Managing Bank Run Risk: The Perils of Discretion

Zongbo Huang*

JOB MARKET PAPER

Latest Version

December 30, 2016

Abstract

This article studies the role of banks' discretion in managing panics in a dynamic model of credit line run. In downturns banks tighten liquidity by cutting credit lines. Anticipating this, borrowers run to draw down credit lines in the first place, which imposes further pressure on banks. Thus liquidity rationing and credit line runs form a feedback loop that amplifies bank distress. I fit the model to the U.S. commercial bank data and find that the feedback effects contribute to more than a half of the liquidity contraction in downturns. From a normative perspective, a commitment tax on bank cutting credit lines is effective in mitigating runs.

Keywords: Credit lines; Liquidity risk; Financial crisis; Runs; Risk management.

*Princeton University, Email: zongboh@princeton.edu. I am especially indebted to my advisor Markus Brunnermeier for his help and support. I am also grateful to Mark Aguiar, Olivier Darmouni, Maryam Farboodi, Ji Huang, Stephen Morris, Jason Ravit, Michael Sockin, Chang Sun, David Thesmar, Wei Xiong, Motohiro Yogo, and Yao Zeng for many helpful comments and discussions. All remaining errors are my sole responsibility.

1 Introduction

Runs on financial institutions played a central role in the 2007-2009 financial crisis. Most existing theories of bank runs focus on the strategic complementarities among depositors but overlook the important role of banks as strategic players. In practice, however, banks not only respond to runs but also control run risk actively as a part of liquidity management. This article underscores banks' strategic role and targets the following questions. First, how do banks control liquidity risk given the possibility of bank runs, and how do banks' strategies in turn affect the incentives to run? Second, how do liquidity provision and run risk respond to policies? In particular, can properly designed policies mitigate and eliminate run risk?

To address these questions, I focus on bank lending through credit lines. A credit line is a flexible loan from a bank to a borrower that permits the borrower to borrow up to a certain limit and repay on an unscheduled basis until the contract ends. Credit lines are commonly used in practice by borrowers to meet immediate liquidity shock and fund daily operations. According to the Federal Reserve Survey of Terms of Business Lending, about 80% of all commercial and industrial loans (C&I loans) in the United States were made under credit lines.

I develop a dynamic banking model of credit line lending and identify an important amplification mechanism through the feedback between liquidity rationing and credit line runs. The model is based on two key features of credit lines. First, credit lines are not fully committed. Banks have legal rights to limit borrowers' access to credit lines, especially when the associated covenants are violated (see, e.g., Sufi (2009) and Roberts and Sufi (2009)); moreover, banks also use this discretion to withhold funds when themselves experiencing liquidity shortages (see, e.g., Acharya, Almeida, Ippolito, and Perez (2014b) and Chaderina and Tengulov (2015)). Second, the use of credit lines is flexible. Thus, firms are able to draw down credit lines preemptively, which is effectively a run on the asset side of banks (see, e.g., Ivashina and Scharfstein (2010), Campello, Graham, and Harvey (2010), and Ippolito, Peydro, Polo, and Sette (2015)).

In the model, borrowers have a demand for liquidity. Each borrower invests in a long-term project and suffers liquidity risk, which would force the borrower to terminate her project unless she can obtain additional liquidity. For example, firms receive trade receivables instead of cash, but they need cash to cover immediate operating expenses. As a result, borrowers arrange credit lines from a bank to protect themselves from such risks. The bank raises deposits to cover the drawdowns of credit lines and earns a premium by pooling the borrowers' liquidity risk and intermediating funds.

The bank is also exposed to its own liquidity shock, which imposes an additional cost of raising deposits and thus makes lending more costly. Therefore, once hit by the shock the bank has an incentive to ration liquidity by cutting credit lines. This liquidity rationing induces borrowers to draw down preemptively in the first place in case their access to credit lines is limited. The preemptive drawdowns, in turn, impose pressure on banks and lead banks to tighten liquidity further. The process repeats and becomes a downward spiral that amplifies the impact of the initial shock. Furthermore, a dynamic inconsistency problem emerges from this environment. The bank may want to commit to a rule-based liquidity policy. If this commitment were credible, it would dissuade borrowers from running. However, the bank may not be able to commit credibly. If the borrowers do in fact run, the bank's future self would respond to it actively by rationing liquidity, instead of acting according to the plan.

I then fit the model to the U.S. commercial bank data and explore the feedback effects and the dangers of bank discretion quantitatively. I find that having a full commitment by the bank mitigates runs significantly, as the bank internalizes the impact of liquidity rationing on the borrowers. The results have new implications for financial stability policies. I first study the effects of leverage ratio requirements. A tighter leverage ratio requirement alleviates the severity of banking distress at the cost of slower credit growth. The bank liquidity risk is mitigated as a result of the bigger equity buffer. In additional, there is a new insight that the restrictions on leverage ratio also dampen the amplification effects of liquidity rationing and credit line runs. I then consider a commitment tax on bank cutting credit lines. As an effective commitment device, a tax of 0.4% is enough to eliminate credit line runs. The tax is also a countercyclical policy by design. Especially, unlike quantity requirements, it does not curb credit growth in economic upturns.

The model takes the contractual properties of credit lines as given and sidesteps the deeper reasons. The following two fundamental frictions may be at work, as indicated by the empirical evidences. First, only borrowers themselves can observe whether they are hit by a liquidity shock. Therefore, contracts cannot be contingent on this information, and borrowers are able to tap credit lines at will . In particular, they can do so preemptively when they are not hit, which leads to a run on credit lines. The second friction comes from borrowers' moral hazard. Although the bank can observe whether the borrowers are behaving, it is difficult to verify this information to the outsiders, such as courts. To deter misbehaving, the bank reserves the right to repudiate the contracts, instead of committing fully to them.

Though my modeling of runs is in the spirit of Diamond and Dybvig (1983), it is different from classic bank run models in two aspects. First, instead of focusing on the coordination failure among borrowers, my model underlines the strategic complementarities between a bank and a group of borrowers and traces out the associated amplification mechanism. Second, in my model both the bank and the borrowers optimize over a long rather than a three-period horizon. In particular, a borrower's run decision depends on the expected value of having a credit line from the bank, which in turn depends on the borrower's belief on the bank's future decisions in this fully dynamic model. In addition, bank liquidity supply is endogenously determined and affect the severity of credit line runs.

This article is related to the literature on credit lines. Sufi (2009) find that credit lines are contingent but not committed sources of liquidity insurance, and shows that firms use both cash and credit lines in managing liquidity because of the risk of credit line revocation. Roberts and Sufi (2009) explore the consequences of financial covenant violations. They show that creditors obtain the control rights after violations to tight lending terms. On the bank side, Kashyap, Rajan, and Stein (2002) study the synergy between credit lines and deposits regarding liquidity provision by banks. Acharya and

Mora (2015) show that this synergy broke down in the first year of the 07-09 crisis and that banks are exposed to double pressures on assets and liabilities. Acharya, Almeida, Ippolito, and Perez (2016) examine the role of bank health and economic conditions in determining the accessibility of credit lines. Recent studies also reveal the existence of credit line runs during the Great Recession. Ivashina and Scharfstein (2010) provide evidence that firms increased the use of credit lines after the failure of Lehman Brothers. Ippolito, Peydro, Polo, and Sette (2015) use Italian Credit Register data to document that firms with multiple credit lines draw down especially from banks with higher exposure to the wholesale funding market. Based on these empirical findings, I identify a new amplification effect in lending through credit lines.

There are also extensive theoretical studies on bank credit lines. Holmstrom and Tirole (1998) demonstrate that banks can use credit lines to provide liquidity insurance to firms as an implementation of the optimal dynamic contract. They also show that when firms' shocks are correlated bank credit lines may not be sufficient. Similarly, in my model, the bank's own risk is not diversifiable and hence disrupts liquidity insurance. The discretion in credit line availability is also highlighted as a fundamental difference from term loan. In Boot, Greenbaum, and Thakor (1993), banks use contracts with discretion to manage jointly financial and reputational capital and to overcome asymmetric information problems by signaling. More recently, Acharya, Almeida, Ippolito, and Perez (2014a) propose a model in which credit line revocation arises endogenously as a result of monitoring. Yet, although the run incentives of credit line borrowers are supported by the empirical studies, it is largely overlooked in the theoretical literature. To fill this gap, my model combines credit line runs and bank discretion into one framework and quantify the isolated strategic effect.

This article is also related to the vast bank run literature, including the seminal work of Bryant (1980) and Diamond and Dybvig (1983), and more recently Cooper and Ross (1998), Allen and Gale (1998), Peck and Shell (2003), Rochet and Vives (2004), He and Xiong (2012), and Vives (2014). This literature largely focuses on the coordination failure among depositors, whereas treats banks passively once runs start. A few exceptions include Ennis and Keister (2009, 2010), Cheng and Milbradt (2012), Gertler and Kiyotaki (2015), and Zeng (2016). Ennis and Keister (2009, 2010) show that when policymakers have limited commitment power suspension of convertibility cannot prevent runs. The benevolent Policymakers would postpone interventions to serve the impatient depositors who haven't withdrawn. In contrast, reducing liquidity supply is ex-post optimal for the self-interested bank in my model, but not ex-ante optimal as it induces runs by the borrowers. Engineer (1989) also show that suspension of convertibility cannot eliminate runs. Furthermore, Cipriani, Martin, McCabe, and Parigi (2014) highlight the danger of suspension that it may create runs. Gertler and Kiyotaki (2015) develop an infinite-horizon model that features financial accelerator effects and roll-over bank runs. The dependency of run probability on fire-sale prices amplifies the aggregate disturbances, even beyond the amplification from the conventional financial accelerator. Considering fire sales resulting from shifts in the composition of assets instead of deleveraging, Zeng (2016) show that cash re-building policies of mutual funds generate a first-mover advantage that leads to shareholder runs. My paper differs from these studies in two key aspects. First, liquidity rationing on credit lines induces runs directly, whereas fire-selling prompts runs indirectly through a decrease in net worth. This leads to different policy implications. For example, a tax on credit line cuts is effective in my context. However a tax on fire-selling may exacerbate the disturbances. Second, credit lines are long-term contracts, hence borrowers' incentive to run is affected by their expectations about bank liquidity policies in the future. This is important as the bulk of financial relationships are long term.

The quantitative analysis follows the precedent of recent papers that estimate structure models of the banking sector, including Corbae and D'Erasmo (2014) and Mankart, Michaelides, and Pagratis (2014) among others. Schroth, Suarez, and Taylor (2014) estimate a dynamic debt run model based on He and Xiong (2012) and show that runs are sensitive to bank's balance sheet composition. My paper is also related to Egan, Hortaçsu, and Matvos (2015), who estimate a bank run model with a differentiated deposit market and explore multiple equilibria.

Layout. The reminder of the paper is organized as follows. Section 2 presents relevant empirical evidences. Section 3 lays out a benchmark model to study bank liquidity management with run-prone borrowers. Section 4 discusses the Markov perfect equilibrium of the benchmark model and analyzes an alternative model in which the bank can commit. The benchmark model is then calibrated in section 5. Section 6 conducts counterfactual experiments and Section 7 concludes.

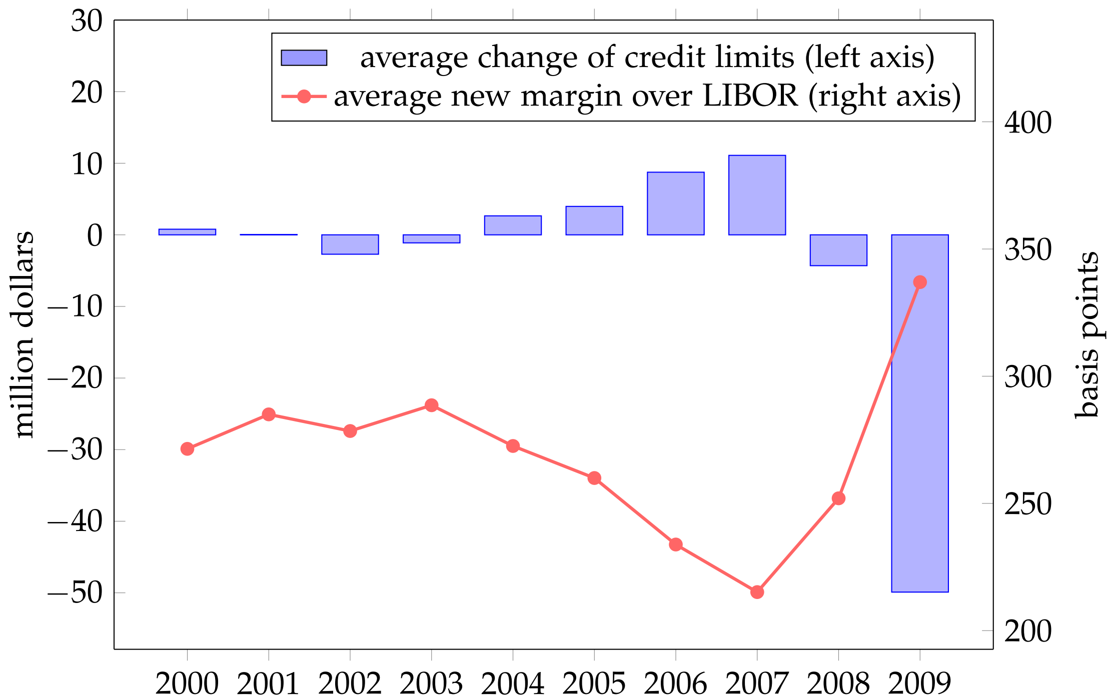
2 Empirical Evidences

I begin by providing empirical evidences about credit line usage and availability, and synthesizing the empirical literature. I emphasize three key aspects: (i) banks tighten liquidity by reducing limits of credit lines during the 07-09 crisis, (ii) credit line borrowers have the incentive to draw down early in case banks restrict credit line access in the future, and (iii) on bank balance sheets total loans contracted less than total credit, the sum of loans and unused credit lines, after the Lehman failure.

Credit Line Availability. Figure 1 shows that banks reduce credit line accessibility during the Great Recession. It plots the two most common outcomes of loan amendments in Dealscan database: credit limit reduction and interest rate increase. The average reduction in credit limits of all amendments in 2009 is 53 million dollars. At the same time, the new margin over LIBOR is about 340 basis points on average in 2009, which makes borrowing more costly and thus limits the use of credit lines indirectly. Consistent with the idea of rationing, the credit limit reductions in 2009 are more prominent than the increases in interest rates.

The presence of covenants in credit line contracts gives creditors the right to limit access conditional on covenant violations. Importantly, banks have the discretion of how to use the right and hence determining the consequences of violations. Acharya, Almeida, Ippolito, and Perez (2014b) find evidence on that accessibility to credit lines, following violations, depends on bank health. In particular, banks are more likely to

Figure 1: Amendments from Dealscan dataset

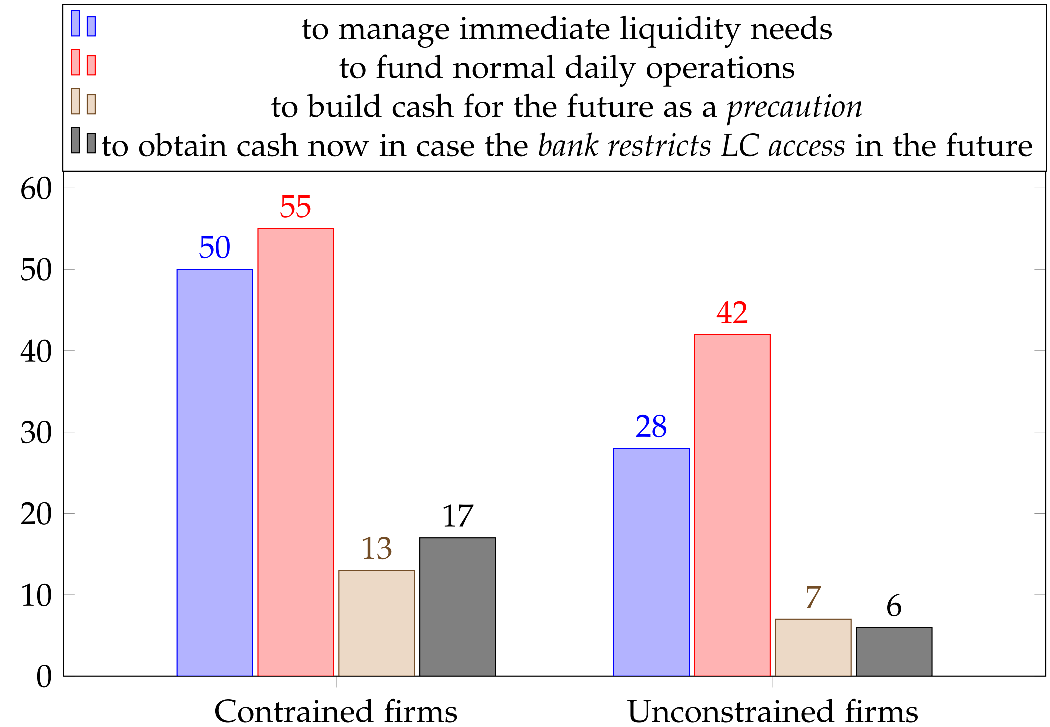


Note: The average change of credit limits is the average of all amendments, the average new margin over LIBOR is the average of all amendments with a new margin and based on LIBOR. Source: Dealscan database from Thomson Reuters

withdraw credit lines instead of waiving covenant violations during crises. Moreover, most credit lines have the material adverse change covenant, which provides lenders the discretion to determine whether a borrower's credit quality deteriorates significantly enough to trigger a violation.

Credit Line Usage. Ivashina and Scharfstein (2010) is the first to point out that the increase in total lending in 2008Q4 is driven by an increase in drawdowns by existing credit line borrowers. They also provide some evidence on preemptive drawdowns by examining the SEC filings. To identify the effect of expected decline in liquidity supply on credit line usage, Ippolito, Peydro, Polo, and Sette (2015) compare drawdowns by the same firm from different banks. They find that higher exposure to the interbank market leads to more drawdowns. Figure 2 presents a more direct evidence on preemptive

Figure 2: Reasons to draw cash from credit lines (%)



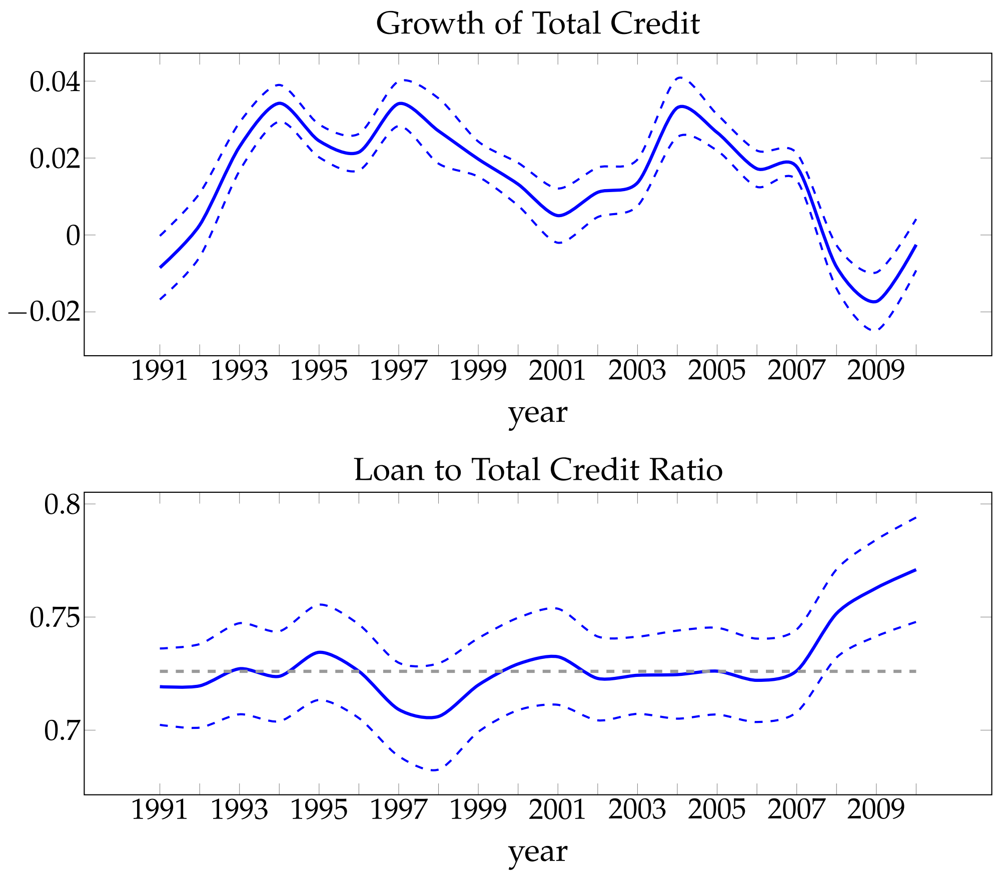
Note: CFO survey of 569 US firms in 2008Q4. Source: Table 8 of Campello, Graham, and Harvey (2010)

drawdowns using the CFO survey data provided in Table 8 of Campello, Graham, and Harvey (2010). The 2008Q4 survey explicitly asks about firms' reasons to draw credit lines. There are 17% of constrained firms and 8% of unconstrained firms reporting that they draw down credit lines in case the bank restricts line access in the future.[1] Although unfortunately this question is only asked in the 2008Q4 survey hence we cannot tell if firms draw down preemptively beyond the crisis, this fact, nonetheless, shows that firms indeed have the incentives to run.

Bank Balance Sheets. Last, I document facts on bank balance sheets using quarterly data from the Consolidated Report of Condition and Income (known as Call Reports). These bank-level facts are consistent with the observations discussed above. Given the

[1]The survey also asks whether firms' operations are "not affected", "somewhat affected" , or "very affected" by difficulties in accessing the credit markets. Firms that are "very affected" are considered as constrained. The question on reasons to draw credit lines allows multiple choices.

Figure 3: Average Quarterly Growth Rate of Total Credit (Upper Panel) and Average Loan to Total Credit Ratio (Lower Panel)



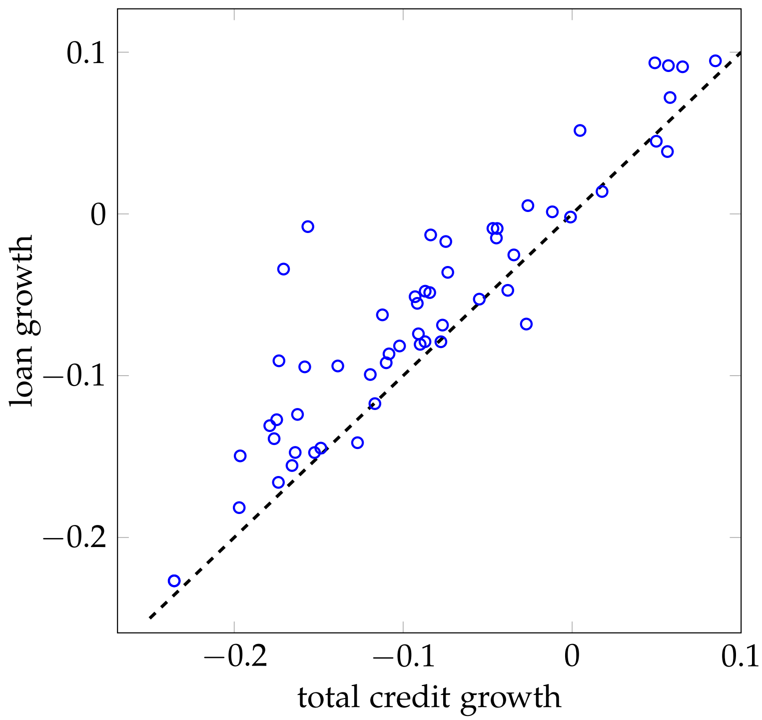
Note: Total credit is the sum of loans and unused credit lines. Dashed blue lines correspond to the 95% confidence interval for the averages. The gray dashed line in the lower panel represents the average loan to total credit ratio between 2000 and 2008. Source: Consolidated Report of Condition and Income.

interest in the effects of liquidity rationing, I first explore the evolution of *bank total credit*, defined as the sum of loans and unused credit lines.[2] The upper panel of Figure 3 presents the average total credit growth of banking holding companies (BHCs) with asset more than 10 billion dollars over time. There is a clear cyclical pattern; moreover, the quarterly growth rate drops to about -2% in 2009, lower than in previous recessions.

Once borrowers tap credit lines, the drawn part appears on balance sheets as loans. To shed light on the usage of credit lines, I next look at the dynamics of bank loans. I

[2]Loans consist of credit line drawdowns and term loans. For the purpose of measuring liquidity rationing through credit line cuts, it is without loss of generality to treat term loans as a form of credit lines with fixed borrowing and repayment schedules.

Figure 4: Growth of Loans and Total Credit from 2008Q3 to 2009Q3



Note: Dashed line is the 45° line. Source: Consolidated Report of Condition and Income.

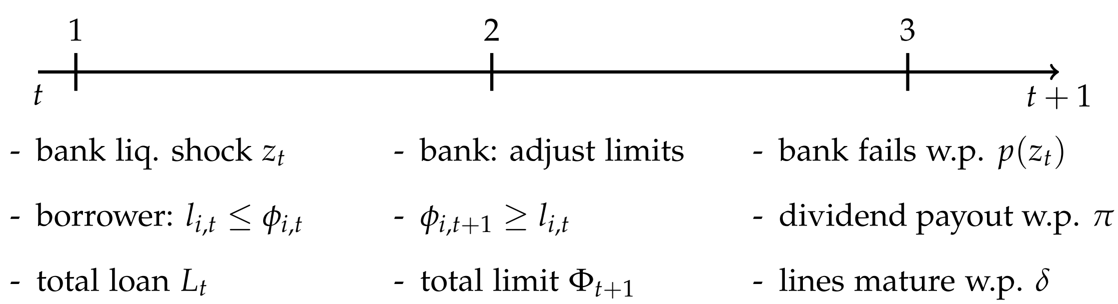
normalize loans by total credit to remove the cyclical component and plot the time series of this ratio in the lower panel of Figure 3. The ratio is pretty much stable until it shoots up in 2008. This increase in the loan-to-credit ratio is consistent with the evidence of preemptive drawdowns.

While Figure 3 presents the cross-sectional averages across time, Figure 4 plots the growth of loans against the growth of total credit of individual BHCs. It shows that loans contract much less than total credit in the year following Lehman failure (above the dashed 45% line). It is also statistical significant that the gap between the two growth rates widens as total credit shrinks, which is consistent with my model.

3 The Model

In this section I develop a framework with strategic interactions between a bank and a group of borrowers. The model is motivated by the key institutional settings of credit lines: flexibility and bank discretion. Although I consider the specific context of credit lines, the amplification mechanism is fairly general and can be readily extended to other settings with a single large player and a continuum of small players.

Figure 5: Timeline



3.1 Setup

Time is discrete and infinite. There is a continuum of borrowers having credit lines from a single bank. The mass of borrowers varies over time as borrowers enter and exit.

Timeline. Figure 5 shows the timeline. In period t, the set of borrowers with access to credit lines is denoted as $\mathcal{I}_t$. Each borrower i has access to her credit line with limit $\phi_{i,t}$, and Φ_t denotes the total available credit. At the beginning of each period, a publicly observed shock hits the bank. This bank liquidity shock $z_t \in \{z_b, z_g\}$ ("bad" and "good" states) evolves as a Markov process $F(z', z) = prob(z_{t+1} = z' | z_t = z)$. As a consequence, the bank fails at the end of the period with probability $p(z_t)$ such that $p(z_g) \leq p(z_b)$.

In each period, each borrower decides how much to draw down simultaneously with other borrowers. Drawn credit lines show up as loans on the asset side of bank balance sheet, whereas the unused portion remains off balance sheet. I denote the amount tapped by borrower i as $l_{i,t}$ and the total drawdowns as L_t. The bank starts with equity E_t and raises deposits to finance the credit line drawdowns. After that, it may alter the liquidity provision for the next period. On the one hand, the bank may issue new credit lines to new borrowers. On the other hand, the bank may reduce the credit limits of existing borrowers to $\{\phi_{i,t+1}\}_{i\in\mathcal{I}_t}$ (and total credit to Φ_{t+1}) but not below the outstanding balances because the bank cannot force borrowers to repay. Borrowers exit if they keeps a zero balance and their credit limits are reduced to zero.

At the end of the period, three events occur in sequence. First, the bank may fail with probability $p(z_t)$, which is given exogenously as a function of the bank's liquidity shock z_t. This assumption can be micro-founded by a deposit run model, in which $p(z_t)$ is determined by a specific equilibrium selection mechanism. Thus, my model can be viewed as a model of double bank runs – a deposit run and a credit line run. To focus on the feedback effects associated with credit line runs I model bank failure exogenously in this paper and treat it as a shock to bank health. I am also working on an extension with endogenous bank failure. In that setting, the two types of bank runs may reinforce each other.

When the bank fails shareholders receive nothing, and borrowers can no longer tap funds from the credit lines. Therefore, borrowers who keeps a zero balance after the bank fails will exit because effectively their credit limits are reduced to zero. Whereas those with positive outstanding balances may keep the drawn funds to finance future needs instead of repaying them immediately.

Second, if the bank does not fail, with an exogenous probability π the bank pays out the whole equity to existing shareholders and raises the same amount of equity from new shareholders. This is a simple way to model dividend payouts. Although the shareholders are replaced, the bank continues to function and the borrowers are not affected.

Finally, each credit line matures independently with probability δ. When their credit lines mature borrowers need to repay their balances and then exit. To sum up, new borrowers enter passively when the bank issues new credit lines. Whereas existing borrowers may exit for two reasons: their credit lines mature, and their credit limits are reduced to zero. If a borrower exits, she obtains a constant fraction, $1-\eta$, of what she would otherwise receive.

Borrowers. Each borrower operates a long-term project of constant size that generates return R in each period. At the beginning of each period there is an idiosyncratic and privately-observed liquidity shock that hits each borrower with probability Λ_t . The shock requires borrowers to inject additional funds into the project to avoid their projects being liquidated. Funds are required for one period only and become fully liquid afterward. I normalize the size of the needed funds to be 1\$, and let $\lambda_{i,t} \in \{0,1\}$ denote whether borrower i is affected at period t.

To insure against this liquidity shock, each borrower obtains a credit line from the bank which specifies a credit limit. Borrowers pay a fixed maintenance fee r^{ϕ} on the credit line limits, regardless of whether the lines are drawn or not. Besides, they also pay a usage fee r_t^l on the amount that they borrow from the bank. The usage fee is set at a fixed margin r_x over a reference rate r_t, such as LIBOR rate.

Borrowers can draw funds up to their credit limits at will, even when they are not hit by the shock. When not hit by the shock, borrowers face a trade-off in making drawdown decisions. The benefit of drawdown is that the drawn funds will be available to borrowers in the next period because the bank cannot force them to payback. In contrast, undrawn credit lines may become unavailable if the access to credit lines is restricted by the bank. The cost of drawdown comes from the usage fee. To generate realistic credit line runs, I also assume that if the funds are drawn preemptively they generate an idiosyncratic and privately-observed return $\kappa_{i,t} \in [\underline{\kappa}, \overline{\kappa}]$. Thus, the effective cost is the difference between the usage fee and the stochastic return. The return is drawn at the beginning of the period from distribution $\Omega(\cdot)$, and it can be negative as a liquidity storage cost.

The assumption of 0-or-1 liquidity shock is essential for tractability. Because of it, borrowers either borrower 1\$ from the bank or do not borrow at all and hold a zero balance. Furthermore, borrowers would obtain credit lines with the limit of 1$ exactly. A higher limit imposes unnecessary maintenance fees, while credit lines with a limit less than 1$ are entirely useless. Therefore, this assumption allows us to focus on borrowers' drawdown decisions and abstract away the heterogeneity in the credit line limits.

The bank. The bank lends to borrowers through credit lines. Each period the bank starts with equity E_t and total credit Φ_t, the sum of the limits of all credit lines. The drawn portion of credit lines appears on the balance sheet as loans L_t. The bank then raises deposits D_t to finance the loans, hence the feasibility constraint is as follows:

$$L_t = D_t + E_t. \tag{1}$$

Because every credit line has the same limit and the drawdown decision is binary, it is sufficient for the bank to take into account the total credit Φ_t and the total amount drawn L_t, instead of the distribution of credit line limits and usage.

Let Π_t denote the bank's profit from intermediating funds, which is a function of loans L_t, deposits D_t, and total credit Φ_t given by

$$\Pi_t = r^{\phi}\Phi_t + r_t^l L_t - (r_t + p(z_t))D_t - c(D_t, \Phi_t). \tag{2}$$

The first two terms of Equation 2 are the maintenance fee and the usage fee received by the bank. As mentioned above, borrowers need to pay this maintenance fee regardless of whether they borrow from the bank or not. The third term represents interest expenses on deposits. The bank pays a base rate r_t and a premium $p(z_t)$ to compensate depositors for the risk of bank failure.[3] The fourth term captures non-interest expenses, such as employee compensation and maintenance of facilities. I assume that $c(D_t, \Phi_t)$ is convex in deposit D_t to introduce curvature into the model.

The bank's profit margin is given by the difference between the return from lending

[3]The usage fee is based on the same interest rate r_t. In practice, the most common reference rate used in credit line contracts is the LIBOR rate, which is the average of interest rates estimated by each of the leading banks that it would be charged were it to borrow from other banks.

$r^{\phi} + r_t^l$ and the cost of raising deposits $r_t + p(z_t)$. The margin, together with the convex cost $c(D_t, \Phi_t)$, determines the optimal leverage of the bank. In bad times, because the additional cost that compensates for the bank failure risk, the profit margin is thinner, and in turn the optimal leverage is lower than that in good times. Therefore, in general the bank would prefer to deleverage when hit by the negative bank liquidity shock and leverage up during recoveries.

An increase in drawdowns may lead to an additional pressure on deleveraging, which is a key component of the amplification mechanism. When the marginal non-interest expense of lending exceeds the profit margin, an increase in drawdowns would decrease bank profit and, in turn, reduce next-period equity. Thus, the bank faces further pressure on deleveraging. This may happen when bank leverage is high and, in particular, when the bank is hit by a negative shock after staying in the good state for a long time. In the latter case, lending may become costly because the cost of raising deposit increases when a negative shock arrives.

Different from a typical banking model, the bank cannot deleverage directly by adjusting total loans, which is determined by borrower drawdown decisions. Instead, the bank may reduce next-period credit limits to control leverage indirectly. In doing so, there are three cases to consider. First, a borrower draw down up to the limit of 1$ and the bank cannot reduce her limit. Second, a borrower does not draw her credit line, and the bank chooses not to reduce her credit limit. Third, a borrower does not draw her credit line, and the bank decides to cut her limit to 0 since any positive amount between 0 and 1 is meaningless. To sum up the three cases, because the only heterogeneity among borrowers is in the binary drawdown decisions, it is sufficient for the bank to choose how many undrawn credit lines to cut. For the same reason, it is also sufficient to decide how many new credit lines to originate if the bank wants to leverage up. In addition, the bank would not cut credit lines and issue new credit lines at the same time.

Let Δ denote the change of total credit, which is negative if the bank cuts credit lines and positive if it issues new credit lines. The law of motion of total credit is thus given by

$$\Phi' = (1-\delta)\Phi + \Delta, \tag{3}$$

where δ is the fraction of credit lines maturing at the end of the period. Adjusting total crediti incurs a quadratic cost, denoted as $f(\Phi_{t+1}, \Phi_t)$.

I assume that the bank can only accumulate equity via retained profits as in Gertler and Kiyotaki (2015). While this assumption is a reasonable approximation of reality, I do not explicitly model the underlying frictions. Moreover, the bank is risk neutral and only pays out all of its equity as dividends with probability π. The bank maximizes the expected utility of shareholders at the end of period t, which is given by

$$V_t = \mathbb{E}_t\Bigg[\sum_{i=1}^{\infty}\beta^i \underbrace{\pi\,(1-\pi)^{i-1}\prod_{j=1}^{i}(1-p(z_j))}_{\text{prob. of exiting at period } t+i} E_{t+i}\Bigg]. \tag{4}$$

3.2 Discussions

The differences from classic bank runs. Classic bank run models emphasize on the strategic complementarities among depositors. Although the similar complementarities also exist in my model among borrowers, the focus of my model is instead on the strategic complementarities between the bank, as a large player, and the borrowers, as a group of small players. More specifically, in classic bank run models liquidity supply is fixed or pre-determined before depositors making withdrawal decisions. In contrast, in my model the bank controls liquidity supply and reacts to borrower runs optimally. This interaction between the liquidity supply and credit line runs leads to a new amplification mechanism that I explore in this paper.

Model assumptions. First, the bank can reduce credit line limits but cannot force borrowers to pay off outstanding balances. This rules out the case of payment acceleration, which is rarely observed in the data (see, e.g., Roberts and Sufi (2009)). This assumption also captures the idea that borrowers may gain bargaining power in renegotiation by tapping credit lines, and thus rationalizes borrowers' run incentives. Moreover, I also abstract away other dimensions of credit line renegotiation for tractability, such as fees, maturity, and collateral. As long as preemptive drawdowns grant borrowers more bargaining power, the exact channel through which the bank tighten liquidity is nonessential for the existence of the feedback effects between the bank and the borrowers. Incorporating these realistic features of credit lines would greatly complicate the model so that I need to keep track of the distribution of borrowers, but would not undermine the key mechanism.

I also assume that the bank lends through credit lines only, which is a reasonable approximation of reality. In practice, about 80% of all C&I loans are made under credit lines. Moreover, in most times term loans are issued together with credit lines to the same borrowers. Adding term loans into the model does not change my qualitative results because term loan borrowers are passive and do not respond to the bank's decisions. Incorporating term loans would depress the sensitivity of preemptive drawdowns to liquidity rationing, but doing so would not affect my quantitative results in any significant way because I directly calibrate that sensitivity to the data.

Last, the stochastic returns on the credit lines drawn preemptively are introduced to generate realistic equilibrium bank runs. A run is necessarily partial in the data, with only some borrowers participating. This assumption helps to pin down the fraction of borrowers who run.

4 Equilibrium Analysis

In this section, I analyze the game backward in time, first consider the bank's problem and then discuss borrowers' optimal decisions. Since I will use recursive methods to solve a Markov perfect equilibrium, let any variable x_t be denoted by x and x_{t+1} be denoted by x'. All key proofs are left to the appendix.

4.1 Bank Decision Making

Before diving into the bank's problem, I first define the borrowers' strategy. Borrowers make drawdown decisions based on their information at the beginning of the period; therefore borrowers' strategy, denoted by σ, is a function of bank equity E, total credit Φ, bank liquidity shock z, and borrower idiosyncratic shocks λ_i and κ_i. Formally, borrower i's decision is given by $l_i = \sigma(E, \Phi, z; \lambda_i, \kappa_i)$.

Since the bank cannot observe borrowers' idiosyncratic shocks nor infer them from drawdown decisions, the only heterogeneity that the bank takes into account is whether a borrower draws down her credit line or not. Moreover, given that the drawdown decision is binary, the total amount of drawdowns is a sufficient statistics for the bank. After integrating l_i over the two idiosyncratic shocks the total amount of drawdowns L is a function of E, Φ, and z only.

At the beginning of the period, the bank starts with equity E and provides total credit Φ, the sum of its credit line limits. After borrowers make drawdown decisions, the bank chooses the next-period total credit Φ' to maximize expected discounted dividends, as defined in Equation 4. The bank's problem can be represented recursively,

$$
\begin{aligned}
V(E, \Phi, z) = & \max_{\Phi' \geq (1-\delta)L} \beta\pi(1-p)E' + \beta(1-\pi)(1-p)\mathbb{E}_{z'|z} V(E', \Phi', z') \\
\text{s.t.} \quad & D = L(E, \Phi, z) - E \\
& E' = E + \Pi - f(\Phi', \Phi),
\end{aligned} \tag{5}
$$

where $\beta\pi E'$ represents the expected value from dividend payouts, and $(1-\pi)(1-p)$ is the probability of continuing without dividend payouts. The next-period equity E' is given by equity E plus profit Π and minus the adjustment cost $f(\Phi', \Phi)$. In addition, since the bank cannot force repayment of drawn credit lines, Φ' has to exceed the unmatured outstanding balances $(1-\delta)L$.

Let σ^B denote the bank's strategy, then the bank's decision can be expressed as $\Phi' =$

$\sigma^B(E, \Phi, z)$. The optimal strategy satisfies the following first-order condition,

$$\underbrace{\left[\pi + (1-\pi)\frac{\partial \mathbb{E}V(E', \Phi', z')}{\partial E'}\right]\frac{\partial E'}{\partial \Phi'}}_{\text{adjustment costs (in utils)}} + \underbrace{\left[(1-\pi)\frac{\partial \mathbb{E}V(E', \Phi', z')}{\partial \Phi'}\right]}_{\text{benefit from (de)leveraging}} = 0. \tag{6}$$

First, note that since dividends are paid out only after the bank exits there is no intertemporal trade-off on dividends as in Gertler and Kiyotaki (2015). Second, the choice of the next-period total credit is determined by two opposing forces. On the one hand, the adjustment cost reduces next-period equity, which in turn decreases both the continuation value and the dividend payouts. On the other hand, choosing Φ' allows the bank to expand or shrink. In particular, decreasing Φ' in bad times allows the bank to tighten liquidity supply.

4.2 Borrower Runs

In this subsection I investigate the borrowers' problem, showing that borrowers have incentives to run by drawing down preemptively. Moreover, credit line runs are more severe when the credit lines are less secure.

The flow payoffs simply depend on borrowers' liquidity shocks and the drawdown decisions, as shown in Table 1. Conditional on hit by the shock, a borrower obtains return R from the project and pays maintenance fee r^ϕ and usage fee r^l if she draws down. Otherwise she receives zero as the project being liquidated. If there is no liquidity shock, a borrower gets $R - r^\phi - (r^l - \kappa_i)$ if she taps the credit line, and $R - r^\phi$ if not. The cost of preemptive drawdown lies in the difference in flow payoffs $r^l - \kappa_i$, whereas the benefit comes from her gain in the continuation value.

Borrowers' continuation values depend on whether the bank fails and whether borrowers exit. First, if the bank continues and borrower i's does not exit, the value at the beginning of the next period W' is determined by bank equity E', total credit Φ', bank shock z', and her own idiosyncratic shocks λ_i' and κ_i'. Looking forward at the end of the current period, borrower i is uncertain about the realizations of the shocks, hence

Table 1: Borrower i's Flow Payoffs and Continuation Values

Liq. Shock	Drawdown	Flow Payoff	Continuation Value
Y	Y	$R - r^{\phi} - r^{l}$	$(1-p)(1-\eta\delta)\hat{W}' + p(1-\eta\delta)W_0$
Y	N	0	W_L
N	Y	$R - r^{\phi} - (r^{l} - \kappa_i)$	$(1-p)(1-\eta\delta)\hat{W}' + p(1-\eta\delta)W_0$
N	N	$R - r^{\phi}$	$(1-p)(1-\eta\delta-\eta q)\hat{W}' + p(1-\eta)W_0$

Note: $\hat{W}'$ represents the expected value at the end of the period if borrower i has access to credit line in the next period. W_0 denotes the expected value of a remaining borrower i after bank failure. $r^l - \kappa_i$ is the effective cost of preemptive drawdowns, and q stands for the endogenous probability of an undrawn credit line being cut.

her value is the expectation of W' taken with respect to z', λ_i', and κ_i'. Formally, the end-of-period value is given by

$$\hat{W}'(E', \Phi') = \mathbb{E}_{z'|z,\lambda_i',\kappa_i'}[W'(E', \Phi', z'; \lambda_i', \kappa_i')]. \tag{7}$$

If the bank continues and borrower i exits, she can only obtain a fraction of this value, $(1-\eta)\hat{W}'(E', \Phi')$. Let q denote the endogenous probability of exit as a result of bank cutting undrawn credit lines. The total probability of exit is thus δ if borrower i draws down and $\delta + q$ otherwise.

The probability endogenous probability of exit is determined at the equilibrium, which equals 0 if the bank does not cut credit line at all ($\Delta > 0$) and

$$q = \min\left\{\frac{-\Delta}{\Phi - L}, 1\right\} \quad \text{if } \Delta < 0, \tag{8}$$

where $-\Delta$ is the total credit lines being cut when $\Delta < 0$, and $\Phi - L$ is the total amount of undrawn credit lines. Because each borrower is atomless, borrower i's decision has no direct impact on q, thus she takes this probability as given when making decisions.

Second, if the bank fails and borrower i does not exit, which happens if she holds a positive balance, borrower i may still hold the borrowed funds to finance future needs until she pays them off upon exit.[4] In doing so, her end-of-period value can be formulated recursively as

$$W_0 = \underbrace{\mathbb{E}_{\lambda_i',\kappa_i'}[R - r^{\Phi} - r^l + (1-\lambda_i')\kappa_i']}_{\text{expected next-period flow payoff}} + \beta(1-\eta\delta)W_0, \tag{9}$$

where the first term represents the expected next-period flow payoff. If borrower i exits after the bank fails, her value drops to $(1-\eta)W_0$.

The total probability of exit also depends on drawdown decisions. If borrower i draws down in the period, she exits only if the credit line matures with probability δ. Whereas if she doesn't draw down, the probability of exit is 1 because her balance is zero and she cannot borrow from the failed bank any more.

To sum up, as shown in Table 1 borrowers' continuation values are functions of the two end-of-period values $\hat{W}'$ and W_0, and the probabilities of exit. I also denote the continuation value after liquidation as W_L. Therefore, the value of borrower i at the beginning of the period after observing the shocks is given by

$$\begin{aligned} W(E,\Phi,z;\lambda_i,\kappa_i) = \max_{l_i}\, & l_i[R - r^{\phi} - r^l + (1-\lambda_i)\kappa_i + \beta(1-p)(1-\eta\delta)\hat{W}' + \beta p(1-\eta\delta)W_0] \\ & + (1-l_i)\{(1-\lambda_i)[R - r^{\phi} + \beta(1-p)(1-\eta\delta-\eta q)\hat{W}' + \beta p(1-\eta)W_0] + \lambda_i\beta\omega_L\}, \end{aligned} \tag{10}$$

where $\hat{W}'$ and W_0 are defined as in Equations 7 and 9, $l_i \in \{0,1\}$ denotes the drawdown decision, and $\lambda_i \in \{0,1\}$ represents whether borrower i is hit by the liquidity shock.

To solve for the borrowers' optimal choices, I first assume that W_L is low enough that borrowers would always draw credit lines if hit by the shock to avoid liquidation. When not hit by the shock borrowers face a key trade-off between the borrowing cost and the loss in continuation value as presented in Table 1 and Equation 10. Borrower i taps her line if and only if the loss in continuation value exceeds the borrowing cost.

[4] The borrower may also repay the balance voluntarily and exit at the end of the period, but she will not choose to do so because odf f the loss in continuation value. Allowing borrowers to make voluntary exit decisions after observing their shocks would not lead to any significant change.

Proposition 1 *Given the value functions and strategies of the bank and other borrowers, the drawdown decision of borrower i who has no liquidity need is given by*

$$l_i^* = \begin{cases} 1 & \textit{if} \;\; r^l - \kappa_i < \beta[(1-p)\eta q \hat{W}' + p\eta(1-\delta)W_0] \\ 0 & \textit{otherwise} \end{cases}. \tag{11}$$

Moreover, l_i^* is increasing in q.

There exists a threshold value κ^* such that borrower i taps her line if and only if $\kappa_i \geq \kappa^*$. Thus, the stochastic return helps to determine the fraction of borrowers who draw down credit lines.

Finally, since λ_i and κ_i are both i.i.d, the total amount of drawdowns L can be expressed as

$$\begin{aligned} L(E, \Phi, z) &= \mathbb{E}_{\lambda_i, \kappa_i}[\sigma(E, \Phi, z; \lambda_i, \kappa_i)] \; \Phi \\ &= [\Lambda + (1-\Lambda)(1-\Omega(\kappa^*))] \, \Phi, \end{aligned} \tag{12}$$

where Λ is the probability that a borrower being hit by the shock, and $\Omega(\cdot)$ is the cumulative density function of the return of drawdowns. Therefore, $\Lambda + (1-\Lambda)(1-\Omega(\kappa^*))$ is the ex-ante probability of drawdown before the idiosyncratic shocks realize, which also represents the fraction of borrowers who draw credit lines.

Following Equations 8, 11, and 12, given the bank's strategy borrowers may coordinate to multiple levels of total drawdowns in some cases. Hence we need a selection mechanism. The exact way in which the stable equilibrium is selected is not crucial to the main point about the amplification mechanism, but in order to show the feedback effects formally I assume the resulting total drawdowns $L(E, \Phi, z)$ being a smooth function. Moreover, in the numerical analysis, to be consistent with the data I assume that the borrowers will coordinate to the best equilibrium whenever multiple equilibria occur.

4.3 Definition of Equilibrium

I solve for a Markov perfect equilibrium. A Markov perfect equilibrium is a subgame perfect equilibrium in which the strategies depend only on the payoff-relevant history. The payoff-relevant history can be summarized by the states $\{E_t, \Phi_t, z_t\}$, and both the borrowers and the bank act according to the states. Denote the states as $\{E_t, \Phi_t, z_t\} \in \mathcal{S}$ and borrowers' idiosyncratic shocks as $\{\lambda_{i,t}, \kappa_{i,t}\} \in \mathcal{H}$.

Definition 1 *A Markov perfect equilibrium of the model is a pair of value functions* $(V : \mathcal{S} \to \mathbb{R}^+, W : \mathcal{S} \times \mathcal{H} \to \mathbb{R}^+)$ *and strategies* $(\sigma^B : \mathcal{S} \to \mathbb{R}, \sigma : \mathcal{S} \times \mathcal{H} \to \{0, 1\})$ *such that*

1. *given the policy functions,* (V, W) *solve the Bellman equations 5 and 10;*
2. *for any borrower,* $\hat{\sigma} = \sigma$ *is optimal given* W, σ^B*, and that all other borrowers follow* σ*;*
3. *given* σ *and* V, σ^B *is optimal for the bank.*

The Markov perfect equilibrium defined above is subgame perfect, and thus it is dynamically consistent. There is, however, a dynamic inconsistency problem on the bank side. Dynamic inconsistency is a situation where a player's best plan for future periods will not be optimal when the future periods arrive. In my model, the bank wants to commit to not cut credit lines even if in bad times and under the pressure of massive drawdowns. If this commitment were credible, it would stop the borrowers from drawing down in the first place. However, the bank might not be able to commit its future self to the plan because if the borrowers do in fact tapping their lines aggressively, the bank's future self would respond to it actively, instead of sticking to the plan.

Because of this dynamic inconsistency problem, the Markov perfect equilibrium defined above is not constrained efficient. In the rest of this section, I first analyze the feedback effects that lead to this inefficiency and then consider a model in which the bank has commitment power in section 4.5.

4.4 A Positive Feedback Loop

In this subsection, I demonstrate that a positive feedback loop emerges and exacerbates financial fragility, as a result of strategic complementarities between the bank and the borrowers. In particular, I analyze the impulse responses to a negative bank liquidity shock.

A negative shock to z_t increases the probability of bank failure and, in turn, the cost of raising deposits. Because the shock is persistent, it is also more likely that the cost will also be high in the future. Overall, this negative shock leads the bank to deleverage and ration liquidity by controlling the next-period total credit Φ'. In the following, I discuss how this negative shock and the resulting liquidity contraction is amplified through the feedback loop.

As demonstrated in Proposition 1, A reduction in total credit Φ' affects drawdown decisions through two channels. First, there is a short-run effect working through the probability of credit line being cut q. A reduction in total credit increases the probability and leads to more preemptive drawdowns. There is also a subtle long-run effect through borrowers' continuation value $\hat{W}$. A reduction in total credit decreases bank leverage. Looking forward the bank becomes healthier and the credit lines are more secure. Thus, the continuation value of having a credit line increases, which in turn induces borrowers to run.

The incentive to run of each borrower results in an increase in total drawdowns. In particular, Proposition 2 considers how more credit line cuts impact the selected stable equilibrium of the coordination stage-game among borrowers. The condition that $\hat{W}'$ increases in credit line cuts guarantees that the long-run effect is in the right direction. In most cases, the short-run effect would dominate; hence, that condition is not essential.

Proposition 2 *If $\hat{W}'$ increases in credit line cuts, borrowers' drawdown L increases in the amount of credit lines being cut at the equilibrium.*

Borrower runs, in turn, may impose further deleveraging pressure on the bank. The bank's trade-off lies between the adjustment cost and the benefit of deleveraging, but how would the bank react to an increase in drawdowns? To facilitate discussion, I first present the normalized bank problem.

Assumption 1 *Both cost functions $c(D, \Phi)$ and $g(\Phi', \Phi)$ are homogeneous of degree one. Moreover, both functions are convex in their first argument.*

Lemma 1 *Under Assumption 1, bank value function is homogeneous of degree one in equity E and total credit Φ. It can be normalized by Φ as*

$$V(E, \Phi, z) = \Phi\, v(e, z),$$

where e denotes the ratio of equity to total credit. In addition, borrowers' value function can be simplified to

$$W(E, C, z; \lambda_i, \kappa_i) = w(e, z; \lambda_i, \kappa_i).$$

By normalization, essentially the bank is divided into multiple identical banks, each with a total credit of 1. From the borrowers' perspective, since bank strategies are invariant to normalization, having a credit line from one of these identical small banks is the same as having a credit line from the original bank. Therefore, borrowers' value function is homogeneous of degree zero in E and Φ.

When the total drawdowns shoot up, both the cost and the benefit of deleveraging increase. Proposition 3 provides sufficient conditions under which the increase in benefit outweighs the increase in cost. It follows that the bank reduces next-period total credit when drawdowns increase.

Proposition 3 *If bank profit $\Pi(E, \Phi, z)$ is concave in E and $\partial \Pi(E, \Phi, z)/\partial E > -1$ given the borrower strategy $L(E, \Phi, z)$, then (i) the normalized value function v is increasing and strictly concave in equity ratio e, and (ii) the next-period total credit Φ' decreases in drawdowns whenever the marginal non-interest expenses exceed the profit margin.*

Whether the conditions are satisfied depends on the state variables. For example, the marginal non-interest expenses exceeds the profit margin when the bank's leverage is high and, in particular, when the bank is hit by a negative shock after staying in the good state for a long time.

To summarize, after a negative shock to z_t the bank reduces total credit. This reduction induces the first wave of borrower runs. In turn, the preemptive drawdowns push the bank to reduce total credit further, which leads to the second wave of runs. This process repeats and becomes a downward spiral that amplifies the impact of the initial shock. The spiral may lead to multiple equilibria if the strategic complementarity is strong enough; however, it turns out that this is not the case given the calibrated parameters.

4.5 A Model with Commitment

In this subsection I consider an alternative model in which the bank can commit to its plan. The main difference of this setting to the benchmark model is that now the bank can influence borrower decisions through the promised bank choices.

In the benchmark model, borrowers make decisions based on their beliefs on the bank's choices, which are determined at the equilibrium as a function of the state variables. In contrast, if the bank can commit to its policy, borrowers would take the promised plan directly into account. Therefore, both borrowers' value function and strategy depend on the promised next-period total credit directly. In particular, the total amount of drawdowns in the model with commitment is given by $L(E, \Phi, z; \Phi')$ instead of $L(E, \Phi, z)$. Taking this into consideration, the bank solves the following problem,

$$V(E, \Phi, z) = \max_{\Phi' \geq (1-\delta)L} \beta\pi(1-p)E' + \beta(1-\pi)(1-p)\mathbb{E}_{z'|z}V(E', \Phi', z'), \tag{13}$$

$$\text{s.t.} \quad D = L(E, \Phi, z;\, \Phi') - E$$

$$E' = E + \Pi - f(\Phi', \Phi).$$

This bank problem is the same as the bank problem in 5, except that the total amount of drawdowns is determined differently and, in particular, depends on the bank's choice of next-period total credit Φ'. Formally, a Markov perfect equilibrium in this model with commitment is defined as follows.

Definition 2 *A Markov perfect equilibrium of the model in which the bank can commit is a pair of value functions* $(V : \mathcal{S} \to \mathbb{R}^+, W : \mathcal{S} \times \mathcal{H} \times \mathbb{R} \to \mathbb{R}^+)$ *and strategies* $(\sigma^B : \mathcal{S} \to \mathbb{R}, \sigma : \mathcal{S} \times \mathcal{H} \times \mathbb{R} \to \{0, 1\})$ *such that*

1. *given the policy functions,* (V, W) *solve the Bellman equations 10 and 13;*
2. *for any borrower,* $\hat{\sigma} = \sigma$ *is optimal given* W, σ^B, *and that all other borrowers follow* σ*;*
3. *given* σ *and* V, σ^B *is optimal for the bank.*

Since the bank can promise to its plan and not react to drawdowns, the feedback loop between the bank and the borrowers collapses. Thus, comparing the benchmark model and the model with commitment allows us to tease out the mechanism and better understand the danger of lack of commitment.

Proposition 4 *Assume that drawdowns* $L(E, \Phi, z; \Phi')$ *is a smooth function. If the bank can commit, (i) it cuts fewer credit lines whenever the marginal non-interest expenses exceeds the profit margin, (ii) there are less preemptive drawdowns, and (iii) both the bank and the borrowers are better off.*

The bank cuts fewer credit lines after internalizing the indirect cost coming from credit line runs, as indicated by the first-order condition given by

$$\left[\pi + (1-\pi)\frac{\partial \mathbb{E} V(E', \Phi', z')}{\partial E'}\right]\left[\frac{\partial E'}{\partial \Phi'} + \underbrace{\frac{\partial E'}{\partial L}\frac{\partial L}{\partial \Phi'}}_{\text{ind. cost}}\right] + \underbrace{\left[(1-\pi)\frac{\partial \mathbb{E} V(E', \Phi', z')}{\partial \Phi'}\right]}_{\text{benefit from (de)leveraging}} = 0. \quad (14)$$

The new term $\frac{\partial E'}{\partial L}\frac{\partial L}{\partial \Phi'}$ represents the indirect cost from borrower drawdown decisions L as a response to the bank's choice of Φ', which is absent in the first-order condition 6. The bank is better off because promising the strategy at the equilibrium of the benchmark model is still feasible. Borrowers are better off because their credit lines are more secure.

5 Numerical Results

5.1 Calibration

I fit the model to U.S. commercial bank data during the 1993-2009 period. The main data source is the bank-level data from the Consolidated Report of Condition and Income (Call Reports). It contains balance sheet information of all U.S. commercial banks at the quarterly frequency. Following the literature, I aggregate the bank-level data to bank holding company level because these ownership ties could foster liquidity sharing across subsidiaries. I drop banks with asset growth greater than 10% during a quarter to mitigate the effect of large mergers and winsorize all variables at 5th and 95th percentile to control the outliers. Also, I focus on large banks with total assets more than 10 billion dollars.

I also use loan-level information from Dealscan dataset, which covers the syndicated corporate loan market in the United States and contains detailed information on credit line issuance.

Parametrization. The prices are exogenously determined and depend on the bank liquidity shock z_t. The usage fee is given by

$$r_t^l = r_t + r_x = r(z_t) + r_x, \tag{15}$$

where r denotes the base rate and r_x is the constant premium of usage fee.

I parametrize the non-interest expenses as follows,

$$c(D_t, \Phi_t) = \gamma D_t^2 / \Phi_t. \tag{16}$$

This quadratic cost introduces curvature into the model and also leads to well defined bank leverage. The adjustment cost is also quadratic given by

$$g(\Phi_{t+1}, \Phi_t) = \mu \Delta_t^2 / \Phi_t, \tag{17}$$

where $\Delta_t = \Phi_{t+1} - (1-\delta)\Phi_t$ from Equation 3. It generates gradual adjustments of total credit and hence gives the model enough flexibility to match the growth of total credit in the sample.

The stochastic process for the return of preemptive drawdown $\Omega(\cdot)$ is simply taken to be the uniform distribution on $[\underline{\kappa}, \overline{\kappa}]$. $\overline{\kappa}$ is normalized to equal to $r^l(z_b)$, such that the highest return cancels with the usage fee in bad times. Given the assumption of uniform distribution, there are two implications. First, the sensitivity of borrowers' response to bank choices depends on the density function, which in turn is determined by the lower bound $\underline{\kappa}$. However, $\underline{\kappa}$ is not directly relevant because borrowers with low returns choose not to run. Second, the stage game among borrowers admits at most two stable equilibria given the bank's strategy. Moreover, if there are two equilibria, one of them corresponds to runs by more than a half of borrowers without liquidity shock, and the other corresponds to runs by less than a half of those borrowers. I select the second equilibrium because in the data I only observe runs by a small fraction of borrowers.

Calibration. A model period is set to be one quarter. I reduce the processes for z_t to a two-state Markov process that $z_t \in \{z_g, z_b\}$. To calibrate the stochastic process $F(z', z)$, I use the NBER recession dates and match the average duration of recessions of 6 quarters and the average time periods between two recessions of 10 years.

I then divide the sample into two subsamples before and after the third quarter of 2008. I map the periods without bank liquidity shock (the good state) in the simulated data to the subsample before 2008Q3, and the bad state to the subsample after 2008Q3.

I calibrate r_t using the average LIBOR rate. The maintenance fee r^ϕ and the premium of usage fee r^l over base rate r_t are set to the corresponding average levels in the Dealscan dataset. Borrower's return R is calibrated to the average profit to liability ratio of U.S. non-financial corporate sector. The loss of continuation values upon exits is taken to be 40% as a normalization. An alternative loss fraction leads to different calibrated parameters, but would not change the quantitative results in any significant way.

The probability of borrower liquidity shock Λ_t in the good state is estimated as the

Table 2: Model calibration

Parameter		Value	Targets
Transition probability	π_{gg}	0.96	NBER data
Transition probability	π_{bb}	0.81	NBER data
Funding cost (annual)	$r(z_g)$	0.03	Avg. LIBOR pre-08Q3
Funding cost (annual)	$r(z_b)$	0.01	Avg. LIBOR post-08Q3
Usage fee premium (annual)	r_x	224	Avg. usage fee premium (bps)
Maintenance fee (annual)	r^{ϕ}	37	Avg. maintenance fee (bps)
Bank failure prob. (annual)	$p(z_b)$	0.02	Avg. CDS spread post-08Q3
Liquidity shock	$\Lambda(z_g)$	0.725	Loan-to-credit ratio pre-08Q3
Liquidity shock	$\Lambda(z_b)$	0.74	Loan-to-credit ratio
Discount rate	β	0.99	Conventional value
Prob. of Maturing	δ	0.005	Dealscan
Bank exit rate	π	0.05	Gertler and Kiyotaki (2015)
Min. return on preemptive drawdowns	$\underline{\kappa}$	-0.072	Loan to credit ratio post-08Q3
Adj. cost	μ	0.156	Total credit growth
Non-interest Expenses	γ	0.006	Leverage pre-08Q3

Note: $\Lambda(z_b)$ is computed as the average of loan-to-credit ratio after 2008Q3 of banks with liquidity ratios above the median.

average loan to total credit ratio before 2008Q3, whereas calibrating Λ_t in the bad state requires extra care. During crises, loans may increase because of an increase in the probability of borrower liquidity shock, as well as preemptive drawdowns. To disentangle these two effects, ideally one would compare two banks with different probabilities of failure, but lending to identical borrowers, which requires a sample of firms with multiple simultaneous credit lines held separately at different banks. In the United States, however, most firms borrow from multiple banks through the syndicated loan market. Drawdowns of syndicated credit lines are distributed among the banks according to their shares in the syndication so that firms cannot draw down from a particular bank only.

Because of this data limitation, I instead look at the loan-to-credit ratio of "healthy" banks based on the following criteria: (i) banks with liquidity ratios above the median, (ii) banks during the 2001 recession, (iii) banks that co-syndicated credit lines with Lehman but with exposures below the first quartile. These measures are chosen conservatively, yet even based these measures, less than a half of the additional drawdowns after 2008Q3 is due to an increase in Λ_t.

On the bank side, I set the probability of exit $\pi = 5\%$ following Gertler and Kiyotaki (2015). The probability of maturing δ is set to match the net new credit issuance between 2008Q3 and 2009Q4 using the loan origination data from Dealscan. More precisely, it is the effective probability that a credit line matures without being refinanced.

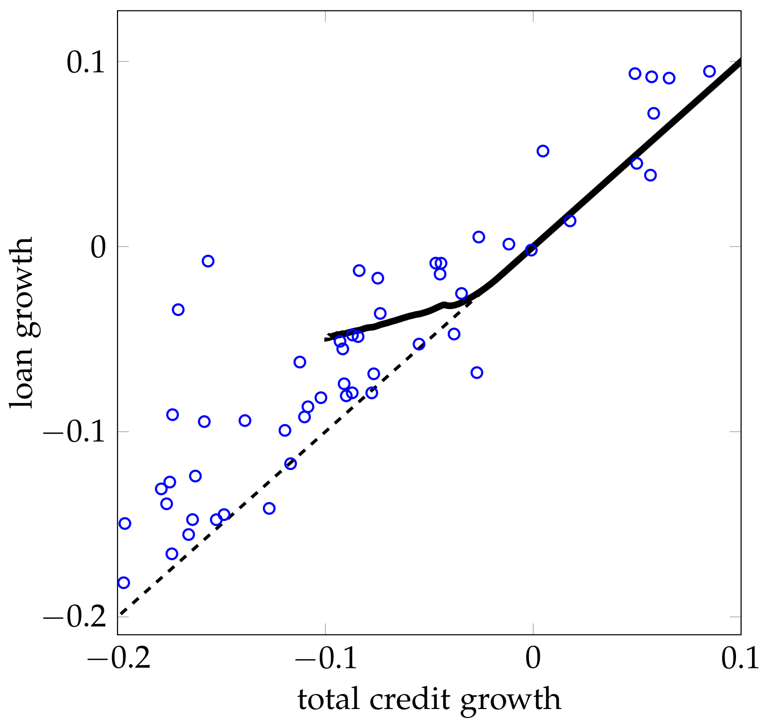
We are left with three parameters $\{\underline{\kappa}, \mu, \gamma\}$. I calibrate these parameters by matching the following moments. The average growth rate of total credit after 2008Q3 (-1.75%), the average equity to total credit ratio (12.2%), and the average loan to total credit ratio after 2008Q3 (76.2%). Table 2 shows the calibrated parameters, and Table 3 provides the moments generated by the model relative to the data.

Table 3: Calibration targets

Panel A: Empirical Targets		
Statistic (%)	Data	Model
Avg. total credit growth post-2008Q3	-1.75	-1.75
Avg. bank equity ratio pre-2008Q3	11.2	11.2
Avg. Loan-to-credit ratio post-2008Q3	76.3	76.3
Panel B: Unmatched Moments		
Avg. total credit growth pre-2008Q3	2.06	2.20
Avg. bank equity ratio post-2008Q3	14.2	14.5

Model Fit. The first and second columns of Table 3 show that the model has good in-sample fit. It is important to stress that although the parameters are jointly calibrated to match all the targets there is a strong one-to-one link between the targets and the parameters. For example, the average bank equity ratio is mainly determined by the quadratic non-interest cost. Furthermore, $\overline{\kappa}$ controls how sensitive borrowers respond to bank policies, thus impacts the loan-to-credit ratio directly.

Figure 6: Growth of Loans and Total Credit

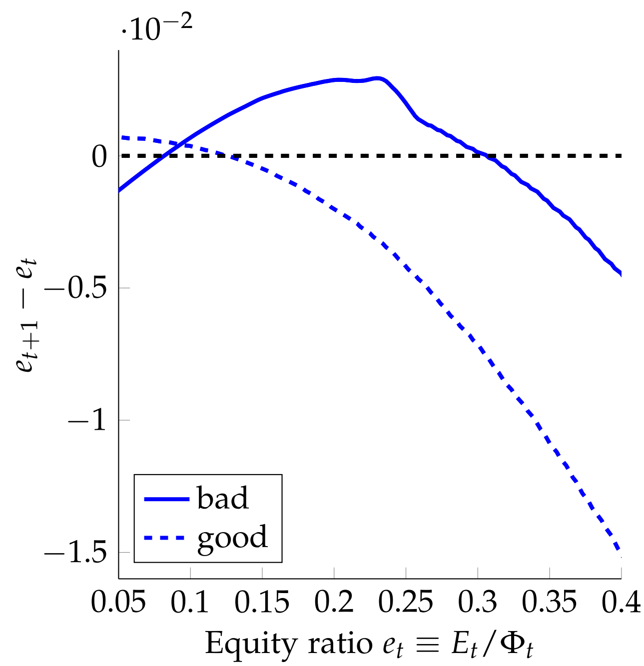


Note: Dashed line is the 45° line. The blue empty circles are from the data as shown in Figure 4. The black solid line represents the simulated data. Source: Consolidated Report of Condition and Income.

The model is calibrated to match the average statistics across time, but it also does a reasonable job to capture the cross-sectional patterns. In particular, the model can generate similar patterns presented in Figure 4. I simulate a panel of banks from the model starting from a good state and with different leverage ratios. A negative bank liquidity shock hits all banks at the beginning of the second period and lasts for six periods. The solid line in Figure 6 reports the relation between the growth of loans and the growth of total credit in the first four quarters calculated from the simulated data. The relation between the two growth rates is nonlinear. Banks with enough equity have little incentive to deleverage; therefore, borrowers have no reason to run, and the growth rates are identical in the model. Whereas banks with low equity ratio choose to ration liquidity, which induces borrower runs. As a result of preemptive drawdowns, loans decline less relative to the total credit.

Although the model is able to capture the non-linearity, it cannot, nor is designed to match the cross-section distribution of bank statistics, given the only heterogeneity in equity ratio. For instance, the minimum total credit growth is -10% in the simulated data, much higher than that in the sample. Additional heterogeneities are required to match the cross-section distributions, which is out of the scope of this paper.

Figure 7: Law of Motion of equity ratio

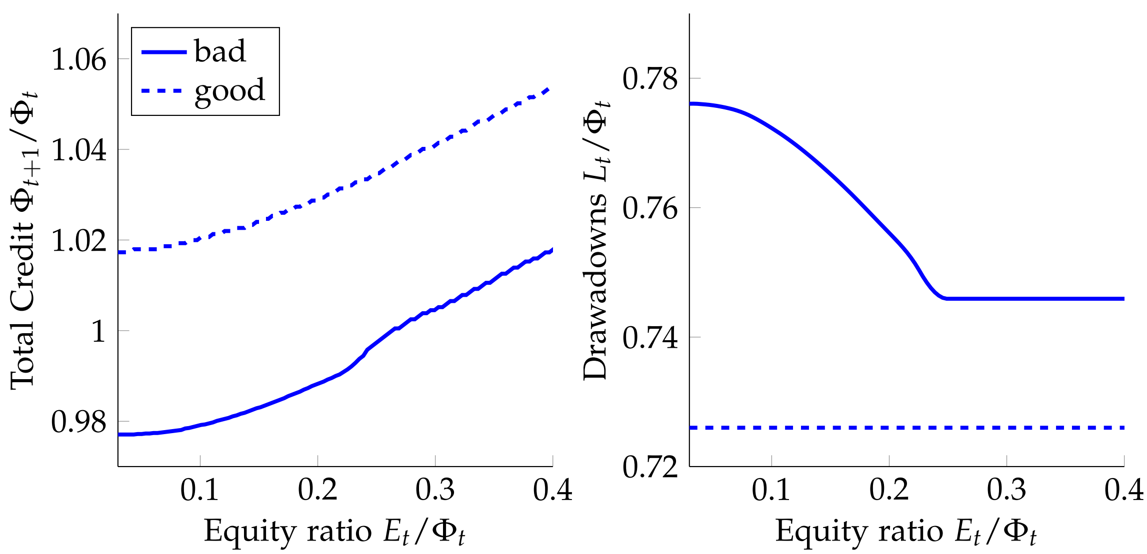


5.2 Equilibrium Decision Rules

For the parameter values in Table 2, I find an equilibrium where borrowers run along the equilibrium path. To understand the equilibrium, I first describe the law of motion of the state variables, and Lemma 1 allows us to focus on the equity ratio e_t as the only endogenous state variable. Figure 7 shows the law of motion of bank equity ratio. For disposition purpose, I plot the change of bank equity ratio $e_{t+1} - e_t$ against e_t in both the good and the bad state.

The figure shows that equity ratio converges to the optimal level 0.112 if the bank stays in the good state for a long time. Equity ratio increase when below this level, and decreases when above. Above the optimal level, a negative liquidity shock leads to an increase in equity ratio. Thus, bank leverage is pro-cyclical; that is, the equity ratio is higher in state z_b than in z_g. When the equity ratio is too low, it further decreases in the bad state because the loss in equity dominates bank deleveraging. This last case, however, is only relevant if the bank starts with an initial equity ratio below the optimal level 0.112.

Figure 8: Policy Functions



Next, I turn to characterizing the strategies. The left panel in Figure 8 shows the bank's optimal choice of next-period total credit as a function of the equity ratio. In good times the bank always expands, whereas in bad times the bank reduces liquidity provision when it does not have enough equity buffer. Borrower drawdown decisions are illustrated in the right panel of Figure 8. Borrowers run in the bad state when the equity ratio is low, as a response to the anticipated liquidity rationing. A time-series implication of the strategies is that after a negative liquidity shock bank leverage shoots up immediately, and then decreases gradually if that shock persists.

6 Counterfactuals

In this section, I conduct three counterfactual experiments. First, I solve the alternative model in which the bank can commit with the calibrated parameters. Comparing the equilibrium of the alternative model with that of the benchmark model allows us to quantify the amplification mechanism. Then I use the calibrated benchmark model to study two policy interventions: (i) the effects of bank leverage ratio requirements on borrower runs and liquidity rationing, and (ii) the impact of a commitment tax on credit line cuts.

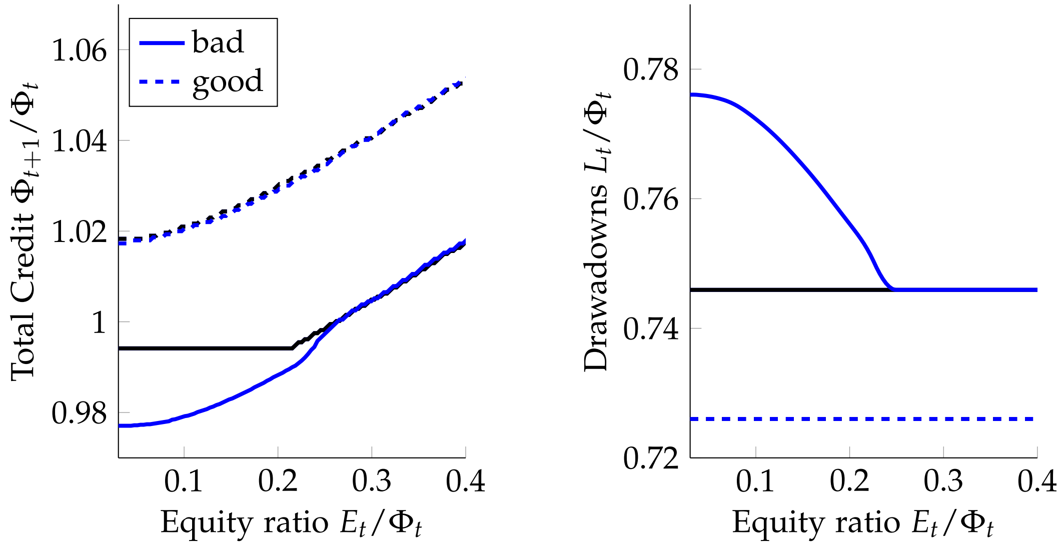
6.1 Commitment

If the bank can credibly commit to the plan, it would not respond to drawdown decisions ex-post. Therefore the feedback loop between the bank and the borrowers collapses. Figure 9 presents a comparison of the strategies for the bank and the borrowers in the benchmark model (blue lines) and the model with bank commitment (black lines).

In the model with commitment, the bank internalizes the indirect cost through the feedback loop and chooses not to cut credit lines at all. Also, because the bank can control the borrower run risk more effectively with credible commitment in the bad state, it also grows faster by issuing more new credit lines in the good state. On the borrower side, credit line drawdowns are entirely determined by borrowers' liquidity shocks, and there is no run on credit lines.

Table 4 compares total credit growth and welfares. Total credit growth is defined in the same way as in Section 5.1. For welfare comparison, I compute measures based on the value functions in the benchmark model and the counterfactuals. I use the stationary distribution of equity ratio to calculate the expected value of the borrows. However, it is not feasible to calculate the bank value in the same way. Although the normalized bank

Figure 9: Policy Functions with and without Commitment



problem is stationary, the original bank problem is not. Therefore, instead, I calculate the value of a bank with 1 unit of equity and at the steady state level of equity ratio.[5] In the model with commitment, the average total credit growth is 2.21% in the good state, higher than its 2.20% counterpart in the benchmark model. The bank also reduces less total credit in the bad state (-0.6% versus -1.75%). Regarding welfare, the bank's value increases by 0.1% if the bank is able to commit. At the same time, the value of borrowers also raises by 0.4%. From a social point of view, there is also an additional welfare gain of commitment as more new credit line borrowers entering on the average when the bank grows faster.

6.2 Leverage Ratio Requirements

Basel III framework introduced a simple, transparent, non-risk based leverage ratio as a credible measure that complements the risk-based capital requirements. In particular, off-balance sheet items, such as unused credit lines, are included in this measure at 10 to 100% conversion factors. In this subsection I ask the question, how much does a leverage ratio requirement affect credit line runs?

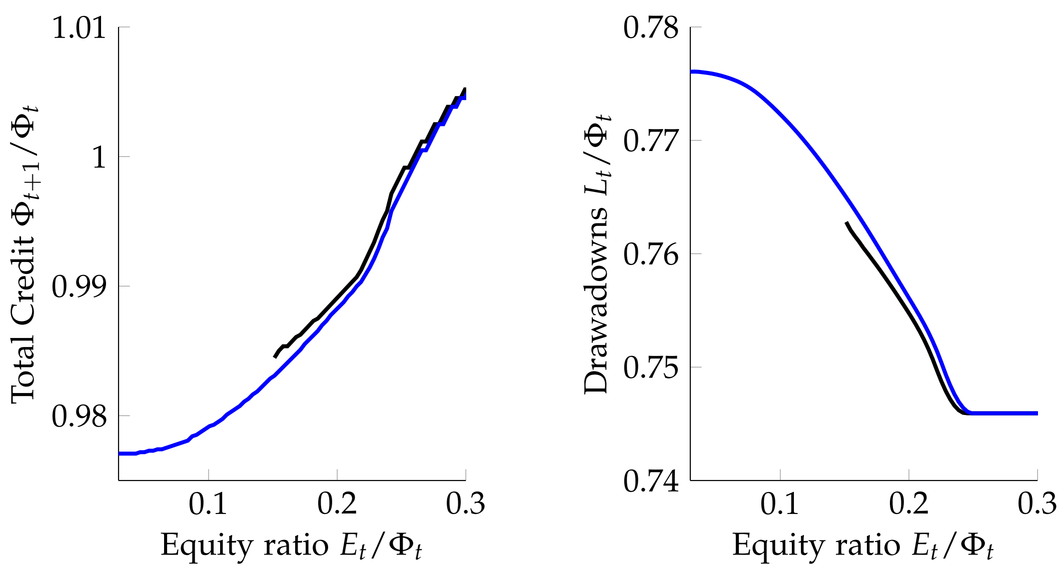
[5]The steady state level is defined as the level of equity ratio after a long history of $z = z_g$.

Table 4: Counterfactuals

	Benchmark	Commitment
Avg. Total Credit Growth in z_b (%)	-1.75	-0.60
Avg. Total Credit Growth in z_g (%)	2.20	2.21
Bank value	1.18	+0.1%
Borrower value	0.26	+0.4%

Specifically, I investigate the effects of a leverage ratio requirement with a 100% conversion factor for unused credit lines. Since in my model this leverage measure and the equity ratio are reciprocals of each other, it is the same to work with the equity ratio. In particular, I impose a lower bound of 15% on the equity ratio.

Figure 10: Leverage ratio Requirements



Note: Only the policy functions in the bad state is plotted to make the difference visible. The black lines plot the policy functions if the leverage ratio requirement is imposed. The blue lines plot the policy functions of the benchmark model.

In the benchmark model, the average equity ratio in the good state is 11.2%, and thus the 15% requirement constraint binds and induces the bank to build more liquidity buffer. Figure 10 presents the results of this counterfactual. In the good state, the bank issues less new credit lines with the requirement, especially when its equity ratio is close to the constraint. At the same time, the bank cuts fewer credit lines in the state z_b. This comparison captures the stability-growth tradeoff of quantity requirements.

The traditional rationale for quantity requirements is that individual banks do not take into account the impact of their own leverage decisions on the vulnerability of the system as a whole. Although my model is ready to be extended to incorporate the general equilibrium effects, this rationale does not present in my model. Instead, the benefit of imposing leverage ratio requirements comes from two aspects. It forces the bank to keep more dry powder against its liquidity shock. There is an additional consideration due to the link between liquidity rationing and borrower runs. In particular, the leverage ratio requirement dampens the amplification mechanism and mitigates runs, as shown in the right panel of Figure 10.

6.3 A Commitment Tax on Revocation

My model also suggests that a commitment tax on cutting credit lines would be efficient to reduce vulnerability. The bank must pay a tax of τ on the amount of reduction in total credit due to credit line cuts.

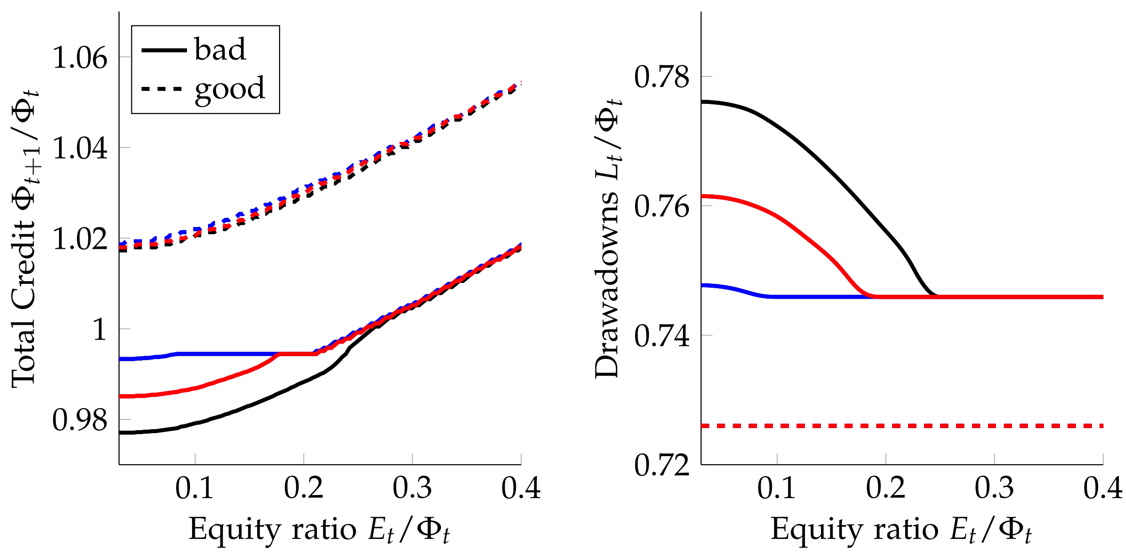
From the bank's perspective, the tax works in the same way as the adjustment cost in discouraging credit line cuts. Now the marginal cost consists of two component,

$$\frac{\partial E'}{\partial \Phi'} = \tau \mathbb{1}[\Delta < 0] + 2\mu|\Delta|. \tag{18}$$

When the tax rate is high enough, the linear cost from taxation alone can deter the bank from cutting credit lines. In Figure 11, I plot the policy functions with different levels of taxes. The commitment tax is effective in dampening the amplification and controlling runs. In particular, a tax of 0.4% is sufficient to eliminate credit line runs.

Commitment taxes are designed to discourage undesirable activities, such as pollution externalities. Recently, Cochrane (2014) suggest a tax on debt to control excessive run-prone liabilities. In a similar spirit, my model proposes a tax on cutting credit lines to mitigate credit line runs. A new insight from the current framework is that the tax provides an efficient commitment device to the bank. Borrowers understand ahead of time that the bank is going to cut fewer credit lines because of the tax. Hence they refrain from preemptive drawdowns. It is also important to note that the tax is a countercyclical policy by design. It reduces run risk in bad times; but, unlike the quantity requirements, it does not curb credit growth in good times.

Figure 11: Tax on Cutting Credit Lines ($\tau = 0$, 0.2%, 0.4%)



In practice, setting the tax rate is challenging. First, the tax rate should respond to general economic conditions and bank liquidity shocks promptly. Second, the tax rate should also depend on bank balance sheet information, which requires effective supervision and information disclosure. Therefore, policy makers have an advantage in designing the tax, and it is much more difficult to achieve the same goal by private contracting.

Extension: Endogenous Probability of Bank Failure. To be added here.

7 Concluding Remarks

I have developed a dynamic model that integrates credit line runs with liquidity rationing. I illustrated how introducing bank liquidity management into bank run models is important for characterizing banking instability. In particular, there is a new strategic complementarity between the liquidity-rationing bank and run-prone credit line borrowers, which leads to a feedback loop that amplifies underlying distress. I also demonstrated the lack of commitment problem behind the feedback effects and proposed a commitment tax on cutting credit line to control run risk.

As discussed in the empirical studies, such as Jiménez, Lopez, and Saurina (2009) and Ippolito, Peydro, Polo, and Sette (2015), the usage and availability of credit lines are affected by both bank characteristics and firm characteristics, and hence bank-firm level data is required to disentangle supply and demand effects. Unfortunately, the call report data does not allow me to include other borrower heterogeneities besides their drawdown decisions. Further investigations with a richer modeling of credit line borrowers would be important. It is also worth to explore general equilibrium effects in the framework. I have focused on one bank, but the model is ready to be extended to study the banking system. Finally, my framework can be used to address other dynamic issues with strategic complementarities, such as fund redemption restrictions and partial defaults on sovereign debt.

References

Acharya, V., H. Almeida, F. Ippolito, and A. Perez (2014a): "Credit lines as monitored liquidity insurance: Theory and evidence," *Journal of Financial Economics*, 112(3), 287–319.

——— (2016): "A new perspective on bank-dependency: The liquidity insurance channel," *Working Paper*.

Acharya, V. V., H. Almeida, F. Ippolito, and A. Perez (2014b): "Bank lines of credit as contingent liquidity: A study of covenant violations and their implications," *ECB Working Paper*.

Acharya, V. V., and N. Mora (2015): "A crisis of banks as liquidity providers," *The Journal of Finance*, 70(1), 1–43.

Allen, F., and D. Gale (1998): "Optimal financial crises," *The Journal of Finance*, 53(4), 1245–1284.

Boot, A. W., S. I. Greenbaum, and A. V. Thakor (1993): "Reputation and discretion in financial contracting," *The American Economic Review*, pp. 1165–1183.

Bryant, J. (1980): "A model of reserves, bank runs, and deposit insurance," *Journal of banking & finance*, 4(4), 335–344.

Campello, M., J. R. Graham, and C. R. Harvey (2010): "The real effects of financial constraints: Evidence from a financial crisis," *Journal of Financial Economics*, 97(3), 470–487.

Chaderina, M., and A. Tengulov (2015): "Discretion and Systemic Risk in Credit Line Contracts: Theory and Evidence," *Available at SSRN 2617223*.

Cheng, H., and K. Milbradt (2012): "The hazards of debt: Rollover freezes, incentives, and bailouts," *Review of Financial Studies*, 25(4), 1070–1110.

Cipriani, M., A. Martin, P. E. McCabe, and B. M. Parigi (2014): "Gates, fees, and preemptive runs," .

Cochrane, J. H. (2014): "Toward a Run-free Financial System," *Across the Great Divide: New Perspectives on the Financial Crisis*, p. 197.

Cooper, R., and T. W. Ross (1998): "Bank runs: Liquidity costs and investment distortions," *Journal of Monetary Economics*, 41(1), 27–38.

Corbae, D., and P. D'Erasmo (2014): "Capital requirements in a quantitative model of banking industry dynamics," .

Diamond, D. W., and P. H. Dybvig (1983): "Bank runs, deposit insurance, and liquidity," *Journal of Political Economy*, pp. 401–419.

EGAN, M., A. HORTAÇSU, AND G. MATVOS (2015): "Deposit competition and financial fragility: Evidence from the us banking sector," *Fama-Miller Working Paper*, pp. 15–33.

ENGINEER, M. (1989): "Bank runs and the suspension of deposit convertibility," *Journal of monetary Economics*, 24(3), 443–454.

ENNIS, H. M., AND T. KEISTER (2009): "Bank runs and institutions: The perils of intervention," *The American Economic Review*, 99(4), 1588–1607.

——— (2010): "Banking panics and policy responses," *Journal of Monetary Economics*, 57(4), 404–419.

GERTLER, M., AND N. KIYOTAKI (2015): "Banking, liquidity, and bank runs in an infinite horizon economy," *The American Economic Review*, 105(7), 2011–2043.

HE, Z., AND W. XIONG (2012): "Dynamic debt runs," *Review of Financial Studies*, 25(6), 1799–1843.

HOLMSTROM, B. R., AND J. TIROLE (1998): "Private and Public Supply of Liquidity," *Journal of Political Economy*, 106(1), 1–40.

IPPOLITO, F., J.-L. PEYDRO, A. POLO, AND E. SETTE (2015): "Double Bank Runs and Liquidity Risk Management," *Journal of Financial Economics (JFE), Forthcoming*.

IVASHINA, V., AND D. SCHARFSTEIN (2010): "Bank lending during the financial crisis of 2008," *Journal of Financial economics*, 97(3), 319–338.

JIMÉNEZ, G., J. A. LOPEZ, AND J. SAURINA (2009): "Empirical analysis of corporate credit lines," *Review of Financial Studies*, 22(12), 5069–5098.

KASHYAP, A. K., R. RAJAN, AND J. C. STEIN (2002): "Banks as Liquidity Providers: An Explanation for the Coexistence of Lending and Deposit-Taking," *The Journal of Finance*, pp. 33–73.

MANKART, J., A. MICHAELIDES, AND S. PAGRATIS (2014): "A Dynamic Model of Banking with Uninsurable Risks and Regulatory Constraints," .

PECK, J., AND K. SHELL (2003): "Equilibrium bank runs," *Journal of political Economy*, 111(1), 103–123.

ROBERTS, M. R., AND A. SUFI (2009): "Control rights and capital structure: An empirical investigation," *The Journal of Finance*, 64(4), 1657–1695.

ROCHET, J.-C., AND X. VIVES (2004): "Coordination failures and the lender of last resort: was Bagehot right after all?," *Journal of the European Economic Association*, 2(6), 1116–1147.

SCHROTH, E., G. A. SUAREZ, AND L. A. TAYLOR (2014): "Dynamic debt runs and financial fragility: Evidence from the 2007 ABCP crisis," *Journal of Financial Economics*, 112(2), 164–189.

Sufi, A. (2009): "Bank lines of credit in corporate finance: An empirical analysis," *Review of Financial Studies*, 22(3), 1057–1088.

Vives, X. (2014): "Strategic complementarity, fragility, and regulation," *Review of Financial Studies*, 27(12), 3547–3592.

Zeng, Y. (2016): "A Dynamic Theory of Mutual Fund Runs and Liquidity Management," .

A Proofs

Proof of Proposition 2. Given Equations 8, 11, and 12, the total drawdowns L at the equilibrium given the bank's strategy can be solved from the following equations.

$$\kappa^* = r^l - \beta \left[(1-p)\eta \hat{W}' \frac{\Delta^-}{\Phi - L} + p\eta(1-\delta)W_0 \right] \tag{19}$$

$$L = \hat{L} \equiv \left[\Lambda + (1-\Lambda)(1-\Omega(\kappa^*)) \right] \Phi, \tag{20}$$

where $\Delta^- \equiv -\min\{\Delta, 0\}$. Since the equilibrium is stable, we have $\frac{\partial \hat{L}}{\partial \kappa^*}\frac{\partial \kappa^*}{\partial L} < 1$. Thus, the equilibrium drawdown L increases in Δ^-.

Proof of Lemma 1. Assumption 1 assumes that both $c(D,\Phi)$ and $f(\Phi',\Phi)$ are homogeneous of degress one. Therefore, by an abuse of notation we have $c(D,\Phi) = \Phi\, c(D/\Phi, 1) = \Phi\, c(D/\Phi)$ and $f(\Phi',\Phi) = \Phi\, f(\Phi'/\Phi, 1) = \Phi\, f(\Phi'/\Phi)$. Also, drawdowns L is linear in Φ as in Equation 12.

Dividing both sides of the original bellman equation 5 by Φ, we have

$$\begin{aligned} v(e,z) = & \max_{\phi' \geq (1-\delta)l} \beta\pi(1-p)\hat{e} + \beta(1-\pi)(1-p)\phi' \mathbb{E}[v(\hat{e}/\phi', z')] \\ \text{s.t.} \quad & \hat{e} = e + r^\phi + r^l l - (r+p)(l-e) - c(l-e) - f(\phi') \end{aligned} \tag{21}$$

where $e \equiv \frac{E}{\Phi}, \hat{e} \equiv \frac{E'}{\Phi}, \phi' \equiv \frac{\Phi'}{\Phi}$, and $l \equiv \frac{L}{\Phi}$.

Proof of Proposition 3. I first show that the normalized value function $v(e,z)$ is unique, increasing, and concave in e given the borrower strategy. Then I consider how bank optimal policy depends on drawdowns.

Lemma 2 *Assume that there is an upper bound $\overline{\phi}$ of total credit growth ϕ'. If $\overline{\phi}\beta(1-\pi) \leq 1$, the bank value function $v(e,z)$ is unique given borrower strategy σ.*

Proof. Apply Contraction Mapping Theorem to the bank problem 21, and check the Blackwell's sufficient conditions.

First, assume that there is a function $u(e,z) \geq v(e,z)$ for all $v(e,z)$. Let ϕ'_v denote the optimal policy with V.

$$
\begin{aligned}
T[v(e,z)] &= \max_{\phi'} \beta\pi(1-p)\hat{e} + \beta(1-\pi)(1-p)\phi' \,\mathbb{E}[v(\hat{e}/\phi', z')] \\
&\leq \beta\pi(1-p)\hat{e}(\phi'_v) + \beta(1-\pi)(1-p)\phi'_v \,\mathbb{E}[u(\hat{e}(\phi'_v)/\phi'_v, z')] \\
&\leq \max_{\phi'} \beta\pi(1-p)\hat{e} + \beta(1-\pi)(1-p)\phi' \,\mathbb{E}[u(\hat{e}/\phi', z')] = T[u(e,z)].
\end{aligned}
$$

Then we check the discounting condition given $\overline{\phi}\beta(1-\pi) \leq 1$.

$$
\begin{aligned}
T[v(e,z)+a] &= \max_{\phi'} \beta\pi(1-p)\hat{e} + \beta(1-\pi)(1-p)\phi' \,\mathbb{E}[v(\hat{e}/\phi', z') + a] \\
&\leq \max_{\phi'} \beta\pi(1-p)\hat{e} + \beta(1-\pi)(1-p)\phi' \,\mathbb{E}[v(\hat{e}/\phi', z')] + \phi'_{v+a}\beta(1-\pi)(1-p)a \\
&= T[v(e,\phi,z)] + \phi'_{v+a}\beta(1-\pi)(1-p)a.
\end{aligned}
$$

The constraint that $\overline{\phi}\beta(1-\pi) \leq 1$ can be relaxed with a variant of the Blackwell's sufficient conditions for unbounded functions. ■

Lemma 3 *If $\partial\Pi(E,\Phi,z)/\partial E > -1$, bank value function $v(e,z)$ is increasing in e.*

Proof. Suppose $e_1 < e_2$ and let ϕ'_1 denote the optimal choice associated with e_1, need to show that $T[v(e_1,z)] < T[v(e_2,z)]$ if $v(e,z)$ is increasing in e. If $\partial\Pi(E,\Phi,z)/\partial E > -1$, we have that $\hat{e}$ increases in e for the same ϕ'. Thus,

$$
\begin{aligned}
T[v(e_2,z)] &= \max_{\phi'} \beta\pi(1-p)\hat{e} + \beta(1-\pi)(1-p)\phi' \,\mathbb{E}[v(\hat{e}/\phi', z')] \\
&\geq \beta\pi(1-p)\hat{e}(\phi' = \phi'_1) + \beta(1-\pi)(1-p)\phi'_1 \,\mathbb{E}[v(\hat{e}(\phi' = \phi'_1)/\phi'_1, z')] \\
&\geq T[v(e_1,\phi,z)].
\end{aligned}
$$

■

Lemma 4 *If $f(\Phi',\Phi)$ is concave in Φ' and $\Pi(E,\Phi,z)$ is concave in E, bank value function $v(e,z)$ is concave in e.*

Proof. Need to show that the bellman equation maps concave functions into concave functions. Assume that there are two equity ratios $e_1 < e_2$ and the corresponding optimal policies are ϕ'_1 and ϕ'_2. Denote $e_0 = \eta e_1 + (1-\eta)e_2$ and $\phi'_0 = \eta\phi'_1 + (1-\eta)\phi'_2$ for $\eta \in (0,1)$, we want to show that if v is concave and increasing in e,

$$T[v(e_0,z)] \geq \eta T[v(e_1,z)] + (1-\eta)T[v(e_2,z)]. \tag{22}$$

The concavity of $f(\Phi',\Phi)$ guarantees that $\hat{e}$ is concave in ϕ'. At the same time since $\Pi(E,\Phi,z)$ is concave in E, $\hat{e}$ is also concave in e. Therefore, as e and ϕ' are separated, $\hat{e}(e_0,\phi'_0) \geq \eta\hat{e}(e_1,\phi'_1) + (1-\eta)\hat{e}(e_2,\phi'_2)$. Next, since v is increasing in e,

$$\begin{aligned}
\frac{\hat{e}(e_0,\phi'_0)}{\phi'_0} &\geq \frac{\eta\phi'_1}{\phi'_0}\frac{\hat{e}(e_1,\phi'_1)}{\phi'_1} + \frac{(1-\eta)\phi'_2}{\phi'_0}\frac{\hat{e}(e_2,\phi'_2)}{\phi'_2} \\
\Rightarrow v\left(\frac{\hat{e}(e_0,\phi'_0)}{\phi'_0},z'\right) &\geq \frac{\eta\phi'_1}{\phi'_0} v\left(\frac{\hat{e}(e_1,\phi'_1)}{\phi'_1},z'\right) + \frac{(1-\eta)\phi'_2}{\phi'_0} v\left(\frac{\hat{e}(e_2,\phi'_2)}{\phi'_2},z'\right).
\end{aligned}$$

The condition that $\Pi(E,\Phi,z)$ is concave in E is satisfied when $L(E,\Phi,z)$ is not too convex in E. ■

The above Lemmas also guarantee that the value function is continuous and differentiable. Now we are ready to prove the second part of Proposition 3. I only consider the interior case when there is a credit line run and the bank withdraws credit lines. I also assume that l is smooth so that the value function is differentiable. The first-order condition is thus given by,

$$-\left[\pi + (1-\pi)\frac{\partial\mathbb{E}[v(\hat{e}/\phi',z')]}{\partial\hat{e}}\right] f'(\phi') + (1-\pi)\left[\mathbb{E}[v(\hat{e}/\phi',z')] + \phi'\frac{\partial\mathbb{E}[v(\hat{e}/\phi',z')]}{\partial\phi'}\right] = 0.$$

Let v' denote the first-order derivative of $v(\hat{e}/\phi',z')$ with respect to its first argument. The condition can be rewritten as

$$-\left(\pi\phi' + (1-\pi)\mathbb{E}[v']\right) f'(\phi') + (1-\pi)\left(\phi'\mathbb{E}[v] - \hat{e}\mathbb{E}[v']\right) = 0.$$

Hence we have $\hat{e} + f'(\phi') > 0$.

To derive the effect of an increase in drawdowns, I total differentiate the first-order condition at the optimal choice, which gives us,

$$\frac{d\phi'}{dl} = -\frac{(1-\pi)\left[-\frac{\partial^2\mathbb{E}[v']}{\partial(\hat{e})^2}f'(\phi') + \frac{\partial\mathbb{E}[v']}{\partial\hat{e}} + \phi'\frac{\partial^2\mathbb{E}[v']}{\partial\phi'\partial\hat{e}}\right][r^l - r - p - c'(l-e)]}{-\left[\pi + (1-\pi)\frac{\partial\mathbb{E}[v']}{\partial\hat{e}}\right]f''(\phi') + (1-\pi)\left[\frac{\partial^2\mathbb{E}[v']}{\partial(\hat{e})^2}(f'(\phi'))^2 - \frac{\partial^2\mathbb{E}[v']}{\partial\phi'\partial\hat{e}}f'(\phi') + \phi'\frac{\partial^2\mathbb{E}[v']}{\partial(\phi')^2} + \frac{\partial\mathbb{E}[v']}{\partial\phi'}\right]}.$$

First, note that the denominator is negative, since the bank solves a maximization problem. Second, the numerator can be simplified to

$$-(1-\pi)\frac{\partial^2\mathbb{E}[v']}{\partial(e')^2}\frac{f'(\phi')+\hat{e}}{\phi'}[r^l - r - p - c'(l-e)].$$

Since v is concave, when the profit margin $r^l - r - p$ is smaller than the marginal non-interest cost, i.e. $r^l - r - p - c'(l-e) < 0$, the numerator is negative as well. Hence bank choice ϕ' decreases in drawdowns l.

Proof of Proposition 4. Below I show that the bank cuts fewer credit lines when it can commit. The other results follow. I focus on the case when the bank chooses to cut credit lines in the model with commitment in the bad state. Otherwise, the Proposition is trivial. The first-order condition in the model with commitment is as follows:

$$\left[\pi + (1-\pi)\frac{\partial\mathbb{E}[v_C(e',z')]}{\phi'\partial e'}\right]\left[-f'(\phi') + \frac{\partial\hat{e}}{\partial l}\frac{\partial l}{\partial\phi'}\right] + (1-\pi)\left[\mathbb{E}[v_C(e',z')] - e'\frac{\partial\mathbb{E}[v_C(e',z')]}{\partial e'}\right] = 0.$$

Since the value functions in the two models are different, it is hard to compare the two policy functions directly. Instead, I consider a continuous change in bank's commitment power. In particular, I assume that α fraction of borrowers believe that the bank is going to stick to the plan. The comparative statics with respect to α is consistent for all $\alpha \in (0,1)$.

$$\left[\pi + (1-\pi)\frac{\partial\mathbb{E}[v_\alpha(e',z')]}{\phi'\partial e'}\right]\left[-f'(\phi') + \alpha\frac{\partial\hat{e}}{\partial l}\frac{\partial l}{\partial\phi'}\right] + (1-\pi)\left[\mathbb{E}[v_\alpha(e',z')] - e'\frac{\partial\mathbb{E}[v_\alpha(e',z')]}{\partial e'}\right] = 0.$$

When α increases, the optimal bank choice ϕ' increases. This can be shown by total differentiating the first-order condition. The numerator of $d\phi'/d\alpha$ is positive since $\frac{\partial \hat{e}}{\partial l} < 0$ and $\frac{\partial l}{\partial \phi'} < 0$. Given that the value functions are continuous, we can take α to the limits, and $\phi'_{\alpha=1} \geq \phi'_{\alpha=0}$ follows.