \frac{4(N-1)\Gamma'}{(-A-\Gamma)N} \tag{B.18}$$

In the case $N > 1$ and $\rho = 1$, $A = \infty$, and hence the above equation implies[13]

$$0 = \frac{d}{dt}\left[\log\left(\Gamma'\Gamma^{2-\frac{4}{N}}\right)\right].$$

[12] The absolute-value operator is needed for the case $\rho < 0$, which implies that the term inside the absolute value operator is negative. Also, we remark that the expected-profit formula can be alternatively derived by taking expectation (at time 0) of Expression 3.26.

[13] To be completely formal and to avoid dividing by 0, we should have directly derived the desired equation below. But this is a straightforward exercise by using the argument for obtaining Equation B.18.

Thus,

$$\Gamma' \, \Gamma^{2-\frac{4}{N}} = C_0 \text{ for some constant } C_0 > 0,$$

which in turns implies

$$\Sigma(t) = \frac{1}{\Gamma(t)} = (C_1 t + C_2)^{\frac{-1}{3-\frac{4}{N}}} \text{ for some constants } C_1 \text{ and } C_2. \tag{B.19}$$

But when $N > 1$, there is no constants $C_1 = C_0(3 - 4/N) > 0$ and C_2 which can make the above $\Sigma(t)$ satisfy either $\Sigma(1) = 0$ or $\lim_{t \to 1} \bar{\lambda}(t) = \sqrt{-\Sigma'(1)} = +\infty$, as required by Equation 3.25. This completes the proof that a linear equilibrium does not exist for $N > 1$ and $\rho = 1$.

In the rest of the proof, we assume either $\rho \neq 1$ or $N = 1$. Under these assumptions, we prove that Equation B.18 has a unique solution of $\Sigma(t)$ as described in the theorem. Now, the only possible case with $\rho = 1$ happens is when $N = 1$. But when $N = 1$, there is no competing informed traders, and ρ is irrelevant. Without loss of generality, we make the additional assumption $\rho \neq 1$. This will ensure a finite A in the rest of the proof.

By Equation B.18,

$$0 = \frac{d}{dt} \left[\log \left(\Gamma' \Gamma^{2-\frac{4}{N}} (\Gamma + A)^{-\frac{4(N-1)}{N}} \right) \right].$$

Hence,

$$\Gamma' \, \Gamma^{2-\frac{4}{N}} \, (\Gamma + A)^{\frac{-4(N-1)}{N}} = C_3 \text{ for some constant } C_3,$$

which in turns implies

$$(\Gamma)^{2-\frac{4}{N}} \, (\Gamma + A)^{\frac{4(1-N)}{N}} \, \Gamma' = C_4 \text{ for some constant } C_4. \tag{B.20}$$

In the case of $\rho = 0$, we have $A = 0$. Hence, the above equation is equivalent to $\Gamma^{-2}\Gamma' = C_4$, which implies that $\Sigma = 1/\Gamma$ is linear in t. Hence, the desired formula for Σ follows immediately from the boundary condition $\Sigma(1) = 0$.

For the case of $\rho \neq 0$, we can make a change of variable as $\Gamma = A\frac{r}{1-r}$, the above equation becomes

$$r^{2-\frac{4}{N}} r' = C_4.$$

From this and the boundary condition on $r(0)$ and $r(1)$, we obtain

$$\frac{1}{A\Sigma(t) + 1} = \left(\left[\left(\frac{1}{A\Sigma(1) + 1} \right)^{3-\frac{4}{N}} - B \right] t + B \right)^{\frac{1}{3-\frac{4}{N}}}. \tag{B.21}$$

Taking derivatives with respect to t in the above equation, we know that $\Sigma'(1)$ is bounded. Hence, from the proved formula $\bar{\lambda}(t) = \sqrt{-\Sigma'(t)}$, we know that $\lim_{t \to 1} \bar{\lambda}$ is finite. Hence, from Equation 3.25, we must have $\Sigma(1) = 0$. Plugging $\Sigma(1) = 0$ into Equation B.21, we immediately arrive at the claimed formula for $\Sigma(t)$. ∎

C Proofs for Section 5

Proof of Theorem 5.1 This is covered in the proof of Theorem 2.1. □

Proof of Thereom 5.2 This is covered in the proof of Theorem 3.1. □

Proof of Theorem 5.3 The proof is in the Appendix of Back, Cao, and Willard (2000).

Proof of Corollary 5.1 We first prove corollaries in section 5 and then corollaries in section 4 for most of them are just special case to corollary 5.1.

(i) From equations B.12, 3.18, 3.17 and B.13, we have $\sum_{1\leq i\leq N}\text{var}(dz^i(t)) = \text{var}(dz^0(t))$, which means informed investors contribute half of the total trading volume $\sum_{1\leq i\leq N}\text{var}(dz^i(t)) + \text{var}(dz^0(t))$.

(ii) From equation B.19, when $N = 1$, we have $\Sigma(t) = C_1 t + C_2$ for some constants C_1, C_2. And only $C_1 = -\Sigma_0, C_2 = \Sigma_0$ satisfies the initial condition and the condition $\lim_{t\to 1}\Sigma(t) = 0$ or $\lim_{t\to 1}\hat{\lambda}(t) = +\infty$, required by Equation 3.25. Thus, we show

$$\Sigma(t) = \Sigma(0)(1-t) = \hat{\Sigma}(t).$$

and it's trivial to show $\beta(t) = \hat{\beta}(t), \lambda(t) = \hat{\lambda}(t)/2$.

(iii) Without loss of generality, we fix $\sigma_v = 1$. Define

$$a_N = 3 - 4/N \geq 1, N \geq 2 \tag{C.1}$$

$$b_{N\phi} = 2(1-\phi)/(N\phi) \tag{C.2}$$

$$\phi = \frac{1}{N} + \frac{N-1}{N}\rho \geq 0 \tag{C.3}$$

$$\rho = \frac{\sigma_v^2 - \sigma_\epsilon^2/(N-1)}{\sigma_v^2 + \sigma_\epsilon^2} = \frac{1 - \frac{\sigma_\epsilon^2/\sigma_v^2}{N-1}}{1 + \sigma_\epsilon^2/\sigma_v^2} = \frac{1 - \frac{\sigma_\epsilon^2}{N-1}}{1 + \sigma_\epsilon^2} \tag{C.4}$$

$$B = \left(1 + N\frac{\rho}{1-\rho}\right)^{-a_N} = \left(1 + N\Bigg/\left(\frac{1+\sigma_\epsilon^2}{1 - \frac{\sigma_\epsilon^2}{N-1}} - 1\right)\right)^{-a_N} = \left(\frac{\sigma_\epsilon^2}{N-1}\right)^{-a_N} \tag{C.5}$$

Write $\Sigma(t)$ as

$$\Sigma(t) = \Sigma(0)N\left(\frac{1+\sigma_\epsilon^2}{1-\frac{\sigma_\epsilon^2}{N-1}} - 1\right)(((1-B)t + B)^{-1/a_N} - 1) \tag{C.6}$$

$$= \frac{((1-B)t+B)^{-1/a_N} - 1}{(N-1)/\sigma_\epsilon^2 - 1} \tag{C.7}$$

$$= \frac{((1-B)t+B)^{-1/a_N} - 1}{B^{-1/a_N} - 1} \tag{C.8}$$

and its derivative with respect to t is:

$$\frac{\partial\Sigma(t)}{\partial t} = -\frac{\Sigma(0)(1-B)((1-B)t+B)^{-1-1/a_N}}{a_N(B^{-1/a_N} - 1)}$$

Differentiating both sides of equation 5.57 gives

$$\frac{\partial\hat{\Sigma}(t)}{\partial t} = -\kappa(\hat{\Sigma}(0))^{1-a_N}(\hat{\Sigma}(t))^{1+a_N}e^{b_N\phi\hat{\Sigma}(0)/\hat{\Sigma}(t)}$$

As $t \to 1$, both $\Sigma(t)$ and $\hat{\Sigma}(t)$ goes to 0. So we using L'Hospital's Rule to calculate the following limit:

$$\begin{aligned}
\lim_{t\to 1}\frac{\Sigma(t)}{\hat{\Sigma}(t)} &= \lim_{t\to 1}\frac{\partial\Sigma(t)/\partial t}{\partial\hat{\Sigma}(t)/\partial t} && \text{(C.9)}\\
&= \lim_{t\to 1}\frac{-\Sigma(0)(1-B)((1-B)t+B)^{-1-1/a_N}/[a_N(B^{-1/a_N}-1)]}{-\kappa(\hat{\Sigma}(0))^{1-a_N}(\hat{\Sigma}(t))^{1+a_N}e^{b_N\phi\hat{\Sigma}(0)/\hat{\Sigma}(t)}} && \text{(C.10)}\\
&= \frac{\Sigma(0)(\hat{\Sigma}(0))^{a_N}(1-B)}{\kappa a_N(B^{-1/a_N}-1)}\lim_{t\to 1}\frac{((1-B)t+B)^{-1-1/a_N}-1}{(\hat{\Sigma}(t))^{1+a_N}e^{b_N\phi\hat{\Sigma}(0)/\hat{\Sigma}(t)}} && \text{(C.11)}\\
&= 0 && \text{(C.12)}
\end{aligned}$$

Because the exponential function $e^{b_N\phi\hat{\Sigma}(0)/\hat{\Sigma}(t)}$ grows much faster than the polynomial function $(1/\hat{\Sigma}(t))^{1+a_N}$ as $1/\hat{\Sigma}(t)$ goes to ∞. So the denominator $(\hat{\Sigma}(t))^{1+a_N}e^{b_N\phi\hat{\Sigma}(0)/\hat{\Sigma}(t)}$ goes to ∞ and at the same time the numerator $((1-B)t+B)^{-1-1/a_N}-1$ goes to 0 as time $t \to 1$, which proves the last equation.

Similarly, by L'Hospital's Rule, we have following result

$$\begin{aligned}
\lim_{t\to 1}\frac{\beta(t)}{\hat{\beta}(t)} &= \lim_{t\to 1}\frac{\sqrt{-\Sigma'}/\Sigma}{\sqrt{-\hat{\Sigma}'}/\hat{\Sigma}} && \text{(C.13)}\\
&= \lim_{t\to 1}\frac{\sqrt{\Sigma'/\hat{\Sigma}'}}{\Sigma/\hat{\Sigma}} && \text{(C.14)}\\
&= \frac{\lim_{t\to 1}\sqrt{\Sigma'/\hat{\Sigma}'}}{\lim_{t\to 1}\Sigma/\hat{\Sigma}} && \text{(C.15)}\\
&= \frac{\lim_{t\to 1}\sqrt{\Sigma'/\hat{\Sigma}'}}{\lim_{t\to 1}\Sigma'/\hat{\Sigma}'} \quad \text{(By L'Hospital's Rule)} && \text{(C.16)}\\
&= \lim_{t\to 1}(\Sigma'/\hat{\Sigma}')^{-\frac{1}{2}} && \text{(C.17)}\\
&= \infty \quad \text{(By Equation C.12)} && \text{(C.18)}
\end{aligned}$$

(iv)

$$\begin{aligned}
\lim_{t\to 1}\frac{1/\lambda}{1/\hat{\lambda}} &= \lim_{t\to 1}\frac{2/\sqrt{-\Sigma'}}{1/\sqrt{-\hat{\Sigma}'}} && \text{(C.19)}\\
&= \lim_{t\to 1}\frac{2}{\sqrt{\Sigma'/\hat{\Sigma}'}} && \text{(C.20)}\\
&= \infty \quad \text{(By Equation C.12)} && \text{(C.21)}
\end{aligned}$$

(v) Given $a_N \geq 1$, $B-1$ and $B^{-1/a_N}-1$ take different signs, so we have

$$\frac{\partial\Sigma(t)}{\partial t} = \frac{\Sigma(0)(B-1)((1-B)t+B)^{-1-1/a_N}}{a_N(B^{-1/a_N}-1)} < 0$$

Taking derivative to $\Sigma(t)$ with B

$$\frac{\partial \Sigma(t)}{\partial B} = \frac{\Sigma(0)((1-B)t+B)^{-1-1/a_N}}{a_N(B^{-1/a_N}-1)^2}[(1-B^{-1-1/a_N})(1-t)+B^{-1-1/a_N}-(t/B+1-t)^{1+1/a_N}]$$

Taking derivative to $[(1-B^{-1-1/a_N})(1-t)+B^{-1-1/a_N}-(t/B+1-t)^{1+1/a_N}]$ with respect to B gives $(1/a_N+1)B^{-2-1/a_N}t[(t/B+1-t)^{1/a_N}-1]$, which is larger than 0 if $B \geq 1$ and smaller than 0 if $B<1$. So $[(1-B^{-1-1/a_N})(1-t)+B^{-1-1/a_N}-(t/B+1-t)^{1+/a_N}]$ reaches its minimum 0 at $B=1$, so we have $\partial\Sigma(t)/\partial B \geq 0$ for all $B>0$.

We express $\lambda(t)$ as function of B rather than ρ or σ_ϵ^2,

$$\lambda(t) = \sqrt{\Sigma'}/2 = \frac{1}{2}\sqrt{\frac{(1-B)((1-B)t+B)^{-1-1/a_N}}{a_N(B^{-1/a_N}-1)}} \tag{C.22}$$

$$\lambda(0) = \frac{1}{2\sqrt{a_N}}\sqrt{\frac{(1-B)B^{-1-1/a_N}}{B^{-1/a_N}-1}} \tag{C.23}$$

$$\lambda(1) = \frac{1}{2\sqrt{a_N}}\sqrt{\frac{1-B}{B^{-1/a_N}-1}} \tag{C.24}$$

Taking derivative to $\lambda(0)$ and $\lambda(1)$ with B gives:

$$\frac{\partial\lambda(0)}{\partial B} = \frac{(1-B)B^{1/a_N}+a_N(B^{1/a_N}-1)}{8\lambda(0)B^2(1-B^{1/a_N})^2} \tag{C.25}$$

$$\propto (1-B)B^{1/a_N}+a_N(B^{1/a_N}-1) \tag{C.26}$$

$$\frac{\partial\lambda(1)}{\partial B} = \frac{B^{1/a_N-1}}{8\lambda(1)(1-B^{1/a_N})^2}[1-(1+a_N)B+a_N B^{1+1/a_N}] \tag{C.27}$$

$$\propto 1-(1+a_N)B+a_N B^{1+1/a_N} \tag{C.28}$$

The derivative of $(1-B)B^{1/a_N}+a_N(B^{1/a_N}-1)$ with respect to B is $(1-1/a_N)(1-B)B^{1/a_N-1}$, which is larger than 0 if $B \leq 1$ and smaller than 0 if $B>1$, so $\partial\lambda(0)/\partial B$ reaches its maximum 0 at $B=1$, i.e., $\partial\lambda(0)/\partial B \leq 0$. $\frac{\partial}{\partial B}[1-(1+a_N)B+a_N B^{1+1/a_N}]$, the derivative of $1-(1+a_N)B+a_N B^{1+1/a_N}$ with respect to B, is larger than 0 if $B \geq 1$ and smaller than 0 if $B<1$, so $\frac{\partial\lambda(1)}{\partial B}$ reaches its minimum 0 at $B=1$, i.e., $\partial\lambda(1)/\partial B \geq 0$.

$$\frac{\partial\lambda(t)}{\partial t} = \frac{\partial(-\Sigma')/\partial t}{2\lambda(t)} = \frac{-\Sigma''}{2\lambda(t)} \tag{C.29}$$

$$= \frac{(1+1/a_N)(1-B)^2((1-B)t+B)^{-1-1/a_N}}{2\lambda(t)a_N(1-B^{1/a_N})} \tag{C.30}$$

$$\propto 1/(1-B^{1/a_N}) \tag{C.31}$$

So, $\lambda(t)$ is increasing in t when $B<1$ and decreasing in t when $B \geq 1$.

(vi) When $\sigma_\epsilon^2 = (N-1)\sigma_v^2$, we have

$$\Sigma(t) = \Sigma(0)(1-t)$$

and hence

$$\beta = \sqrt{\Sigma(0)}/(1-t), \lambda = \frac{1}{2}, \bar{\lambda} = 1.$$

and the profits of informed traders are

$$\int_0^1 \lambda(t)dt = \int_0^1 \frac{1}{2}dt = \frac{1}{2}$$

Therefore, market efficiency, market liquidity, and profit are the same as if there exists a monopolistic informed investor with all the signals in the market. And from equation B.5 and equation 40 (i.e., $\delta(t) = (1+(N-1)\rho(t))/N$) in BCW, we know $\rho(t) = \rho\Omega(t)/\Omega(0)$ and hence the conditional correlation between private signals $\rho(t)$ remains 0 throughout the trading period.

When $\sigma_\epsilon^2 \neq (N-1)\sigma_v^2$, we first have

$$N\delta(t) = 1 + (N-1)\rho\frac{\Omega(t)}{\Omega(0)} \to 1$$

for $\Omega(t) \leq \Sigma(t) \to 0$ as time t goes to 1. Further,

$$\begin{aligned}
\lim_{t\to 1}\frac{\Sigma(t)}{1-t} &= \lim_{t\to 1} -\Sigma'(t) \quad \text{(By L'Hospital's Rule)} && \text{(C.32)}\\
&= \frac{(1-\rho)\Sigma(0)(1-B)}{\rho N a_N}\lim_{t\to 1}((1-B)t+B)^{-1-1/a_N} && \text{(C.33)}\\
&= \frac{(1-\rho)\Sigma(0)(1-B)}{\rho N a_N} && \text{(C.34)}\\
&= S_0 && \text{(C.35)}
\end{aligned}$$

from here we also have $\lim_{t\to 1}\Sigma^{'}(t) = -S_0$.

$$\begin{aligned}
\lim_{t\to 1}\beta(t)(1-t) &= \lim_{t\to 1}\frac{\sqrt{-\Sigma'(t)}}{\Sigma(t)/(1-t)} && \text{(C.36)}\\
&= \frac{\lim_{t\to 1}\sqrt{-\Sigma'(t)}}{\lim_{t\to 1}\Sigma(t)/(1-t)} && \text{(C.37)}\\
&= \frac{\sqrt{S_0}}{S_0} && \text{(C.38)}\\
&= 1/\sqrt{S_0} && \text{(C.39)}
\end{aligned}$$

$$\begin{aligned}
\lim_{t\to 1}\lambda(t) &= \lim_{t\to 1}\frac{\sqrt{-\Sigma'(t)}}{2} && \text{(C.40)}\\
&= \frac{\lim_{t\to 1}\sqrt{-\Sigma'(t)}}{2} && \text{(C.41)}\\
&= \frac{\sqrt{S_0}}{2} && \text{(C.42)}
\end{aligned}$$

We can express the profit $\pi(0)$ in B rather than ρ,

$$\pi(0) = \sqrt{\frac{a_N\Sigma(0)}{4(N-2)^2}\frac{(1-B^{(1-a_N)/2})^2}{(1-B)(B^{-1/a_N}-1)}}$$

Taking derivative with B gives

$$\begin{aligned}
\frac{\partial\pi(0)}{\partial B} &\propto \frac{\partial}{\partial B}\left[\frac{(1-B^{(1-1/a_N)/2})^2}{(1-B)(B^{-1/a_N}-1)}\right] && \text{(C.43)}\\
&= -\frac{B^{\frac{1}{2a_N}-\frac{1}{2}}(1-B^{\frac{1}{2}-\frac{1}{2a_N}})(1-B^{\frac{1}{2}+\frac{1}{2a_N}})}{a_N(1-B)^2(1-B^{1/a_N})^2}\left[(B-1)B^{\frac{1}{2a_N}-\frac{1}{2}}-a_N(B^{1/a_N}-1)\right] && \text{(C.44)}\\
&\propto a_N(B^{1/a_N}-1)-(B-1)B^{\frac{1}{2a_N}-\frac{1}{2}} && \text{(C.45)}
\end{aligned}$$

Again taking derivative to $a_N(B^{1/a_N}-1)-(B-1)B^{\frac{1}{2a_N}-\frac{1}{2}}$ with B gives $B^{\frac{1}{2a_N}-\frac{3}{2}}[-(\frac{1}{2a_N}+\frac{1}{2})B+B^{\frac{1}{2a_N}+\frac{1}{2}}+(\frac{1}{2a_N}-\frac{1}{2})]$. The derivative of the second term $-(\frac{1}{2a_N}+\frac{1}{2})B+B^{\frac{1}{2a_N}+\frac{1}{2}}+(\frac{1}{2a_N}-\frac{1}{2})$ is $(\frac{1}{2a_N}+\frac{1}{2})(B^{\frac{1}{2a_N}-\frac{1}{2}}-1)$. Given $a_N\geq 1$, $B^{\frac{1}{2a_N}-\frac{1}{2}}-1$ is negative if $B>1$ and positive if $B\leq 1$, so $a_N(B^{1/a_N}-1)-(B-1)B^{\frac{1}{2a_N}-\frac{1}{2}}$ reaches its maximum 0 at $B=1$, which means $a_N(B^{1/a_N}-1)-(B-1)B^{\frac{1}{2a_N}-\frac{1}{2}}$ decreases in B and equals to 0 at $B=1$. So, $\frac{\partial\pi(0)}{\partial B}$ is positive when $B<1$ and negative when $B\geq 1$, i.e., $\pi(0)$ reaches its maximum at $B=1$. □

Proofs of Corollary 5.2 When one of the N informed traders (without loss of generality, assume she is the N-th trader) leaves the market, the remaining $N-1$ traders in aggregate don't know the true value of the asset v. Instead, the variable which the $N-1$ informed traders and the market maker are interested of is the informed traders' expectation of v:

$$v_{N\to N-1} = E[v|s_1,\ldots,s_{N-1}] = \frac{N(N-1)}{(N-1)^2+\sigma_\epsilon^2/\sigma_v^2}\sum_{i=1}^{N-1}s_i$$

Correspondingly, the expected profits each of the remaining $N-1$ informed traders obtains

$$\begin{aligned}
\pi_{N\to N-1} &= \sqrt{\Sigma_{N\to N-1}(0)\left(\frac{1-\rho}{\rho}\right)\left(\frac{3(N-1)-4}{1-B_{N\to N-1}}\right)}\frac{\left|1-B_{N\to N-1}^{\frac{(N-1)-2}{3(N-1)-4}}\right|}{2(N-1)|(N-1)-2|} && \text{(C.46)}\\
&\text{here,} && \text{(C.47)}\\
\Sigma_{N\to N-1}(0) &= \mathrm{var}[v_{N\to N-1}] = \frac{(N-1)^2\sigma_v^2}{(N-1)^2+\sigma_\epsilon^2/\sigma_v^2} && \text{(C.48)}\\
&\to \frac{(N-1)^2\sigma_v^4}{\sigma_\epsilon^2}, \sigma_\epsilon^2\to\infty && \text{(C.49)}\\
B_{N\to N-1} &= \left(\frac{1-\rho}{1-\rho+(N-1)\rho}\right)^{3-4/(N-1)} = \left(\frac{N\sigma_\epsilon^2}{(N-1)^2\sigma_v^2+\sigma_\epsilon^2}\right)^{3-4/(N-1)} && \text{(C.50)}\\
&\to N^{3-4/(N-1)}, \sigma_\epsilon^2\to\infty. && \text{(C.51)}
\end{aligned}$$

Considering the limiting behaviour of the ratio of π_N and $\pi_{N\to N-1}$ when $\sigma_\epsilon^2\to\infty$:

$$\lim_{\sigma_\epsilon^2\to\infty}\frac{\pi_N^2}{\pi_{N\to N-1}^2} \tag{C.52}$$

$$= \lim_{\sigma_\epsilon^2\to\infty} \frac{\lim_{\sigma_\epsilon^2\to\infty}\pi_N^2}{\lim_{\sigma_\epsilon^2\to\infty}\pi_{N\to N-1}^2} \tag{C.53}$$

$$= \lim_{\sigma_\epsilon^2\to\infty} \frac{\Sigma(0)\left(\frac{1-\rho}{\rho}\right)\left(\frac{3N-4}{\sigma_\epsilon^2/(N-1)}\right)/(4N^2(N-2)^2)}{\frac{(N-1)^2\sigma_v^4}{\sigma_\epsilon^2}\left(\frac{1-\rho}{\rho}\right)\left(\frac{3(N-1)-4}{1-N^{3-4/(N-1)}}\right)\left(1-N^{1-2/(N-1)}\right)^2/(4(N-1)^2((N-1)-2)^2)} \tag{C.54}$$

$$= \frac{(3N-4)(N-1)(N-3)^2(1-N^{3-4/(N-1)})}{(3N-7)N^2(N-2)^2(1-N^{1-2/(N-1)})^2} \tag{C.55}$$

$$> 1, \quad N \geq 4 \tag{C.56}$$

the above expression in N decreases to 1 as N goes to ∞.

For the case of $N = 3$, following similar steps, we get

$$\lim_{\sigma_\epsilon^2\to\infty} \frac{\pi_3^2}{\pi_{3\to 2}^2} \tag{C.57}$$

$$= \lim_{\sigma_\epsilon^2\to\infty} \frac{\Sigma(0)\left(\frac{1-\rho}{\rho}\right)\left(\frac{5}{\sigma_\epsilon^2/2}\right)/36}{\frac{4\sigma_v^4}{64\sigma_\epsilon^2}\left(\frac{1-\rho}{\rho}\right)(\log(3))^2} \tag{C.58}$$

$$= \frac{40}{9(\log(3))^2} \tag{C.59}$$

$$= 3.68 > 1 \tag{C.60}$$

So, we have $\pi_N/\pi_{N\to N-1} > 1$ as σ_ϵ^2 grows to ∞ for all $N \geq 3$, which means there exist a large enough $\bar{\sigma}_\epsilon$ such that $\pi_N > \pi_{N\to N-1}$ for all $\sigma_\epsilon > \bar{\sigma}_\epsilon$. The case of $N = 2$ is covered in the proof of Corollary 4.8.

Writing π_N and π_M in σ_ϵ^2 gives us:

$$\lim_{\sigma_\epsilon^2\to\infty} \pi_N/\pi_M = \lim_{\sigma_\epsilon^2\to\infty} \frac{\sqrt{3-4/N}}{2(N-2)} \sqrt{\frac{(1-(\frac{\sigma_\epsilon^2}{N-1})^{1-2/N})^2}{(1-(\frac{\sigma_\epsilon^2}{N-1})^{3-4/N})(\frac{N-1}{\sigma_\epsilon^2}-1)}} \Bigg/ \frac{1}{2\sqrt{1+\sigma_\epsilon^2}} \tag{C.61}$$

$$= \frac{\sqrt{3-4/N}}{2(N-2)} \lim_{\sigma_\epsilon^2\to\infty} \frac{2\sqrt{N-1}(\frac{\sigma_\epsilon^2}{N-1})^{1-2/N}(\frac{\sigma_\epsilon^2}{N-1})^{1/2}}{(\frac{\sigma_\epsilon^2}{N-1})^{3/2-2/N}} \tag{C.62}$$

$$= \frac{\sqrt{(3-4/N)(N-1)}}{N-2} \tag{C.63}$$

$\sqrt{(3-4/N)(N-1)}/N-2$ decrease in N and equals 1.8257 at $N = 3$, 1.2245 at $N = 4$, and 0.9888 at $N = 5$. So, for $N = 3, 4$, there exists $\hat{\sigma}_\epsilon$ such that for $\sigma_\epsilon > \hat{\sigma}_\epsilon$, $\pi_N > \pi_M$. However, for cases $N \geq 5$, we always have $\pi_M > \pi_N$ at the limit of $\sigma_\epsilon \to \infty$, and our numerical results always show that the ratio of π_N/π_M is increasing in σ_ϵ, which means $\pi_M > \pi_N$ holds for all σ_ϵ when $N \geq 5$. □

D Proofs for Section 4

Proofs of Corollary 4.1 This is a special case of part (i) in Corollary (5.1). □

Proofs of Corollary 4.2 This is a special case of part (ii) in Corollary (5.1). □

Proof of Corollary 4.3 It's straightforward to verify that $\beta(t)$ is increasing in t and decreasing in σ_ϵ^2, as $\beta(t) = 1/[\sigma_\epsilon(1-t)]$. □

Proof of Corollary 4.4 This is a special case of part (v) in Corollary (5.1). □

Proofs of Corollary 4.6

The ratio of market liquidity can be decomposed into three components:

$$\frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \frac{2}{1} \times \frac{\hat{\beta}(t)}{\beta(t)} \times \frac{\hat{\Sigma}(t)}{\Sigma(t)} = 2 \times \sqrt{1-t} \times \frac{\sigma_\epsilon^2 + \sigma_v^2 t/(1-t)}{\sigma_\epsilon^2 - \sigma_v^2 \ln(1-t)}$$

As t approaches 1, $\hat{\beta}(t)/\beta(t)$ goes to zero at the order of $\sqrt{1-t}$ but $\hat{\Sigma}(t)/\Sigma(t)$ goes to infinity at the order of $1/[(1-t)\ln(1-t)]$. Thus, we must have

$$\lim_{t\to 1} \frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \infty.$$

When $\sigma_\epsilon \le \sigma_v$, we have

$$\begin{aligned} \frac{\hat{\Sigma}(t)}{\Sigma(t)} &= \frac{\sigma_\epsilon^2 + \sigma_v^2 t/(1-t)}{\sigma_\epsilon^2 - \sigma_v^2 \ln(1-t)} \\ &\ge \frac{1}{(1-t)(1-\ln(1-t))} \\ &\ge \frac{\sqrt{e}}{2\sqrt{1-t}}, \end{aligned}$$

where the last inequality holds because $\sqrt{1-t}[1-\ln(1-t)]$ is maximized at $t = 1 - 1/e$. It follows that

$$\frac{1/\lambda}{1/\hat{\lambda}} \ge 2 \times \sqrt{1-t} \times \frac{\sqrt{e}}{2\sqrt{1-t}} = \sqrt{e} > 1.$$

and

$$\pi(0) = \int_0^1 \lambda(t)dt < \int_0^1 \hat{\lambda}(t)dt = \hat{\pi}(0).$$

□

Proofs of Corollary 4.7

$$\frac{1/\lambda(t)}{1/\hat{\lambda}(t)} = \frac{2}{1} \times \frac{\hat{\beta}(t)}{\beta(t)} \times \frac{\hat{\Sigma}(t)}{\Sigma(t)} = 2 \times \sqrt{1-t} \times \frac{\sigma_\epsilon^2 + \sigma_v^2 t/(1-t)}{\sigma_\epsilon^2 - \sigma_v^2 \ln(1-t)}$$

When $t > 3/4$, $2\sqrt{1-t} < 1$. Moreover, as σ_ϵ increases, $\Sigma(t)/\hat{\Sigma}(t)$ goes to 1 since informed investors have very imprecise signals and thus are reluctant to trade, which causes very little information to be revealed to the market. As a result, market is less liquid in the presence of public disclosure for large σ_ϵ, which means there exists a $\sigma_\epsilon^* > \sigma_v$, such that $1/\lambda(t) < 1/\hat{\lambda}(t)$ for $\sigma_\epsilon^* > \sigma_\epsilon$ and $t > 3/4$.

$$\frac{\hat{\pi}(0)}{\pi(0)} = \int_0^1 \frac{4(\sigma_\epsilon^2 - 1)/\log(\sigma_\epsilon^2)}{[\sigma_\epsilon^2 - \log(1-t)]\sqrt{1-t}}dt \quad \text{(D.1)}$$

$$\leq \int_0^1 \frac{4(\sigma_\epsilon^2 - 1)/\log(\sigma_\epsilon^2)}{\sigma_\epsilon^2\sqrt{1-t}}dt \quad \text{(D.2)}$$

$$= \frac{8(\sigma_\epsilon^2 - 1)}{\sigma_\epsilon^2 \log(\sigma_\epsilon^2)} \quad \text{(D.3)}$$

So, we have $\lim_{\sigma_\epsilon^2 \to \infty} \hat{\pi}(0)/\pi(0) = 0$, which by the definition of limit means there exists a large enough $\sigma_\epsilon^{**} > \sigma_v$ such that for $\sigma_\epsilon > \sigma_\epsilon^{**}$, $\pi(0)/\hat{\pi}(0) > 1$. □

Proof of Corollary 4.8 We have

$$\frac{\pi_D}{\pi_M} = \frac{\sqrt{\Sigma(0)\sigma_\epsilon^2}\log(\sigma_\epsilon^2)}{4(\sigma_\epsilon^2 - 1)} \Bigg/ \frac{1}{\sqrt{1+\sigma_\epsilon^2}} \quad \text{(D.4)}$$

$$= \frac{\sqrt{\sigma_\epsilon^2(1+\sigma_\epsilon^2)}\log(\sigma_\epsilon^2)}{4(\sigma_\epsilon^2 - 1)} \quad \text{(D.5)}$$

Taking derivative to π_D/π_M with respect to σ_ϵ^2 gives

$$\frac{\partial}{\partial \sigma_\epsilon^2}\frac{\pi_D}{\pi_M} = \frac{2(\sigma_\epsilon^4 - 1) - 3(\sigma_\epsilon^2 + 1)\log(\sigma_\epsilon^2)}{8(\sigma_\epsilon^2 - 1)^2\sqrt{\sigma_\epsilon^2(1+\sigma_\epsilon^2)}}$$

Considering the function $2(u^2 - 1) - 3(u+1)\log(u)$, its first derivative with respect to u is

$$\frac{2(u^2-1) - 3(u+1)\log(u)}{\partial u} = 4u - (3 + 1/u) - 3\log u \quad \text{(D.6)}$$

$$\geq 4u - 4u - (3 + 1/u) - 3(u-1) \quad \text{(D.7)}$$

$$= u - 1/u \geq 0, u \geq 1. \quad \text{(D.8)}$$

and its value is 0 at $u = 1$, which means $\partial(\frac{\pi_D}{\pi_M})/\partial\sigma_\epsilon^2 \geq 0$ for all $\sigma_\epsilon^2 \geq 1$. And also we have the ratio of π_D/π_M grows to ∞ as σ_ϵ^2 goes to ∞,

$$\lim_{\sigma_\epsilon^2 \to \infty} \frac{\pi_D}{\pi_M} = \lim_{\sigma_\epsilon^2 \to \infty} \frac{\log(1/\sigma_\epsilon^2)}{4} = \infty.$$

there is a large enough $\hat{\sigma}_\epsilon$ such that for $\sigma_\epsilon > \hat{\sigma}_\epsilon$, we have $\pi_D > \pi_M$. □

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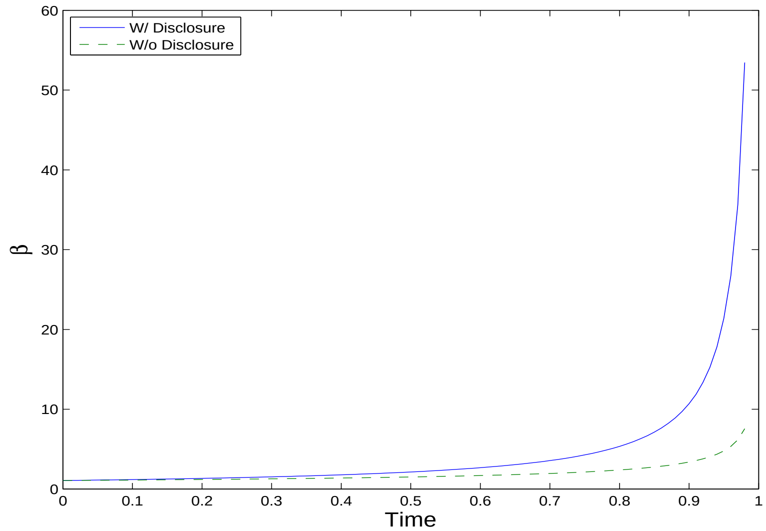
A



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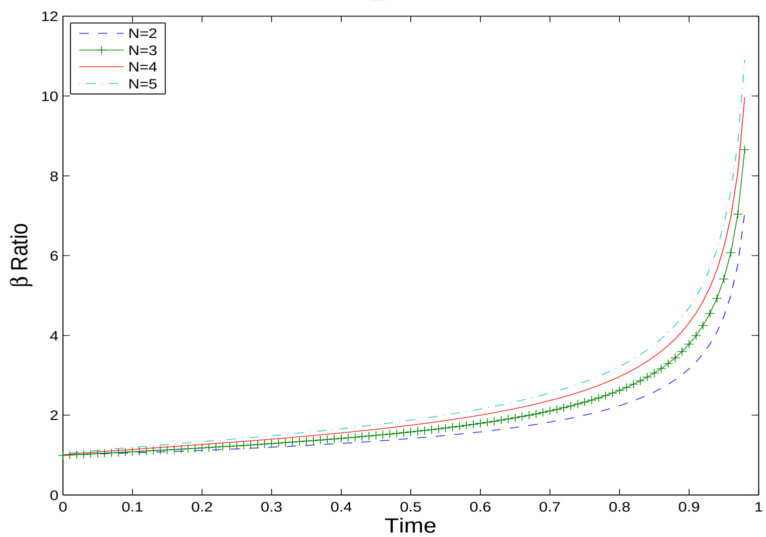

Figure 1: Figure 1.A: Trading intensity β as a function of time for $\sigma_\epsilon^2 = 0.875$ and $N = 2$. The solid line is for the case with disclosure and the dashed line is for the case without disclosure. Figure 1. B: Ratio of β with and without disclosure as a function of t for $\sigma_\epsilon^2 = 0.875$ and $N = 2, 3, 4, 5$.

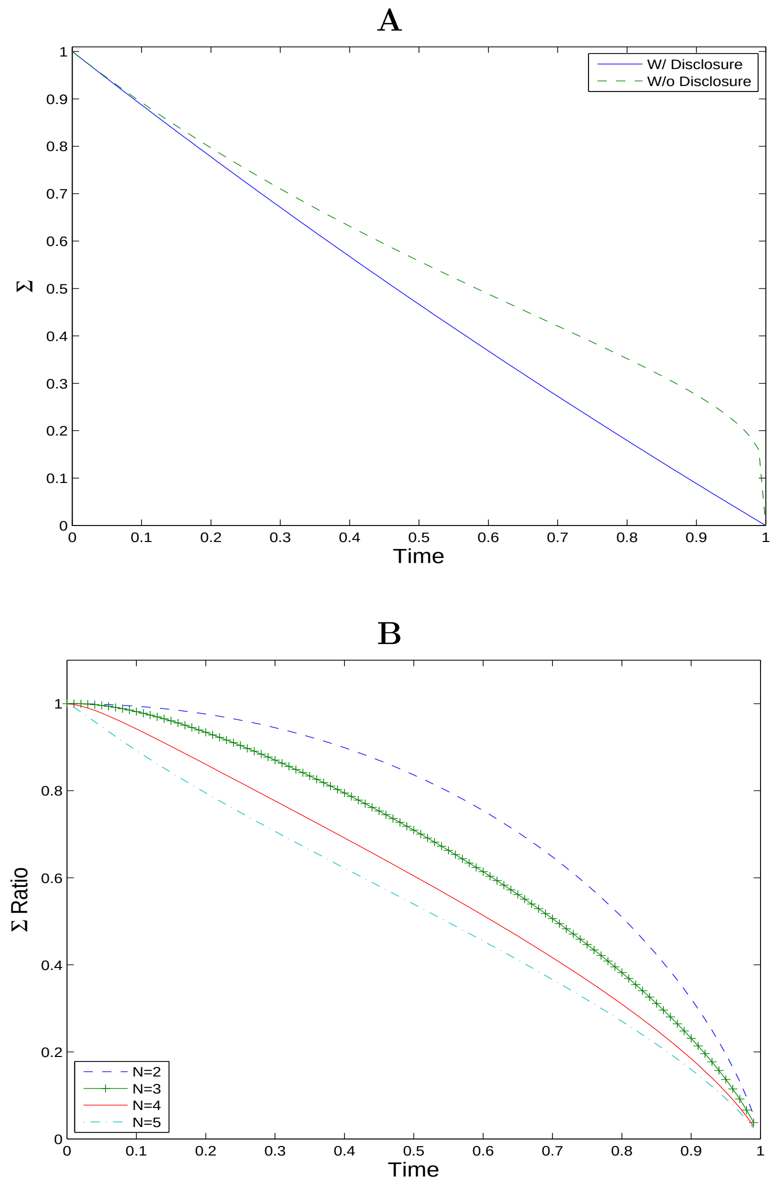

Figure 2: Figure 2A: Residual uncertainty Σ as a function of time for $\sigma_\epsilon^2 = 0.875$ and $N = 2$. The solid line is for the case with disclosure and the dashed line is for the case without disclosure. Figure 2B: Ratio of Σ with and without disclosure as a function of t for $\sigma_\epsilon^2 = 0.875$ and $N = 2, 3, 4, 5$.

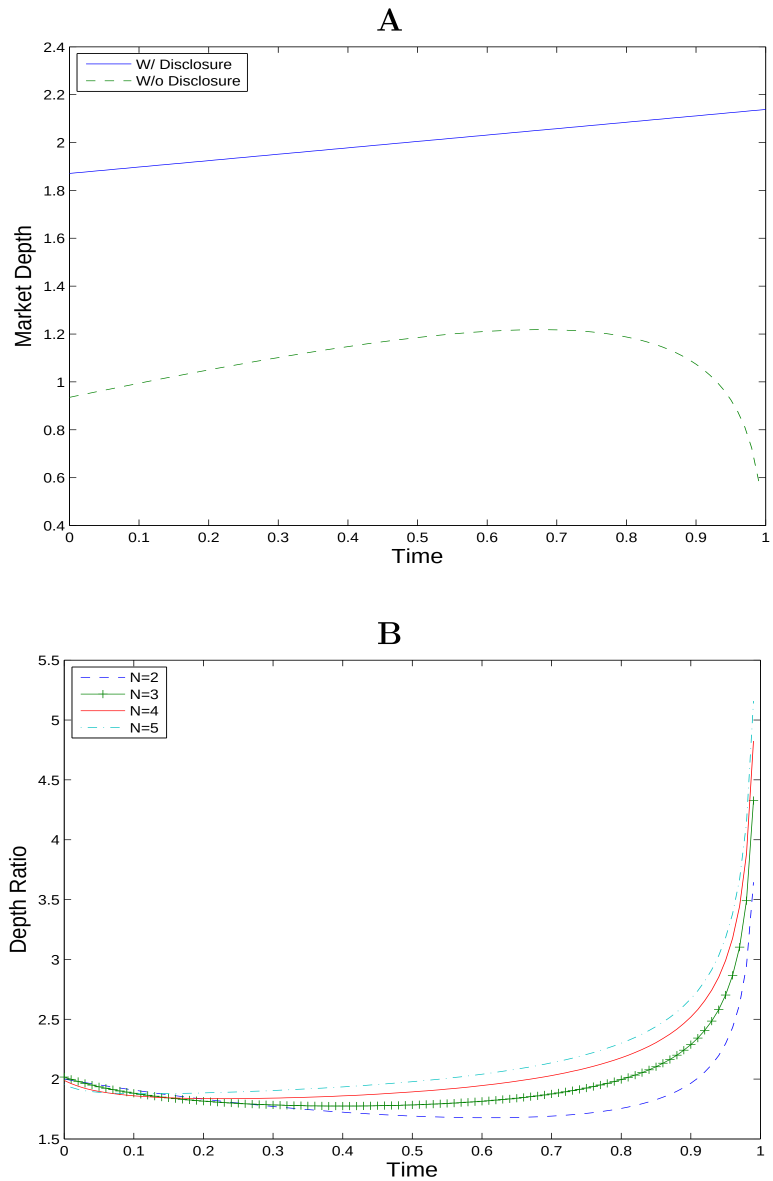

Figure 3: Figure 3A:Market depth $1/\lambda$ as a function of time for $\sigma_\epsilon^2 = 0.875$ and $N = 2$. The solid line is for the case with disclosure and the dashed line is for the case without disclosure. Figure 3B: Market depth ratio with and without disclosure as a function of t for $\sigma_\epsilon^2 = 0.875$ and $N = 2, 3, 4, 5$.

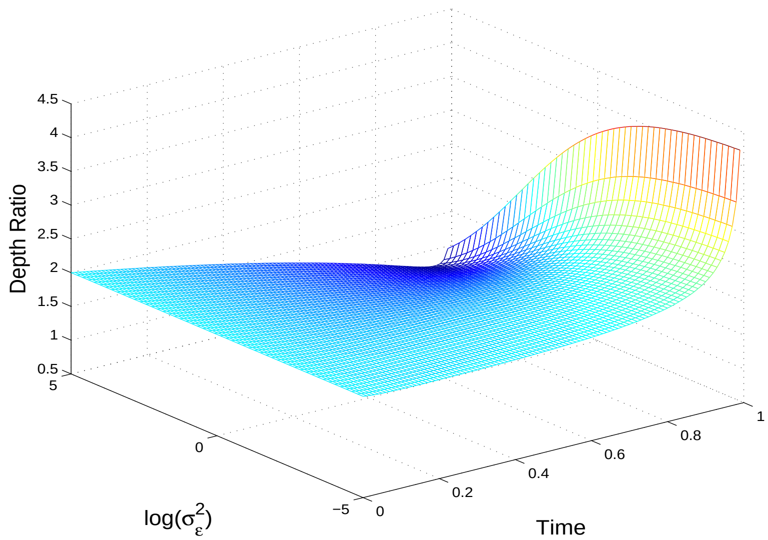

Figure 4: The ratio of market depth $1/\lambda$ with disclosure and without disclosure as a function of $\log(\sigma_{\epsilon}^{2}), t$ for $N = 2$.

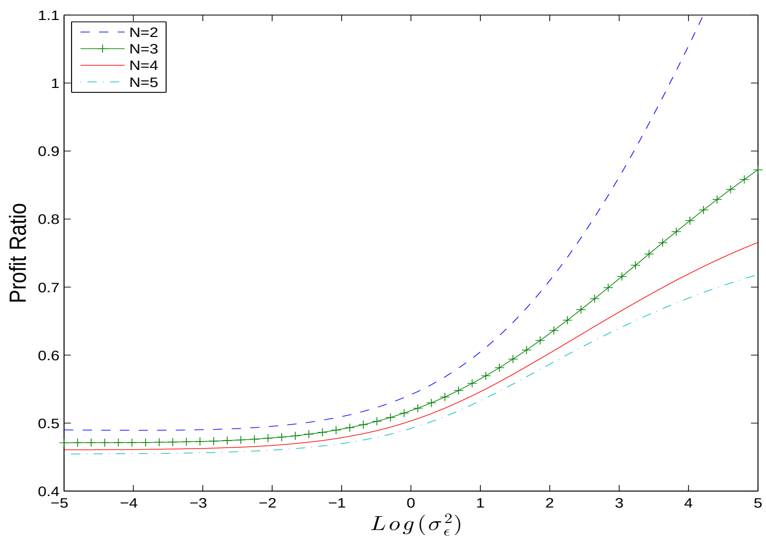

Figure 5: The ratio of informed traders' total profits $\pi(0)$ with disclosure and without disclosure as a function of $\log(\sigma_\epsilon^2)$ for $N = 2, 3, 4, 5$.

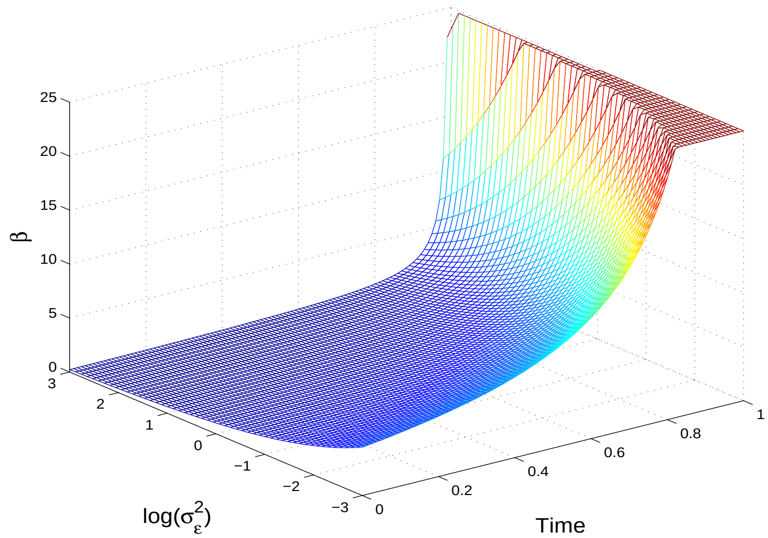

Figure 6: Trading intensity β as a function of $\log(\sigma_\epsilon^2), t$ for $N = 2$ with disclosure

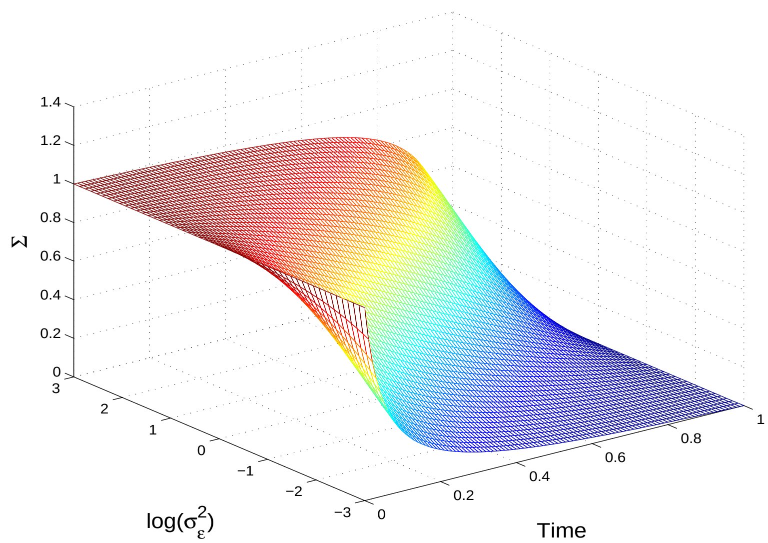

Figure 7: Residual uncertainty Σ as a function of $\log(\sigma_\epsilon^2), t$ for $N = 2$ with disclosure

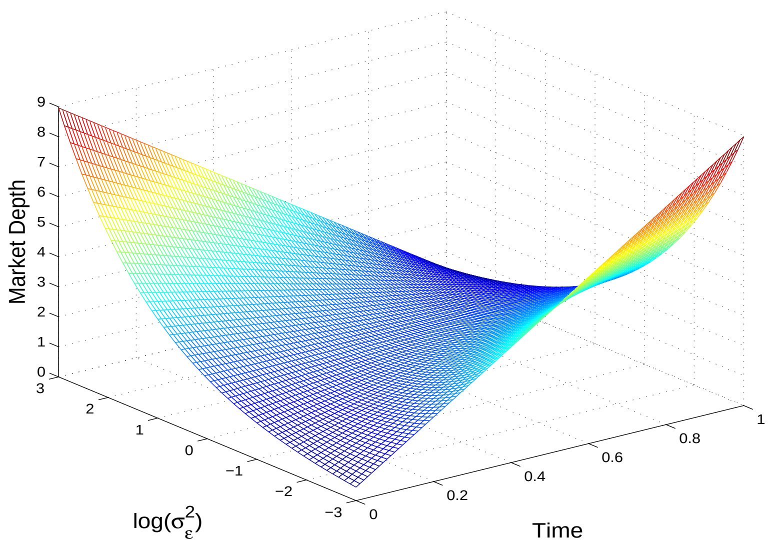

Figure 8: Market depth $1/\lambda$ as a function of $\log(\sigma_\epsilon^2), t$ for $N = 2$ with disclosure.

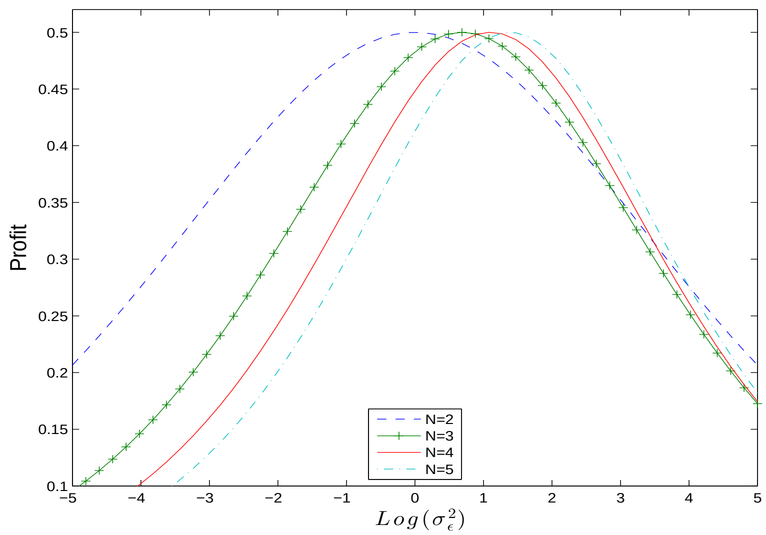

Figure 9: Informed Traders' expected profit $\pi(0)$ as a function of $\log(\sigma_\epsilon^2)$ for $N = 2, 3, 4, 5$ with disclosure.

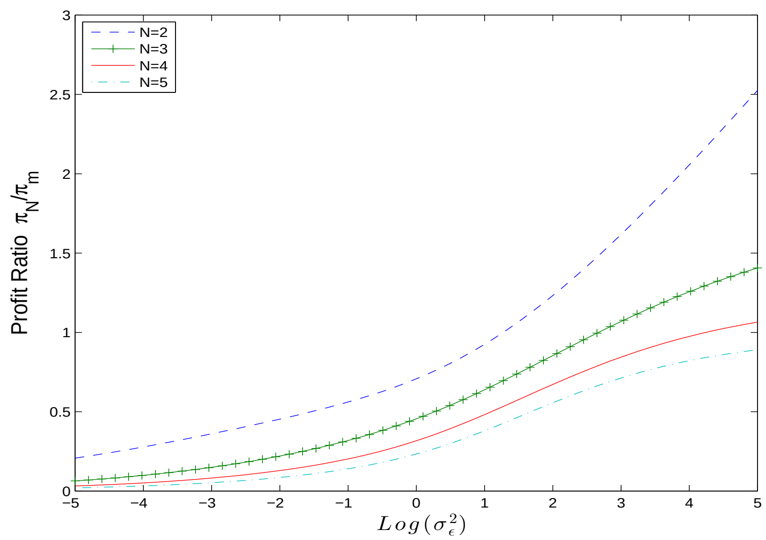

Figure 10: The ratio of informed traders' total profits π_N with many competitive traders and π_M with a monopolistic trader as a function of $\log(\sigma_\epsilon^2)$ for $N = 2, 3, 4, 5$.