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## Investment, idiosyncratic risk, and growth options<sup>☆</sup>

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### ABSTRACT

We provide evidence that growth options play an important role in determining the negative relation between corporate investment and idiosyncratic risk in the absence of agency problem. A simple real options model predicts that the negative relation between corporate investment and idiosyncratic risk is a U-shaped function of the level of idiosyncratic risk: investment responds the most when idiosyncratic risk is at the intermediate level. And the negative relation is stronger when firms possess more growth options. Our results are robust when we control for the effect of managerial risk aversion, supporting the view that firms' optimal response to uncertainty is an important driving force behind the negative investment–idiosyncratic risk relation.

### 1. Introduction

Recent empirical studies document a robust negative relation between investment and firm-specific uncertainty (see, for example, [Panousi and Papanikolaou \(2012\)](#)). However, the underlying mechanism of such a negative investment–idiosyncratic risk relation remains a debatable question. One line of explanation can be derived from the real options models. When investment is irreversible and future is uncertain, the ability to delay investment is valuable. The option value of waiting is high when uncertainty is high, and firms optimally choose to postpone investment (e.g., [Bloom et al. \(2007\)](#), [Bloom \(2009\)](#), [Julio and Yook \(2012\)](#), [Kellogg \(2014\)](#), [Segal et al. \(2015\)](#), and [Gulen and Ion \(2016\)](#)). While the traditional view suggests that only systematic risk should affect investment (e.g., [Craine \(1989\)](#)), the real options models predict that it is total risk, including both systematic and idiosyncratic risks, that should matter for investment since both risks affect the value of the option to wait.

An alternative explanation is derived from the agency theory and managerial risk aversion. If risk-averse managers hold undiversified stakes of the firm due to reasons such as endogenous ownership incentive,<sup>1</sup> the idiosyncratic risk of the firm generates a wedge between the managers' and shareholders' optimal decisions, which leads to underinvestment ([Panousi and Papanikolaou, 2012](#)).

These two alternative explanations have quite different economic interpretations and policy implications. The real options theory suggests that the negative relation between investment and firm-specific uncertainty is a result of the optimal investment decision made by the firm to maximize total firm value. The managerial risk aversion explanation, however, suggests that this negative

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<sup>1</sup> See, [Himmelberg et al. \(2002\)](#), for example.

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relation between investment and idiosyncratic risk is a consequence of the agency problem and reflects a distortion of the firm's first-best investment decision. Thus it is important to understand what is the major driving force underlying the negative relation between investment and firm-specific uncertainty.

In this paper, we provide new evidence that growth options play a crucial role in generating the negative relation between investment and idiosyncratic risk. While most previous real options studies focus on the threshold effect on investment, we investigate the expected investment at the firm level. The expected investment not only depends on the threshold of investment, but also on the probability of reaching the threshold at a given period of time, as suggested by Sarkar (2000). While an increase in firm-specific uncertainty increases the threshold (and thus decreases expected investment), it also increases the probability of reaching the threshold (and thus increases expected investment). We show that these two confounding effects generate interesting and unique patterns between expected investment and the level of firm-specific uncertainty. Based on a simple real options model, we show that (1) there is a negative relation between investment and idiosyncratic risk when the level of idiosyncratic risk is above a minimum threshold and (2) the investment–idiosyncratic risk sensitivity is a U-shaped function of the level of idiosyncratic risk, which means that investment responds the most when idiosyncratic risk is at an intermediate level. Finally, the real options model predicts a composition effect: the investment–idiosyncratic risk relation is stronger for firms with more growth options.<sup>2</sup>

Following Leahy and Whited (1996), Bulan (2005), and Panousi and Papanikolaou (2012), we measure uncertainty using stock return volatility. We decompose stock return volatility into a systematic and an idiosyncratic component, and use idiosyncratic return volatility as a measure of idiosyncratic risk. Using a large panel of US firms from 1967 to 2017, we confirm our theoretical predictions derived from the real options model in the cross section. Our results remain robust after we control for managerial risk aversion, suggesting that firms' optimal decisions under uncertainty play a major role in explaining the negative investment–idiosyncratic risk relation.

Our paper contributes to the literature in several aspects. First, we provide new evidence on the investment–uncertainty relation due to real options effects. While standard real options models as in McDonald and Siegel (1986) and Dixit and Pindyck (1994) suggest a negative relation between investment and total risk, the relation may depend on various factors, including the degree of investment irreversibility,<sup>3</sup> competition,<sup>4</sup> financial constraints, and liquidity.<sup>5</sup> Empirical studies so far have had difficulty providing unambiguous evidence on the investment–uncertainty relation or offer direct support for the real options effects.<sup>6</sup> By focusing on expected investment rather than a pure threshold effect of investment, we demonstrate that the relation between investment and uncertainty has several unique features, such as the U-shaped investment–uncertainty sensitivity, which can be reconciled in a simple real options framework.

Second, we distinguish idiosyncratic risk from systematic risk and establish the importance of firms' optimal decisions in explaining the negative relation between investment and firm-specific uncertainty. Most of previous theoretical work on real options does not make distinction between idiosyncratic and systematic risks. Empirical studies exploring the predictions of these models focus on the relation between investment and total risk. Very few studies test the implications of real option models on the investment–idiosyncratic risk relation. Managerial risk aversion may also generate a negative relation between investment and idiosyncratic risk. Panousi and Papanikolaou (2012) provide evidence that the negative relation between investment and idiosyncratic risk is stronger for firms with higher insider ownership. We present a rich set of results that are better explained by the real options theory but cannot be easily explained by managerial risk aversion. Moreover, the results remain robust after we control for the effect of insider ownership. Our results emphasize the importance of firms' optimal investment decisions in generating the negative relation between investment and firm-specific risk.

The remainder of the paper proceeds as follows. Section 2 presents a simple real options model, which derives the main predictions on the relation between expected investment and idiosyncratic risk. Section 3 describes the data. Section 4 presents empirical tests of model predictions. Section 5 addresses endogeneity concerns and tests alternative explanations. Section 6 concludes the paper.

<sup>2</sup> Grullon et al. (2012) study the composition effect of real options on the relation between return and contemporaneous stock volatility. They find that the positive volatility–return relation is much stronger for firms with more real options.

<sup>3</sup> Abel et al. (1996) develop a more general framework by allowing partial investment irreversibility and suggest that while the option to expand discourages investment, the option to contract encourages investment.

<sup>4</sup> Competition may erode the real option premium. Caballero (1991) claims that the decreasing marginal return to capital due to imperfect competition and/or decreasing returns to scale is crucial for generating the negative relation between uncertainty and investment under asymmetric adjustment costs. Grenadier (2002) further shows that the threat of preemption in a strategic competition drives down the option value to wait and pushes the investment threshold close to the Marshallian present value criteria. Competition may not necessarily eliminate the option value of waiting under certain realistic assumptions. Leahy (1993), Pindyck (1993), and Caballero and Pindyck (1996) show that industry-specific shocks restore the positive opportunity costs associated with irreversible investment in competitive equilibrium. Novy-Marx (2007) suggests that the real options value remains significant under perfect competition in industries with important heterogeneous opportunity costs.

<sup>5</sup> On one hand, firms with high uncertainty may be financially constrained, and limited access to external financing decreases investment, which strengthens the negative effect of uncertainty on investment. On the other hand, financial constraints can erode the real option value and weaken the negative effect of uncertainty on investment. Boyle and Guthrie (2003) extend the real options model by endogenizing financial constraints and find that while the uncertainty of project value increases the option value to wait and decreases investment, the uncertainty of firm liquidity decreases the option value and increases investment for financially constrained firms.

<sup>6</sup> At the aggregate level, Caballero and Pindyck (1996) find a negative effect of uncertainty on investment for industries, but the effect is less than the predictions based on single projects. At the firm level, Leahy and Whited (1996) find a negative relation between investment and uncertainty estimated by finite-order autoregressive representation from daily stock returns. However, the results become insignificant after controlling for Tobin's  $Q$ , and thus the authors conclude that the uncertainty affects investment only through its impact on Tobin's  $Q$ . Chu and Fang (2020) show that firms' labor investment is negatively correlated with economic policy uncertainty.

## 2. Model

In this section, we present a simple real options model, which follows the seminal work by McDonald and Siegel (1986) and Dixit and Pindyck (1994), and is closely related to the work of Sarkar (2000).

A firm starts from time 0 with assets-in-place from a single project (the project of the assets-in-place is normalized to one without loss of generality), which produces a perpetual stochastic cash flow of  $x_t$ . We assume that  $x_t$  follows a Geometric Brownian motion process:

$$dx_t = \alpha x_t dt + \sigma x_t dz_t, \tag{1}$$

where  $z_t$  is a standard Wiener process and represents the project-specific shock.  $\alpha$  is the expected growth rate of the cash flow and  $\sigma$  is the standard deviation of the growth rate.

The value of the market portfolio (dividend-reinvested),  $m_t$ , also follows a Geometric Brownian motion process:

$$dm_t = \mu_m m_t dt + \sigma_m m_t dz_{m,t}, \tag{2}$$

where  $z_{m,t}$  is a standard Wiener process and represents the market-wide aggregate shock.  $\mu^m$  is the expected return of the market portfolio and  $\sigma_m$  is the standard deviation of the market return.

The correlation between the cash flow of the project and the market portfolio is  $\rho$  ( $dz dz_m = \rho dt$ ). Then the risk-adjusted discount rate for the cash flow of the project is determined by the CAPM:

$$\mu = r + \phi \rho \sigma = r + \beta \lambda_m, \tag{3}$$

where  $r$  is a constant risk-free rate,  $\phi = \frac{\lambda_m}{\sigma_m}$  is the market price of risk,  $\lambda_m = \mu_m - r$  is the market risk premium, and  $\beta = \frac{\text{cov}(\frac{dx}{x}, \frac{dm}{m})}{\text{var}(\frac{dm}{m})} = \frac{\sigma \rho}{\sigma_m}$  is the sensitivity of the growth rate of cash flow to the market return.

The value of the assets-in-place at time 0 is given by:

$$V^{AP} = E_0 \left[ \int_0^\infty x_t e^{-\mu t} dt \right] = E_0 \left[ \int_0^\infty x_0 e^{(\alpha - \mu)t} dt \right] = \frac{x_0}{\mu - \alpha} = \frac{x_0}{\delta}, \tag{4}$$

where  $E_0$  denotes the expectation at time 0 under the physical probability measure, and  $\delta = \mu - \alpha > 0$  represents the (implicit or explicit) dividend rate.

The firm also has an opportunity to invest in  $n$  new identical projects. When the firm invests in a new project, it incurs a fixed investment cost  $I_0$  and receives a perpetual stochastic cash flow of  $x_t$  afterwards. The value of the growth option of a single project can be solved by the standard contingent claims valuation (we assume that the market is complete and therefore the stochastic component of the return on the project can be exactly replicated by some traded asset). The following proposition provides the solution of the model (see Appendix A for proof).

**Proposition 2.1.** *The optimal investment strategy is*

$$x^* = \frac{\beta_1}{\beta_1 - 1} \delta I_0, \tag{5}$$

and the value of the growth option of a single project is

$$V_{x_t}^{GO} = A_1 x_t^{\beta_1}, \tag{6}$$

where

$$\beta_1 = \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left( \frac{r - \delta}{\sigma^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma^2}} > 1, \tag{7}$$

$$A_1 = (\beta_1 - 1)^{\beta_1 - 1} I_0^{-(\beta_1 - 1)} (\delta \beta_1)^{-\beta_1}. \tag{8}$$

We are interested in expected investment, which is closely related to the probability of investing, or in other words, the probability of reaching the investment threshold  $x^*$ . Following Sarkar (2000), we derive the probability of investing within a fixed time period  $T$  in the following proposition (see Appendix A for proof).

**Proposition 2.2.** *The probability of reaching the critical level within a fixed time period  $T$  is given by*

$$P_{Inv} = \Phi \left( \frac{-\ln \frac{x^*}{x_0} + (\alpha - \frac{1}{2} \sigma^2) T}{\sigma \sqrt{T}} \right) - \left( \frac{x^*}{x_0} \right)^{\frac{2\alpha}{\sigma^2} - 1} \Phi \left( \frac{-\ln \frac{x^*}{x_0} - (\alpha - \frac{1}{2} \sigma^2) T}{\sigma \sqrt{T}} \right). \tag{9}$$

We are interested in how the expected investment changes with volatility ( $\sigma$ ). Define the expected investment-to-assets ratio as expected investment scaled by assets-in-place

$$i \equiv \frac{E(I)}{V^{AP}} = \frac{n P_{Inv} I_0}{x_0 / \delta} = \frac{n \delta I_0}{x_0} P_{Inv}(\sigma). \tag{10}$$

The investment–volatility sensitivity is given by

$$\frac{\partial i(n, \sigma)}{\partial \sigma} = \frac{n\delta I_0}{x_0} \frac{dP_{Inv}(\sigma)}{d\sigma}. \tag{11}$$

The total volatility of the cash flow ( $\sigma^2$ ) can be written as the sum of systematic volatility ( $\sigma_s^2$ ) and idiosyncratic volatility ( $\sigma_i^2$ ):

$$\sigma^2 = \sigma_s^2 + \sigma_i^2, \tag{12}$$

$$\text{where } \sigma_s^2 = \beta^2 \sigma_m^2. \tag{13}$$

By fixing one and varying the other, we can separately identify the effect of idiosyncratic and systematic volatility on expected investment. Since the sign of  $\frac{dP_{Inv}(\sigma)}{d\sigma}$  cannot be analytically determined unambiguously, we use numerical analysis to illustrate the relation between investment and volatility.

Given that we are particularly interested in the relation between investment and idiosyncratic risk, we vary idiosyncratic risk and fix systematic risk in the first case. We use the following base parameters:  $\alpha = 0$ ;  $r = 0.05$ ;  $\lambda_m = 0.1$ ;  $\sigma_m = 0.2$ ;  $I_0 = 1$ ;  $x_0 = 0.07$ ;  $T = 5$ . We set the drift  $\alpha = 0$  in order to focus on the volatility effect rather than the growth effect. The risk-free rate ( $r$ ), the equity premium ( $\lambda_m$ ), and market volatility ( $\sigma_m$ ) are chosen to be close to the actual numbers. The investment cost ( $I_0$ ) is normalized to one without loss of generality. Different starting value of  $x_0$  will result in different  $P_{Inv}$  but will not affect the relation between  $P_{Inv}$  and  $\sigma$ . Time period  $T$  is set to five years. We fix  $\mu = 0.15$ , which implies that  $\beta = (\mu - r)/\lambda_m = 1$  and  $\sigma_s = \beta\sigma_m = 0.2$ . We then vary  $\sigma_s$  and identify its effect on  $P_{Inv}(\sigma)$  and  $\frac{dP_{Inv}(\sigma)}{d\sigma}$ . The results are plotted in Fig. 1. In Appendix C, we vary the values of  $r$ ,  $\mu$ , and  $T$  and show that our results are robust for a wide range of parameters (Figs. A.1–A.3).

Fig. 1a shows that there is a negative relation between the probability of investing and idiosyncratic volatility given that  $\sigma_i$  is above a minimum threshold (which is 0.55 in this case). Panousi and Papanikolaou (2012) suggest that a negative relation between investment and idiosyncratic risk can arise due to suboptimal investment decisions of risk averse managers. Our model indicates that, in a world without agency problems, investment can be negatively correlated with idiosyncratic risk due to growth options. Below we present the first prediction of the model.

**Prediction 1.** Given that the level of idiosyncratic risk is above a minimal threshold, there is a significant negative relation between investment and idiosyncratic risk in the absence of agency problems.

While the negative investment–idiosyncratic risk relation is intuitive due to the option value of waiting, the possible positive investment–idiosyncratic risk relation when uncertainty is low deserves some elaboration. It is important to note that expected investment (or the probability of investing) is determined by two effects. One is the threshold effect. The higher the trigger level, the lower probability the firm will invest. It can be easily shown that the threshold is a monotonic increasing function of uncertainty. The second effect is the pure probability effect. Given a fixed level of threshold, the probability of reaching the threshold increases with uncertainty. The overall effect of uncertainty on investment depends on the relative importance of the threshold and the pure probability effect. When uncertainty is very low, the pure probability effect is important and therefore we may observe a positive investment–uncertainty relation. After uncertainty passes a minimum threshold, the threshold effect starts to dominate, and we start to observe a negative relation between investment and uncertainty.

Fig. 1b further shows that the investment–idiosyncratic risk sensitivity is a U-shaped function of  $\sigma_i$ . When  $\sigma_i$  is low but above the minimum threshold ( $0.55 \leq \sigma_i \leq 1.00$  in this case), the magnitude of the negative investment–idiosyncratic risk sensitivity increases with  $\sigma_i$ . When  $\sigma_i$  is high ( $\sigma_i > 1.00$  in this case), the magnitude of the negative investment–idiosyncratic risk sensitivity decreases with  $\sigma_i$ . This result is formally stated as our second prediction.

**Prediction 2.** The negative investment–idiosyncratic risk sensitivity is a U-shaped function of the level of idiosyncratic risk. That is, when the level of idiosyncratic risk is low (but above the minimum threshold), the magnitude of the negative investment–idiosyncratic risk sensitivity increases with idiosyncratic risk; when the level of idiosyncratic risk is high, the magnitude of the negative investment–idiosyncratic risk sensitivity decreases with idiosyncratic risk.

The U-shaped relation can be understood by the interaction between the threshold and the pure probability effect discussed above, and an additional saturation effect. When volatility is low, the negative investment–idiosyncratic risk relation generated by the threshold effect is attenuated by the pure probability effect. As the volatility level increases, the threshold effect starts to dominate the pure probability effect, and therefore the negative investment–idiosyncratic risk relation becomes stronger. When volatility is high, an additional saturation effect kicks in. The probability of investment is so low that any further increase in volatility only has a small marginal effect, and the magnitude of the negative investment–idiosyncratic risk relation starts to decrease. Prediction 2 suggests that expected investment should respond the most when idiosyncratic risk is at an intermediate level.

Finally, we are interested in how firm composition affects the investment–idiosyncratic risk relation. Eq. (11) straightforwardly indicates that the magnitude of the investment–idiosyncratic risk relation is an increasing function of the number of growth options ( $n$ ), which gives our third prediction.

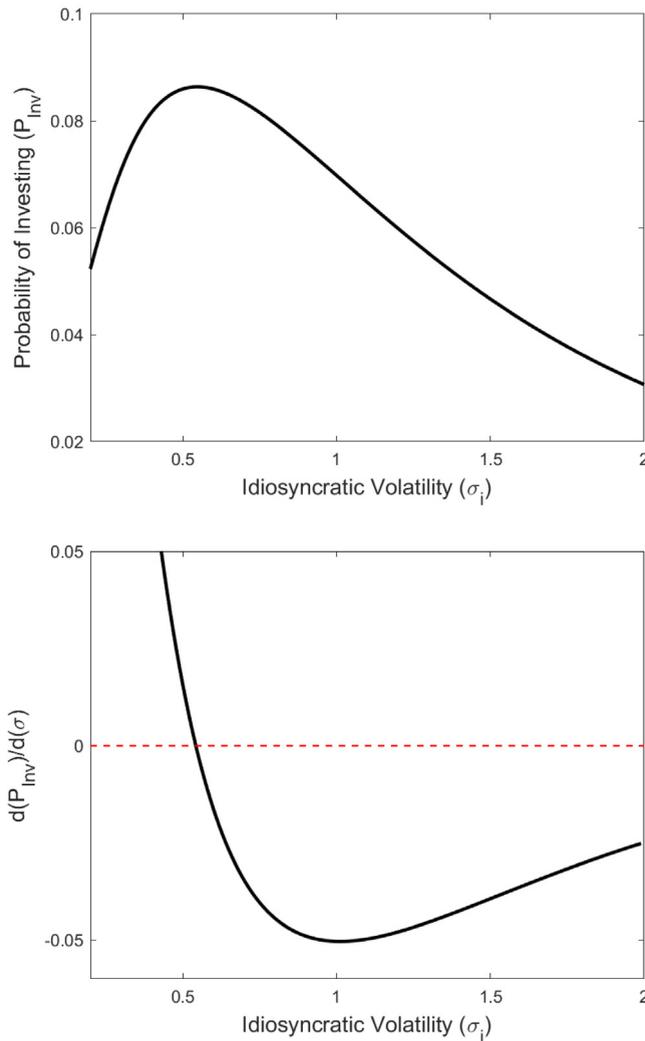


Fig. 1. (a) Probability of investing as a function of idiosyncratic volatility; (b)  $dP/d\sigma$  as a function of idiosyncratic volatility. Parameters are set as follows:  $\alpha = 0$ ;  $r = 0.05$ ;  $\lambda_m = 0.1$ ;  $\sigma_m = 0.2$ ;  $I_0 = 1$ ;  $x_0 = 0.07$ ;  $T = 5$ ;  $\mu = 0.15$ ;  $\sigma_s = 0.2$ .

**Prediction 3.** The negative investment–idiosyncratic risk relation is stronger for firms with more growth options.

It is worth noting that this composition effect does not affect the sign of the investment–uncertainty relation. In other words, whether or not the real options effects generate a positive or negative investment–uncertainty relation, the magnitude of the relation always increases with the number of growth options.

To shed light on the relation between investment and systematic risk, we vary systematic risk and fix idiosyncratic risk in the second case. We set the base parameters the same as case 1:  $\alpha = 0$ ;  $r = 0.05$ ;  $\lambda_m = 0.1$ ;  $\sigma_m = 0.2$ ;  $I_0 = 1$ ;  $x_0 = 0.07$ ;  $T = 5$ . The difference is that in case 2, we fix  $\sigma_i = 0.2$  but vary  $\sigma_s$ . In this case,  $\beta = \sigma_s/\sigma_m$  and  $\mu = r + \beta\lambda_m$  will vary with  $\sigma_s$  at the same time. We can then identify the effect of systematic risk on  $P_{Inv}(\sigma)$  and  $\frac{dP_{Inv}(\sigma)}{d\sigma}$ . The results are plotted in Fig. 2.

We observe that the effects of idiosyncratic and systematic volatility on investment share similarities but also have differences. On one hand, when we vary idiosyncratic volatility, it only changes investment through the channel of total volatility. On the other hand, when we vary systematic volatility, it changes investment through two channels: one is total volatility and the other is discount rate. As a result, idiosyncratic risk captures the pure volatility effect, but systematic risk reflects both the volatility effect and the discount rate effect. This is evident when we compare Figs. 1 and 2. While the investment–idiosyncratic volatility sensitivity is a simple U-shape, the investment–systematic volatility sensitivity is more complicated: it is negative when systematic risk is low and exhibits U-shape only when systematic risk gradually increases. The initial negative relation between investment and systematic volatility is driven by the discount rate effect, which dominates the total volatility effect when systematic risk is low.

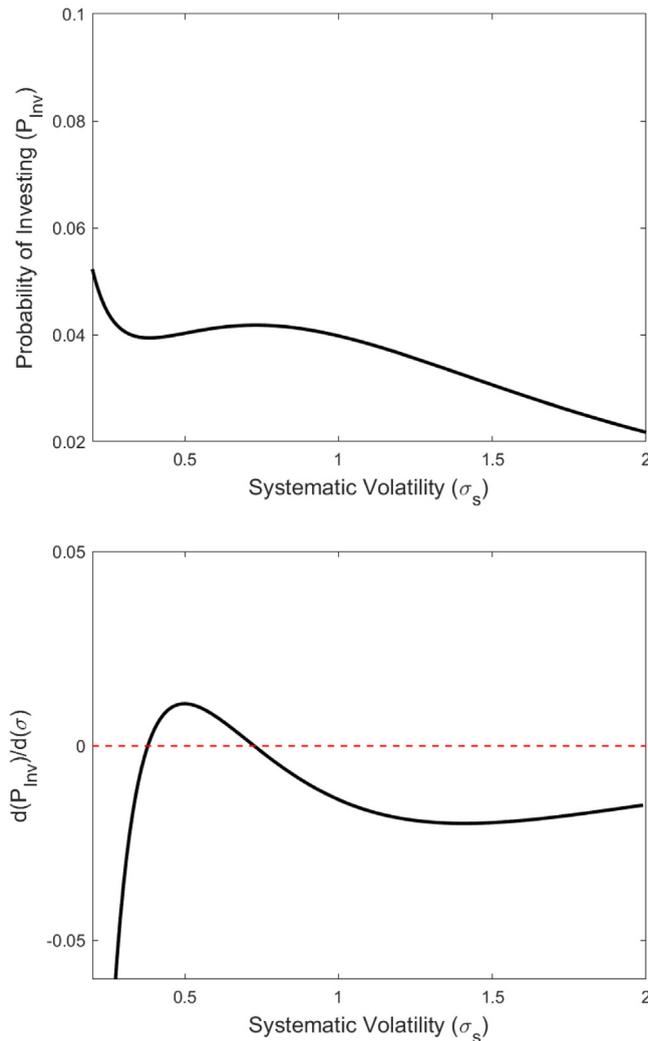


Fig. 2. (a) Probability of investing as a function of systematic volatility; (b)  $dP/d\sigma$  as a function of systematic volatility. Parameters are set as follows:  $\alpha = 0$ ;  $r = 0.05$ ;  $\lambda_m = 0.1$ ;  $\sigma_m = 0.2$ ;  $I_0 = 1$ ;  $x_0 = 0.07$ ;  $T = 5$ ;  $\sigma_i = 0.2$ .

### 3. Data

Our full sample includes all publicly traded firms in Compustat with fiscal years ranging from 1967 to 2017. We delete firms with headquarters outside USA and with format codes in 4, 5 and 6. We also delete firm-years with large acquisitions (the footnote for sales is 'AB'). We require firms to have positive book assets and positive book equity, and have a valid share price and shares outstanding at the fiscal year end. We exclude firms in financial, utility, and government-regulated industries (four-digit standard and industrial classification (SIC) codes between 6000 and 6999 or between 4900 and 4949 or larger than 9000). We drop observations with a missing standard industrial classification code, investment, Tobin's  $Q$ , cash flow, leverage, size, stock returns, idiosyncratic volatility, and systematic volatility. In order to alleviate the effect of extreme values, we winsorize all variables at the 0.5% and 99.5% levels for all analysis. The final sample has 124,314 firm-year observations.

We obtain daily stock return data from the Center for Research in Security Prices (CRSP). We include only common stocks trading on NYSE, AMEX and Nasdaq with available return data.

We obtain insider ownership data from the Thomson Financial Institutional Holdings database of filings derived from Forms 3, 4, and 5. The sample covers the period from 1986 to 2017. After merging with the Compustat data, we have 66,711 observations in total. We obtain monthly Standard & Poors (S&P) Long-Term Domestic Issuer Credit Rating from Compustat, which covers the period from 1985 to 2017. We obtain option data from OptionMetrics, which covers the period from 1996 to 2017.

### 3.1. Measures of uncertainty

We measure uncertainty based on the variation of daily stock returns on an annual basis. We decompose daily stock returns into systematic and idiosyncratic components by performing the following regression every year:

$$r_{itd} = a_{it} + b_{it}^1 f_{itd} + \varepsilon_{itd}, \quad (14)$$

where  $r_{itd}$  is the stock return on trading day  $d$  in year  $t$  for firm  $i$  and  $f_{itd}$  is the daily factor return on day  $d$  in year  $t$ . The total volatility, idiosyncratic volatility, and systematic volatility for stock  $i$  in year  $t$  are defined as:

$$TV_{it} = \sqrt{\frac{252}{n} \sum_{d=1}^n r_{itd}^2}, \quad (15)$$

$$IV_{it} = \sqrt{\frac{252}{n} \sum_{d=1}^n \varepsilon_{itd}^2}, \quad (16)$$

$$SV_{it} = \sqrt{TV_{it}^2 - IV_{it}^2}, \quad (17)$$

where  $n$  is the number of trading days in year  $t$  for firm  $i$  and all the volatility measures are annualized in order to facilitate comparison across different volatility measures. We require at least 100 daily stock return data points to calculate stock return volatility. We consider four sets of systematic factors. The first set is the market factor (MKT), which is the excess return on the value-weighted market portfolio. The second set includes both the market factor and the industry factors, which are the excess returns on the value-weighted industry portfolios based on the Fama and French (1997) 30-industry classification. The third set is the Fama and French (1993) three factors, including the MKT, the small-minus-big (SML) size factor, and the high-minus-low (HML) value factor. The fourth set uses the Carhart (1997) four factors, including MKT, SML, HML and a momentum factor (MOM). These factor returns are from the Kenneth French's website. All our empirical analysis reported in this paper is based on the benchmark market model. Results are quantitatively similar for all alternative idiosyncratic volatility measures.

Stock return volatility captures a wide range of factors that might affect the uncertain environment that firms may face in the real world. Given that the discount rate is relatively stable during a short horizon (one year), return variance is mainly driven by cash flow uncertainty. In the cross section, we find that stock return volatility is significantly positively correlated with earnings volatility. Bloom et al. (2007) also provide both empirical and theoretical evidence that stock return volatility can well capture the underlying uncertainty on firms' investment dynamics. In the time series, Wei and Zhang (2006) provide evidence that the changes in firm idiosyncratic volatility are mainly driven by changes in earnings volatility.<sup>7</sup> Bloom (2009) further shows that many cross-sectional measures of profit uncertainty are highly correlated with time-series stock market volatility.

One concern of using stock return variation to measure fundamental uncertainty is that stock prices may move due to reasons unrelated to a firm's fundamentals, for example, temporary misvaluation or trading noises. In order to alleviate this concern, we also use the variation of weekly stock returns to measure uncertainty, and the results remain qualitatively the same.<sup>8</sup>

### 3.2. Measures of growth options

Growth options are not directly observable, and it is important to find good proxies to measure them. We adopt two measures, including Tobin's  $Q$  and past investment rate, to capture investment growth options. Tobin's  $Q$ , defined as the market value divided by book value of assets, has been widely used as a proxy for growth opportunities. However, firms may have different types of real options, including investment growth options, financial flexibility options, and default options, among others. While Tobin's  $Q$  measures the overall effect of real options, it may not be a perfect proxy for the pure effect of investment growth options. Moreover, investment may take different forms, including not only physical investment but also intangible investment, for example, research and development (R&D) investment. When production requires more than one type of capital, the overall Tobin's  $Q$  is a combination of the marginal  $q$  for each type of capital. For example, if the firm's production requires both physical investment and R&D investment, Tobin's  $Q$  measures the composite effect of the marginal  $q$  for physical investment and the marginal  $q$  for R&D investment (e.g., Lin (2012), Li and Liu (2010), and Peters and Taylor (2017)). Physical investment is determined by its own marginal  $q$ , and its relation with the overall Tobin's  $Q$  depends on the relative weight of physical-investment  $q$ . In unreported tables, we show that the physical investment- $q$  sensitivity significantly decreases with R&D intensity, while the R&D investment- $q$  sensitivity increases with R&D intensity. The results suggest that the link between physical investment and the overall measure of growth options (Tobin's  $Q$ ) depends on the specific nature of the firm's investment. Since our studies in this paper mainly focus on the relation between physical investment and idiosyncratic risk,<sup>9</sup> we need a proxy that can capture growth options closely linked to physical investment. In other words, we need a measure for the marginal  $q$  of physical investment.

<sup>7</sup> Rajgopal and Venkatachalam (2011) and Kong (2019) show that higher stock return volatility is associated with lower earnings quality and more earnings management.

<sup>8</sup> Another concern is that our results might be driven by penny stocks, which may have extreme values of idiosyncratic volatility. To address this concern, we conduct robustness checks by excluding stocks with prices less than \$1 or \$5. The results remain qualitatively the same and are not reported to save space.

<sup>9</sup> The relation between R&D investment and idiosyncratic risk is ambiguous in theory. The progress of R&D activities may help resolve uncertainty (see, Berk et al. (2004), for example), and the strategic nature of R&D may significantly complicate the relation between R&D investment and idiosyncratic risk. Empirically, we do not find a robust relation between R&D and idiosyncratic risk.

Due to the fact that physical investment is only determined by its own marginal  $q$ , we choose to use past physical investment rate to measure growth opportunities that are directly linked to physical investment. In unreported tables, we further establish the link between investment rate and growth options by showing that the investment rate is positively correlated with future sales growth, profit growth, and investment. Our results suggest that past investment rate can serve as a good proxy for future growth opportunities.

### 3.3. Summary statistics

We report the summary statistics of firm characteristics in Table 1. Panel A reports the summary statistics in the full sample. The average investment-to-assets ratio ( $I/A$ , which is also referred to as the investment rate) is 7.6% with a standard deviation of 8.8%. The average annualized idiosyncratic volatility ( $IV$ ) is 50.3% with a standard deviation of 27.8%, and the average annualized systematic volatility ( $SV$ ) is 13.6% with a standard deviation of 7.6%. The average total volatility ( $TV$ ), Tobin's  $Q$  ( $Q$ ), cash flow-to-assets ratio ( $CF$ ), natural logarithm of book assets ( $SIZE$ ), and leverage ( $LEV$ ) are 53.2%, 1.716, 6.2%, 5.703, and 22.3%, respectively. The average sales growth ( $SG$ ) and profit growth ( $PG$ ) are 15.9% and 14.4%. The average R&D investment-to-assets ratio ( $RD/A$ ) is 6.9%. We also report insider ownership based on the Thomson Financial Institutional Holdings database, which has an average of 11.3% and a median of 5.2%.

Panel B presents the time-series average of the median firm characteristics in subsamples split by investment rate, which is a proxy for the composition of growth options. Firms with a high investment rate have high systematic risk, Tobin's  $Q$ , cash flow, leverage, sales growth, and profitability growth. However, firms with a high investment rate tend to have lower R&D investment. The relation between investment rate and insider ownership is not monotonic in this univariate analysis. Insider ownership first decreases and then increases with investment.

Panel C shows the time-series average of the median firm characteristics in subsamples split by idiosyncratic volatility. Firms with high idiosyncratic volatility in general have a low investment rate, low cash flow, small size, high R&D investment and high insider ownership.

## 4. Empirical results

### 4.1. Investment and idiosyncratic risk

In this section, we first establish the negative relation between investment and idiosyncratic risk as stated in Prediction 1. We run the following reduced-form investment regression:

$$\frac{I_{i,t}}{K_{i,t-1}} = \alpha_i + \alpha_t + \beta_1 \sigma_{i,t-1} + \gamma z_{i,t-1} + \varepsilon_{i,t}, \quad (18)$$

where  $\frac{I_{i,t}}{K_{i,t-1}}$  is the investment-to-assets ratio,<sup>10</sup>  $\sigma_{i,t-1}$  is the measure of idiosyncratic risk, and  $z_{i,t-1}$  is a vector of control variables.  $\alpha_i$  and  $\alpha_t$  are firm and time fixed effects, respectively.

We report the estimates of Eq. (18) in Table 2. All the models are estimated with firm and time fixed effects. Model 1 is the investment regression with the idiosyncratic risk as the only explanatory variable. Model 2 includes Tobin's  $Q$  as an additional explanatory variable. Model 3 adds a set of control variables that have been used often in previous studies, including cash flow, firm size, and leverage.<sup>11</sup> Model 4 also includes systematic risk as an additional control variable. The coefficient on idiosyncratic volatility is negative and statistically significant at the 1% level in all specifications, with the magnitude of the coefficient ranging from 0.0201 to 0.0275. The standard deviation of idiosyncratic volatility in the full sample is 0.278, which implies that a one-standard-deviation increase of idiosyncratic volatility leads to 0.6% ~ 0.8% decrease of investment. Given that the average investment rate is 7.6%, our estimate shows that the impact of idiosyncratic risk on firm investment is economically significant. The coefficients on Tobin's  $Q$  and cash flow are all positive and statistically significant, consistent with previous findings.

The coefficient on systematic risk, however, is positive. As discussed by Panousi and Papanikolaou (2012), this might be due to the fact that systematic risk is correlated with a firm's growth opportunities, which has not been fully captured by Tobin's  $Q$ . In Table 5, we provide evidence that after correcting measurement errors in Tobin's  $Q$ , the coefficient on systematic risk becomes negative and statistically significant. These results are consistent with the prediction of real options theory that there should be a negative relation between investment and total volatility, including both systematic and idiosyncratic volatility.

<sup>10</sup> Instead of using book assets as capital stock in the denominator to calculate investment-to-capital ratio and Tobin's  $Q$ , we also use the replacement value of capital as defined in Salinger and Summers (1983) and net PPE as alternative measures of capital stock. The results remain the same. Our results are also quantitatively similar for a number of alternative measures of physical investment, including fixed investment (investment in property, plant, and equipment) and inventory.

<sup>11</sup> In unreported tables, we control for other variables that have been used in the previous literature, including sales growth, past annual return, institutional ownership, insider ownership, credit rating, and liquidity. The results remain qualitatively the same.

**Table 1**

Summary statistics. This table reports summary statistics for our sample. Panel A reports the time-series averages of the number of observations, mean, standard deviation, minimum, 5th percentile, 25th percentile, median, 75th percentile, 95th percentile, and maximum of firm characteristics, including investment-to-assets ratio ( $I/A$ ), idiosyncratic volatility ( $IV$ ), systematic volatility ( $SV$ ), total volatility ( $TV$ ), Tobin's  $Q$  ( $Q$ ), cash flow-to-assets ratio ( $CF$ ), firm size ( $SIZE$ ), leverage ( $LEV$ ), sales growth ( $SG$ ), gross profitability growth ( $PG$ ), R&D investment-to-assets ratio ( $RD/A$ ), and insider ownership ( $InsiderOwn$ ) in the full sample. See Appendix B for the detailed definition of variables. All variables are winsorized at the 0.5% and 99.5% levels. Panel B and C report the time-series averages of the median of firm characteristics in quintile groups split by lagged investment rate and idiosyncratic volatility, respectively. The sample period is from 1967 to 2017.

Panel A. Full sample

	N	Mean	Std	Min	P5	P25	P50	P75	P95	Max
$I/A$	2670	0.076	0.088	0.001	0.007	0.027	0.051	0.092	0.231	0.710
$IV$	2668	0.503	0.278	0.137	0.199	0.308	0.439	0.620	1.039	1.848
$SV$	2668	0.136	0.076	0.009	0.030	0.080	0.127	0.182	0.279	0.390
$TV$	2668	0.532	0.275	0.152	0.227	0.339	0.470	0.650	1.057	1.865
$Q$	2670	1.716	1.353	0.521	0.754	1.017	1.303	1.875	4.077	12.434
$CF$	2669	0.062	0.166	-1.069	-0.225	0.031	0.089	0.139	0.241	0.509
$SIZE$	2670	5.703	1.908	1.440	2.723	4.336	5.602	6.953	9.078	10.931
$LEV$	2670	0.223	0.178	0.000	0.001	0.067	0.201	0.338	0.556	0.771
$SG$	2650	0.159	0.579	-0.800	-0.256	-0.011	0.084	0.204	0.655	7.143
$PG$	2557	0.144	0.612	-2.750	-0.373	-0.034	0.087	0.230	0.760	5.844
$RD/A$	1540	0.069	0.101	0.000	0.000	0.007	0.034	0.092	0.254	0.765
$InsiderOwn$	1606	0.113	0.152	0.000	0.004	0.018	0.052	0.146	0.440	0.969

Panel B. Subsamples split by  $I/A$ 

	Low	2	3	4	High
$I/A$	0.018	0.035	0.051	0.075	0.131
$IV$	0.550	0.441	0.402	0.387	0.439
$SV$	0.115	0.122	0.127	0.128	0.142
$TV$	0.573	0.470	0.434	0.422	0.475
$Q$	1.222	1.265	1.314	1.346	1.391
$CF$	0.046	0.077	0.093	0.109	0.122
$SIZE$	4.600	5.524	5.906	6.082	5.866
$LEV$	0.157	0.188	0.201	0.208	0.246
$SG$	0.055	0.070	0.077	0.089	0.131
$PG$	0.065	0.076	0.081	0.091	0.127
$RD/A$	0.046	0.040	0.036	0.030	0.020
$InsiderOwn$	0.065	0.055	0.047	0.045	0.052

Panel C. Subsamples split by  $IV$ 

	Low	2	3	4	High
$I/A$	0.059	0.057	0.054	0.046	0.035
$IV$	0.241	0.337	0.439	0.555	0.785
$SV$	0.106	0.124	0.137	0.142	0.134
$TV$	0.275	0.370	0.473	0.587	0.808
$Q$	1.385	1.297	1.274	1.277	1.301
$CF$	0.114	0.106	0.094	0.068	0.017
$SIZE$	7.392	6.458	5.658	4.929	3.903
$LEV$	0.220	0.210	0.193	0.180	0.189
$SG$	0.078	0.093	0.099	0.089	0.062
$PG$	0.080	0.095	0.102	0.090	0.069
$RD/A$	0.020	0.022	0.037	0.056	0.067
$InsiderOwn$	0.025	0.046	0.060	0.069	0.081

#### 4.2. The level effect of idiosyncratic risk

In this section, we test the second prediction of the model, which states that the investment–idiosyncratic risk relation is a U-shaped function of the level of idiosyncratic risk. If we are able to observe the investment–idiosyncratic risk sensitivity at each level of idiosyncratic risk, we can test this prediction by regressing the investment–idiosyncratic risk sensitivity on idiosyncratic volatility in a quadratic functional form. However, the investment–idiosyncratic risk sensitivity is not directly observable and has to be estimated first. As a result, our analysis is based on the regression of investment on idiosyncratic risk, which is predicted to have a cubical form of the level of idiosyncratic risk. To avoid testing a highly nonlinear cubical functional form, we instead adopt the following two empirical strategies.

First, we sort firms into quintiles based on their lagged level of idiosyncratic volatility, and run the investment regression in Eq. (18) for each subsample. The coefficients on idiosyncratic volatility are reported in Panel A of Table 3 for different model specifications. Model 1 includes idiosyncratic risk as the only explanatory variable. Model 2 includes Tobin's  $Q$  as an additional explanatory variable. Model 3 adds a set of control variables. Model 4 also includes systematic risk as an additional control variable. The results show that the magnitude of the investment–idiosyncratic risk sensitivity first increases with idiosyncratic volatility when its level is low, and starts to decrease when idiosyncratic volatility gets high. In the lowest quintile, the coefficient is not statistically

**Table 2**

Investment and idiosyncratic risk. This table reports the results of the investment regression:

$$\frac{I_{i,t}}{K_{i,t-1}} = \beta_1 IV_{i,t-1} + \gamma z_{i,t-1} + \alpha_i + \alpha_t + \varepsilon_{i,t},$$

where the dependent variable is the investment rate ( $I/A$ ) at time  $t$ . The main independent variable of interest is idiosyncratic volatility ( $IV$ ). Control variables  $z$  include systematic volatility ( $SV$ ), Tobin's  $Q$  ( $Q$ ), cash flow-to-assets ratio ( $CF$ ), firm size ( $SIZE$ ) and leverage ( $LEV$ ). See Appendix B for the detailed definition of variables. The sample period is from 1967 to 2017. All the regressions are estimated with firm and time fixed effects. Robust  $t$ -statistics based on standard errors clustered at the firm level are reported in parenthesis.

	Model 1	Model 2	Model 3	Model 4
$IV(t-1)$	-0.0239 (-15.32)	-0.0201 (-12.95)	-0.0214 (-13.60)	-0.0275 (-16.33)
$SV(t-1)$				0.0493 (12.48)
$Q(t-1)$		0.0105 (23.08)	0.0089 (20.43)	0.0084 (19.22)
$CF(t-1)$			0.0481 (17.08)	0.0475 (16.96)
$SIZE(t-1)$			-0.0146 (-18.47)	-0.0160 (-19.83)
$LEV(t-1)$			-0.0492 (-14.92)	-0.0474 (-14.40)
Firm fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Observations	136,182	136,182	136,182	136,182
Adjusted $R^2$	0.46	0.48	0.50	0.50

significant in all specifications except for model 4, and the magnitude is the smallest among all subsamples. The magnitude of the coefficient significantly increases as the level of idiosyncratic risk increases and reaches the highest in the third quintile for models 1–3 and in the second quintile for model 4, ranging from 0.049 to 0.071. The coefficient also becomes statistically significant at the 1% level for all model specifications for the third quintile when idiosyncratic risk is at the intermediate level. The magnitude of the coefficient in the intermediate range is more than three times as large as the average magnitude of the coefficient in the full sample. As the level of idiosyncratic volatility further increases, however, the magnitude of the coefficient starts to decrease. The coefficient becomes 0.016 ~ 0.022 for the highest quintile.

To further quantify the effect of idiosyncratic risk level on the investment–idiosyncratic risk relation, we run the regression of investment on linear and quadratic terms of idiosyncratic volatility in low and high idiosyncratic risk subsamples, respectively. For each year, we classify firms into low or high idiosyncratic risk subsamples based on whether the firm's lagged idiosyncratic volatility is below or above the median of the sample. We then run the following regression in low and high idiosyncratic risk subsamples, respectively, by including a quadratic term of idiosyncratic volatility:

$$\frac{I_{i,t}}{K_{i,t-1}} = \alpha_i + \alpha_t + \beta_1 \sigma_{i,t-1} + \beta_2 \sigma_{i,t-1}^2 + \gamma z_{i,t-1} + \varepsilon_{i,t}. \tag{19}$$

The regression results are reported in Panel B of Table 3. In the low idiosyncratic volatility subsample, the coefficient on the quadratic term is significantly negative at the 5% level for all model specifications, ranging from -0.0571 to -0.0339. Given that the standard deviation of idiosyncratic volatility is 0.278, one standard-deviation increase of idiosyncratic volatility will lead to a 0.009 ~ 0.016 increase in the magnitude of the negative investment–idiosyncratic risk sensitivity, which is more than half of the average magnitude in the full sample. In the high idiosyncratic volatility subsample, the coefficient on the quadratic term becomes positive, close to 0.012, and significant at the 1% level for all model specifications, suggesting that one standard-deviation increase of idiosyncratic volatility will lead to 0.003 decrease in the magnitude of the negative investment–idiosyncratic risk sensitivity. It is worth noting that the effect of idiosyncratic risk on the investment–idiosyncratic risk sensitivity is asymmetric. The effect is stronger when idiosyncratic volatility is low. This asymmetry is also predicted by our model. In Fig. 1b, the investment–idiosyncratic risk sensitivity has a steeper slope when the level of idiosyncratic volatility is low, but only starts to increase slowly when idiosyncratic volatility is high. This is due to the fact that while the threshold effect kicks in fairly quickly as idiosyncratic risk increases initially, the saturation effect has a moderate effect when idiosyncratic risk further increases.

In sum, our results in this section provide evidence that the investment–idiosyncratic risk sensitivity is a U-shaped function of the level of idiosyncratic risk and becomes the strongest when the uncertainty level is in the intermediate range: the sensitivity increases with idiosyncratic volatility when its level is low, but decreases with idiosyncratic volatility when its level is high. The empirical evidence is consistent with the theoretical predictions from our real options model.

#### 4.3. The composition effect of growth options

In this section, we investigate how the investment–idiosyncratic risk relation changes with firm growth options ( $GO$ ). A direct implication of our model is that firms with more growth options should have a stronger negative relation between investment and

**Table 3**

Investment, idiosyncratic risk and the effect of idiosyncratic volatility level. Panel A reports the coefficients on idiosyncratic volatility ( $\beta_1$ ) in the following investment regression for quintile subsamples split by lagged level of idiosyncratic volatility:

$$\frac{I_{i,t}}{K_{i,t-1}} = \beta_1 IV_{i,t-1} + \gamma z_{i,t-1} + \alpha_i + \alpha_t + \varepsilon_{i,t}.$$

Model 1 includes idiosyncratic risk as the only explanatory variable. Model 2 includes Tobin's  $Q$  as an additional explanatory variable. Model 3 adds a set of control variables. Model 4 also includes systematic risk as an additional control variable. Panel B reports the results of the following investment regression, which includes a quadratic term of idiosyncratic volatility, in low and high idiosyncratic volatility subsamples:

$$\frac{I_{i,t}}{K_{i,t-1}} = \beta_1 IV_{i,t-1} + \beta_2 IV_{i,t-1}^2 + \gamma z_{i,t-1} + \alpha_i + \alpha_t + \varepsilon_{i,t}.$$

Control variables  $z$  are measured at  $t-1$  and include: systematic volatility ( $SV$ ), Tobin's  $Q$  ( $Q$ ), cash flow-to-assets ratio ( $CF$ ), firm size ( $SIZE$ ) and leverage ( $LEV$ ). See Appendix B for the detailed definition of variables. The sample period is from 1967 to 2017. All the regressions are estimated with firm and time fixed effects. Robust  $t$ -statistics based on standard errors clustered at the firm level are reported in parenthesis.

Panel A								
	Model 1		Model 2		Model 3		Model 4	
Rank <sub>IV</sub>								
Low	-0.0022		-0.0045		-0.0032		-0.0197	
	(-0.22)		(-0.46)		(-0.34)		(-2.00)	
2	-0.0427		-0.0389		-0.0521		-0.0717	
	(-2.44)		(-2.28)		(-3.08)		(-4.07)	
3	-0.0529		-0.0490		-0.0535		-0.0704	
	(-3.05)		(-2.87)		(-3.17)		(-4.11)	
4	-0.0426		-0.0321		-0.0319		-0.0386	
	(-3.16)		(-2.46)		(-2.47)		(-2.97)	
High	-0.0182		-0.0160		-0.0186		-0.0221	
	(-5.88)		(-5.28)		(-5.93)		(-6.76)	
Panel B								
	Low				High			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
$IV(t-1)$	0.0345	0.0324	0.0142	-0.0043	-0.0549	-0.0505	-0.0520	-0.0565
	(2.81)	(2.69)	(1.20)	(-0.36)	(-12.10)	(-11.30)	(-11.55)	(-12.50)
$IV(t-1)^2$	-0.0571	-0.0557	-0.0379	-0.0339	0.0126	0.0123	0.0120	0.0121
	(-3.52)	(-3.50)	(-2.43)	(-2.18)	(7.00)	(6.94)	(6.83)	(6.87)
$SV(t-1)$				0.0520				0.0470
				(10.83)				(10.46)
$Q(t-1)$		0.0140	0.0085	0.0079		0.0094	0.0082	0.0077
		(47.34)	(27.51)	(25.11)		(42.33)	(36.77)	(34.14)
$CF(t-1)$			0.1218	0.1208			0.0345	0.0342
			(33.22)	(32.98)			(18.46)	(18.29)
$SIZE(t-1)$			-0.0155	-0.0166			-0.0159	-0.0178
			(-31.19)	(-32.69)			(-25.65)	(-27.56)
$LEV(t-1)$			-0.0369	-0.0356			-0.0585	-0.0559
			(-15.17)	(-14.65)			(-20.03)	(-19.08)
Firm fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	68,077	68,077	68,077	68,077	68,105	68,105	68,105	68,105
Adjusted $R^2$	0.566	0.58	0.60	0.60	0.42	0.44	0.46	0.46

idiosyncratic risk as stated in Prediction 3. We run the following regression in the full sample by interacting idiosyncratic volatility and growth options:

$$\frac{I_{i,t}}{K_{i,t-1}} = \alpha_i + \alpha_t + \beta_1 \sigma_{i,t-1} + \beta_2 \sigma_{i,t-1} \times GO_{i,t-1} + \gamma z_{i,t-1} + \varepsilon_{i,t}, \tag{20}$$

where  $GO_{i,t-1}$  is measured as the rank of a specific proxy of growth options, which equals 0 for the lowest quintile and 4 for the highest quintile. We use ranks to avoid the possible large effect of a few influential points and to facilitate the coefficient comparison across different measures of growth options.<sup>12</sup> We use two proxies to measure a firm's growth options, including the widely-used Tobin's  $Q$ , and past investment rate. Table 4 reports the estimation results of the investment regression in Eq. (20), which includes the interaction term between idiosyncratic risk and growth options proxies. The coefficient of the interaction term is negative and significant at the 5% level for both measures of growth options in all model specifications. The magnitude of the coefficients varies significantly across different measures of growth options. The magnitude of the coefficient on the interaction term for Tobin's  $Q$  is only 0.002 ~ 0.003. The magnitude is enhanced for past investment rate, reaching 0.009 ~ 0.011. The regression estimates suggest that the magnitude of the investment–idiosyncratic risk sensitivity on average increases by 0.011 for one unit increase in the rank

<sup>12</sup> Our results remain unchanged if we directly use the raw value of variables that measure growth options rather than their ranks.

**Table 4**

Investment, idiosyncratic risk, and the effect of growth options. This table reports the results of the following investment regression, which includes the interaction term between idiosyncratic volatility and growth options:

$$\frac{I_{i,t}}{K_{i,t-1}} = \beta_1 IV_{i,t-1} + \beta_2 IV_{i,t-1} * GO_{i,t-1} + \beta_3 GO_{i,t-1} + \gamma z_{i,t-1} + \alpha_i + \alpha_t + \epsilon_{i,t}.$$

We use Tobin's  $Q$  or past investment rate to measure growth options and then sort firms into quintile portfolios by the measure of growth options in each year.  $GO$  is the rank of growth option measure for each firm, which equals 0 for the lowest quintile and 4 for the highest quintile. Control variables  $z$  are measured at  $t - 1$  and include: systematic volatility ( $SV$ ), Tobin's  $Q$  ( $Q$ ), cash flow-to-assets ratio ( $CF$ ), firm size ( $SIZE$ ) and leverage ( $LEV$ ). See [Appendix B](#) for the detailed definition of variables. The sample period is from 1967 to 2017. All the regressions are estimated with firm and time fixed effects. Robust  $t$ -statistics based on standard errors clustered at the firm level are reported in parenthesis.

	Tobin's $Q$				Past investment rate			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
$IV(t - 1)$	-0.0111 (-5.77)	-0.0085 (-4.45)	-0.0134 (-6.89)	-0.0168 (-8.41)	0.0009 (0.62)	0.0033 (2.34)	-0.0021 (-1.44)	-0.0061 (-3.90)
$IV(t - 1) * GO(t - 1)$	-0.0017 (-1.96)	-0.0032 (-3.85)	-0.0022 (-2.61)	-0.0026 (-3.12)	-0.0106 (-12.57)	-0.0107 (-12.79)	-0.0093 (-11.24)	-0.0097 (-11.62)
$GO(t - 1)$	0.0166 (27.11)	0.0138 (21.57)	0.0117 (18.54)	0.0116 (18.43)	0.0216 (40.37)	0.0203 (38.07)	0.0195 (36.69)	0.0195 (36.72)
$SV(t - 1)$				0.0324 (7.56)				0.0350 (9.37)
$Q(t - 1)$		0.0066 (13.28)	0.0055 (11.39)	0.0053 (10.96)		0.0090 (21.09)	0.0073 (17.94)	0.0069 (17.00)
$CF(t - 1)$			0.0371 (12.38)	0.0370 (12.36)			0.0349 (13.80)	0.0345 (13.68)
$SIZE(t - 1)$			-0.0153 (-17.18)	-0.0163 (-17.92)			-0.0159 (-21.45)	-0.0168 (-22.21)
$LEV(t - 1)$			-0.0528 (-14.46)	-0.0515 (-14.06)			-0.0489 (-16.03)	-0.0476 (-15.62)
Firm fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	136,182	136,182	136,182	136,182	136,182	136,182	136,182	136,182
Adjusted $R^2$	0.49	0.49	0.51	0.51	0.49	0.51	0.52	0.52

of growth options, which is close to 50% of the average magnitude of the coefficient on idiosyncratic risk in the full sample (the coefficient on idiosyncratic risk in the full sample is around  $-0.020$ ). The results are consistent the argument that Tobin's  $Q$  is a noisy proxy for growth options that directly link to firm's physical investment. Past physical investment rate serves as a more accurate measure of the firm's physical investment  $q$  because it is only determined by its own marginal  $q$  and thus isolates the effects from other forms of investment. While the marginal  $q$  of physical investment cannot be directly observed, the physical-investment rate can be measured with accuracy.

Taken together, the results suggest that the negative relation between investment and idiosyncratic risk is stronger for firms with more growth options. The effect of growth options remain significant when we use different proxies for growth options, including Tobin's  $Q$  and past physical investment rate. Past investment rate generates the strongest effect on the investment–idiosyncratic relation as it is directly linked to physical investment  $q$ .

## 5. Endogeneity and alternative explanations

One potential concern about our results is endogeneity. Endogeneity issues may arise if idiosyncratic risk is correlated with omitted variables that are not fully captured in our previous regression analysis. In this section, we aim to control for factors that might affect investment and discuss alternative explanations.

### 5.1. Growth opportunities

If growth opportunities are not fully captured by Tobin's  $Q$ , the investment–idiosyncratic risk relation may exist due to the correlation between idiosyncratic risk and growth opportunities. In this section, we address this issue by controlling for lagged investment in a dynamic panel regression and correcting measurement errors in  $Q$  with high-order moment estimators.

#### 5.1.1. Dynamic panel regression with lagged investment

As we have shown previously, firms with high investment have high sales and profit growth in the future. Past investment contains useful information about firm's growth opportunities. In this section, we include lagged investment as an additional explanatory variable in the investment regression in order to better control for growth opportunities, which may not be fully captured by Tobin's  $Q$ . Due to the fact that the ordinary least squares (OLS) estimator suffers from dynamic panel bias, we use the two-step first-differenced GMM approach suggested by [Arellano and Bond \(1991\)](#) to estimate the dynamic panel regression. The model is estimated using the first differences instrumented by all available lagged investment up to  $t - 2$ . We require at least three observations for a firm to be included in the panel estimation. The regression results are reported in [Table 5](#).

**Table 5**

Dynamic panel regression and measurement error in Tobin's  $Q$ . This table reports the results of the following investment regression estimated by dynamic panel regression and the Erickson and Whited (2012) third-order-moments estimator that corrects measurement errors in Tobin's  $Q$ :

$$\frac{I_{i,t}}{K_{i,t-1}} = \beta_1 \frac{I_{i,t-1}}{K_{i,t-2}} + \beta_2 IV_{i,t-1} + \beta_3 IV_{i,t-1} * GO_{i,t-1} + \gamma z_{i,t-1} + \alpha_i + \alpha_t + \varepsilon_{i,t},$$

$$\frac{I_{i,t}}{K_{i,t-1}} = \beta_1 IV_{i,t-1} + \beta_2 IV_{i,t-1} * GO_{i,t-1} + \beta_3 GO_{i,t-1} + \gamma z_{i,t-1} + \alpha_i + \alpha_t + \varepsilon_{i,t}.$$

We apply the minimum distance technique as developed by Erickson and Whited (2012) to use the high-order moment estimator in our unbalanced panel data. We use past investment rate to measure growth options.  $GO$  is the rank of growth option measure for each firm, which equals 0 for the lowest quintile and 4 for the highest quintile. Control variables  $z$  are measured at  $t-1$  and include: systematic volatility ( $SV$ ), Tobin's  $Q$  ( $Q$ ), cash flow-to-assets ratio ( $CF$ ), firm size ( $SIZE$ ) and leverage ( $LEV$ ). See Appendix B for the detailed definition of variables. The sample period is from 1967 to 2017. All the regressions are estimated with firm and time fixed effects. The  $t$ -statistics are reported in parenthesis.

	Dynamic panel		Erickson and Whited (2012)	
$I/A(t-1)$	0.2974 (28.41)	0.2971 (28.46)		
$IV(t-1)$	-0.0083 (-4.90)	-0.0050 (-2.85)	-0.0066 (-2.16)	0.0032 (0.98)
$IV(t-1) * GO(t-1)$		-0.0031 (-2.33)		-0.0048 (-3.84)
$GO(t-1)$				0.0010 (1.27)
$SV(t-1)$	0.0112 (2.69)	0.0121 (2.91)	-0.0451 (-4.32)	-0.0206 (-2.14)
$Q(t-1)$	0.0041 (8.44)	0.0041 (8.48)	0.0271 (10.29)	-0.0003 (-2.44)
$CF(t-1)$	0.0133 (3.98)	0.0130 (3.91)	0.0357 (11.34)	0.0319 (10.85)
$SIZE(t-1)$	-0.0573 (-31.10)	-0.0573 (-31.23)	-0.0110 (-7.18)	-0.0157 (-11.21)
$LEV(t-1)$	-0.1169 (-20.86)	-0.1166 (-20.81)	-0.0187 (-4.89)	-0.0244 (-6.58)
Firm fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Observations	132,152	132,152	136,182	136,182

After controlling for lagged investment, the coefficient on idiosyncratic volatility remains negative and statistically significant at the 1% level (coefficient = -0.0083 and  $t$ -statistic = -4.90), suggesting a robust negative relation between investment and idiosyncratic risk. The coefficient on the interaction term between idiosyncratic volatility and growth options also remains negative and statistically significant at the 1% level (coefficient = -0.0031 and  $t$ -statistic = -2.33), providing support for our previous findings: the more growth options firms have, the stronger the negative relation between investment and idiosyncratic risk is.

### 5.1.2. Measurement errors in $Q$

The standard  $q$ -theory of investment with quadratic adjustment costs suggests that marginal  $Q$  is a sufficient statistic for investment. However, measurement errors in  $Q$  may lead to a downward bias on the coefficient on  $Q$ , and any variable that is correlated with the measurement errors may have a spurious relation with investment. Following Erickson and Whited (2000, 2012), we apply high-order moment estimator to correct measurement errors in Tobin's  $Q$ . We apply the minimum distance technique as developed by Erickson and Whited (2012) to use the high-order moment estimator in our unbalanced panel data. The results based on the third-order-moments of Erickson and Whited (2012) estimator are reported in Table 5.

After correcting the measurement errors in Tobin's  $Q$ , the coefficient on idiosyncratic volatility remains negative and statistically significant at the 5% level (coefficient = -0.0066 and  $t$ -statistic = -2.16). Furthermore, the coefficient on the interaction term between idiosyncratic volatility and growth options remains negatively and statistically significant at the 1% level (coefficient = -0.0048 and  $t$ -statistic = -3.84). The results suggest that after correcting measurement errors in  $Q$ , idiosyncratic risk continues to have a significant negative effect on investment, and the effect is stronger for firms with more growth options. The coefficient on Tobin's  $Q$  increases from less than 0.01 of the OLS estimator to 0.0271 of the Erickson and Whited (2000) estimator, suggesting the presence of measurement errors in  $Q$ .

## 5.2. Managerial risk aversion

Panousi and Papanikolaou (2012) show that managerial risk aversion can lead to underinvestment when firm-specific uncertainty is high, generating a negative relation between investment and idiosyncratic risk. They further document that this negative relation is more pronounced for firms with high insider ownership, and even more so when firms also have low institutional ownership or

**Table 6**

Investment and idiosyncratic risk: Growth options vs. Managerial risk aversion. Panel A reports the results of the following investment regression, which controls for insider ownership and its interaction with idiosyncratic volatility, in the full sample and three subsamples:

$$\frac{I_{i,t}}{K_{i,t-1}} = \beta_1 IV_{i,t-1} + \beta_2 IV_{i,t-1} * GO_{i,t-1} + \beta_3 GO_{i,t-1} + \beta_4 IV_{i,t-1} * InsiderOwn_{i,t-1} + \beta_5 InsiderOwn_{i,t-1} + \gamma z_{i,t-1} + \alpha_i + \alpha_t + \epsilon_{i,t}$$

In each year, we first sort all firms into high and low insider ownership portfolios by the median insider ownership. We then sort each portfolio into high and low institutional ownership (or *VEGA*) portfolios by the median institutional ownership (or *VEGA*). Results are reported for the full sample (column 1), the subsample with high insider ownership (column 2), the subsample with high insider ownership and low institutional ownership (column 3), and the subsample with high insider ownership and low *VEGA* (column 4). Panel B reports the results of the following investment regression, which includes a quadratic term of idiosyncratic volatility and controls for insider ownership and its interaction with idiosyncratic volatility, in low and high idiosyncratic volatility subsamples:

$$\frac{I_{i,t}}{K_{i,t-1}} = \beta_1 IV_{i,t-1} + \beta_2 IV_{i,t-1}^2 + \beta_3 IV_{i,t-1} * InsiderOwn_{i,t-1} + \beta_4 InsiderOwn_{i,t-1} + \gamma z_{i,t-1} + \alpha_i + \alpha_t + \epsilon_{i,t}$$

Control variables *z* are measured at *t* – 1 and include systematic volatility (*SV*), Tobin's *Q* (*Q*), cash flow-to-assets ratio (*CF*), firm size (*SIZE*) and leverage (*LEV*). See Appendix B for the detailed definition of variables. The sample period is from 1986 to 2017. All the regressions are estimated with firm and time fixed effects. Robust *t*-statistics based on standard errors clustered at the firm level are reported in parenthesis.

Panel A

Sample	Full Sample	High InsiderOwn	High InsiderOwn & Low InstOwn	High InsiderOwn & Low Vega
	(1)	(2)	(3)	(4)
<i>IV</i> ( <i>t</i> – 1)	–0.0056 (–2.56)	–0.0050 (–1.59)	–0.0035 (–0.57)	0.0018 (0.16)
<i>IV</i> ( <i>t</i> – 1) * <i>GO</i> ( <i>t</i> – 1)	–0.0082 (–8.22)	–0.0077 (–5.94)	–0.0135 (–5.73)	–0.0197 (–4.34)
<i>GO</i> ( <i>t</i> – 1)	0.0169 (26.43)	0.0154 (16.29)	0.0165 (12.46)	0.0180 (7.88)
<i>IV</i> ( <i>t</i> – 1) * <i>InsiderOwn</i> ( <i>t</i> – 1)	0.0042 (0.57)	–0.0024 (–0.26)	0.0355 (1.60)	0.0256 (0.87)
<i>InsiderOwn</i> ( <i>t</i> – 1)	0.0092 (1.21)	0.0249 (2.37)	0.0110 (0.63)	–0.0036 (–0.17)
<i>SV</i> ( <i>t</i> – 1)	0.0154 (3.90)	0.0126 (2.16)	0.0165 (1.86)	–0.0034 (–0.30)
<i>Q</i> ( <i>t</i> – 1)	0.0058 (13.93)	0.0057 (9.78)	0.0050 (6.58)	0.0050 (3.93)
<i>CF</i> ( <i>t</i> – 1)	0.0195 (7.84)	0.0176 (5.18)	0.0416 (6.63)	0.0348 (3.25)
<i>SIZE</i> ( <i>t</i> – 1)	–0.0159 (–17.28)	–0.0141 (–11.07)	–0.0155 (–7.89)	–0.0146 (–4.04)
<i>LEV</i> ( <i>t</i> – 1)	–0.004 (–11.70)	–0.005 (–9.23)	–0.005 (–6.65)	–0.004 (–2.94)
Firm fixed effects	Yes	Yes	Yes	Yes
Year fixed effects	Yes	Yes	Yes	Yes
Observations	78,399	38,742	17,585	3636
Adjusted <i>R</i> <sup>2</sup>	0.58	0.59	0.68	0.75

Panel B

	Low				High			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
<i>IV</i> ( <i>t</i> – 1)	0.0624 (4.02)	0.0571 (3.81)	0.0254 (1.75)	0.0131 (0.90)	–0.0349 (–4.69)	–0.0295 (–4.03)	–0.0375 (–4.91)	–0.0404 (–5.20)
<i>IV</i> ( <i>t</i> – 1) <sup>2</sup>	–0.0709 (–3.61)	–0.0731 (–3.81)	–0.0441 (–2.37)	–0.0419 (–2.26)	0.0079 (2.54)	0.0068 (2.22)	0.0085 (2.70)	0.0087 (2.77)
<i>IV</i> ( <i>t</i> – 1) * <i>InsiderOwn</i> ( <i>t</i> – 1)	0.0009 (0.04)	0.0161 (0.66)	0.0126 (0.52)	0.0155 (0.64)	–0.0183 (–1.55)	–0.0172 (–1.48)	–0.0132 (–1.12)	–0.0118 (–1.00)
<i>InsiderOwn</i> ( <i>t</i> – 1)	0.0188 (1.88)	0.0116 (1.19)	0.0023 (0.25)	0.0012 (0.13)	0.0479 (3.04)	0.0434 (2.79)	0.0351 (2.27)	0.0344 (2.22)
<i>SV</i> ( <i>t</i> – 1)				0.0323 (4.60)				0.0212 (3.68)
<i>Q</i> ( <i>t</i> – 1)		0.0121 (16.60)	0.0079 (11.78)	0.0074 (11.07)		0.0074 (13.73)	0.0065 (12.43)	0.0063 (11.90)
<i>CF</i> ( <i>t</i> – 1)			0.0755 (10.66)	0.0758 (10.71)			0.0173 (5.92)	0.0173 (5.92)
<i>SIZE</i> ( <i>t</i> – 1)			–0.0155 (–12.28)	–0.0162 (–12.70)			–0.0137 (–9.47)	–0.0147 (–9.72)
<i>LEV</i> ( <i>t</i> – 1)			–0.005 (–8.89)	–0.005 (–8.60)			–0.005 (–8.35)	–0.005 (–8.13)
Firm fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

(continued on next page)

Table 6 (continued).

Panel B	Low				High			
	Model 1	Model 2	Model 3	Model 4	Model 1	Model 2	Model 3	Model 4
Year fixed effects	Yes							
Observations	41,259	41,259	41,259	41,259	37,140	37,140	37,140	37,140
Adjusted R <sup>2</sup>	0.62	0.64	0.65	0.65	0.47	0.49	0.50	0.50

low convex executive compensation schemes (i.e. low VEGA).<sup>13</sup> In order to control for the effect of managerial risk aversion, we perform our analysis in two ways.

First, we sort firms into quintiles based on the fraction of insider ownership. Within each insider ownership quintile, we further sort firms into quintiles based on their past investment rate. And we pool firms with the same rank of investment together. In this way, we generate subsamples with sufficient dispersion in investment but little difference in insider ownership. Similarly, we generate subsamples with sufficient dispersion in the level of idiosyncratic volatility but little difference in insider ownership. In unreported results, we perform the investment regression in the subsamples and confirm that our previous results remain qualitatively the same after controlling for insider ownership: the negative investment–idiosyncratic risk relation is stronger for firms with more growth options; the negative relation is a U-shaped function of the idiosyncratic volatility level.

Second, instead of performing a subsample analysis, we directly control for the effect of insider ownership by including an additional interaction term between idiosyncratic volatility and insider ownership in the investment regressions. Panel A of Table 6 reports the results of the following investment regression, which controls for insider ownership and its interaction with idiosyncratic volatility, in the full sample and three subsamples:

$$\frac{I_{i,t}}{K_{i,t-1}} = \beta_1 IV_{i,t-1} + \beta_2 IV_{i,t-1} * GO_{i,t-1} + \beta_3 GO_{i,t-1} + \beta_4 IV_{i,t-1} * InsiderOwn_{i,t-1} + \beta_5 InsiderOwn_{i,t-1} + \gamma z_{i,t-1} + \alpha_i + \alpha_t + \varepsilon_{i,t}. \tag{21}$$

The results for the full sample are reported in column 1. It is evident that the effect of growth options remains strong after controlling for insider ownership: the interaction term between idiosyncratic risk and growth options is significantly negative. We further conduct the regression in subsamples where the effect of managerial risk aversion is documented to be more pronounced. Specifically, In each year, we first sort all firms into high and low insider ownership portfolios by the median insider ownership. We then sort each portfolio into high and low institutional ownership (or VEGA) portfolios by the median institutional ownership (or VEGA). Results are reported for the subsample with high insider ownership (column 2), the subsample with high insider ownership and low institutional ownership (column 3), and the subsample with high insider ownership and low VEGA (column 4). It is evident that the effect of growth options remain robust in all of the three subsamples, which is subject to more pronounced effect of managerial risk aversion. Compared with the growth options effect, the effect of insider ownership appears weak. The coefficient on the interaction term between idiosyncratic volatility and insider ownership is not significant in all model specifications.

Panel B of Table 6 further reports the results of the following investment regression, which includes a quadratic term of idiosyncratic volatility and controls for insider ownership and its interaction with idiosyncratic volatility, in low and high idiosyncratic volatility subsamples:

$$\frac{I_{i,t}}{K_{i,t-1}} = \beta_1 IV_{i,t-1} + \beta_2 IV_{i,t-1}^2 + \beta_3 IV_{i,t-1} * InsiderOwn_{i,t-1} + \beta_4 InsiderOwn_{i,t-1} + \gamma z_{i,t-1} + \alpha_i + \alpha_t + \varepsilon_{i,t}. \tag{22}$$

The results clearly show that after controlling for insider ownership, we continue to find a U-shaped investment–idiosyncratic risk sensitivity: the coefficient on the quadratic term of idiosyncratic volatility is negative when idiosyncratic volatility is low, but becomes positive when idiosyncratic volatility is high. In contrast, the coefficient on the interaction term between idiosyncratic volatility and insider ownership is insignificant.

Taken together, our results show that the real options effects remain strong and robust after controlling for the effect of managerial risk aversion but not vice versa. Our results therefore lend strong support to the real options channel but cast doubt on the explanatory power of managerial risk aversion in generating a sizable negative relation between investment and idiosyncratic risk.

## 6. Conclusions

The relation between investment and uncertainty has long been an important but controversial topic. The recent empirical finding of a strong negative relation between investment and firm-specific risk raises a new question: do firms invest optimally when they

<sup>13</sup> In Appendix D, we replicate the baseline results of Panousi and Papanikolaou (2012) in our sample, which shows that the negative investment–idiosyncratic volatility relation is stronger when insider ownership is larger.

face idiosyncratic risk? Whether firms postpone investment optimally due to the option value of waiting or underinvest due to managerial risk aversion has important implications for corporate decisions and policy making.

Motivated by a simple real options model, we provide new insights into the role of growth options in shaping the relation between expected investment and idiosyncratic risk. We show both theoretically and empirically that there is a negative investment–idiosyncratic risk relation when the level of idiosyncratic risk is above a minimum threshold, and the magnitude of the relation is a U-shaped function of the level of idiosyncratic risk. Moreover, the investment–idiosyncratic risk relation is stronger for firms with more growth options relative to their assets in place.

Separating idiosyncratic risk from systematic risk help distinguish the real options explanation from other standard investment theories. Our results remain robust after we control for managerial risk aversion, suggesting the importance of firms’ optimal decisions under uncertainty in explaining the negative relation between corporate investment and idiosyncratic risk.

**CRedit authorship contribution statement**

**Clark Liu:** Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing - original draft, Writing - review & editing. **Shujing Wang:** Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing - original draft, Writing - review & editing.

**Appendix A. Proof**

**Proof of Proposition 2.1.** Assume there is a traded asset  $x$  whose price is perfectly correlated with the cash flow of the project, and there is a risk-free asset whose price evolves according to

$$dy_t = ry_t dt.$$

The value of the growth option of a single project can be replicated by the following dynamic self-financing trading strategy

$$\begin{aligned} V_t^{GO} &= ax + by \\ dV_t^{GO} &= a(dx + x\delta dt) + bdy = (ax\alpha + ax\delta + byr)dt + ax\sigma dz. \end{aligned}$$

Ito’s Lemma gives

$$dV_t^{GO} = (V_x^{GO}\alpha x + \frac{1}{2}V_{xx}^{GO}x^2\sigma^2)dt + V_x^{GO}x\sigma dz.$$

Comparing the coefficients of  $dt$  and  $dz$  for  $dV_t^{GO}$ , we get the following PDE

$$\frac{1}{2}\sigma^2x^2V_{xx}^{GO} + (r - \delta)xV_x^{GO} - rV^{GO} = 0.$$

This is a homogeneous linear equation of second order, which has the following general solution

$$V^{GO}(x) = A_1x^{\beta_1} + A_2x^{\beta_2},$$

where  $\beta_1$  and  $\beta_2$  are the roots of the following quadratic equation:

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + (r - \delta)\beta - r = 0$$

The two roots are

$$\begin{aligned} \beta_1 &= \frac{1}{2} - \frac{r - \delta}{\sigma^2} + \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} > 1 \\ \beta_2 &= \frac{1}{2} - \frac{r - \delta}{\sigma^2} - \sqrt{\left(\frac{r - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} < 0 \end{aligned}$$

And we have the following boundary conditions

$$V^{GO}(0) = 0 \Rightarrow A_2 = 0$$

$$\text{Value-matching condition : } V^{GO}(x^*) = V^{AP}(x^*) - I_0 \Rightarrow A_1(x^*)^{\beta_1} = \frac{x^*}{\delta} - I_0$$

$$\text{Smooth-pasting condition : } V_x^{GO}(x^*) = V_x^{AP}(x^*) \Rightarrow A_1\beta_1(x^*)^{\beta_1-1} = \frac{1}{\delta}$$

Therefore we can solve for  $A_1$ ,  $A_2$ , and  $x^*$

$$\begin{aligned} A_1 &= (\beta_1 - 1)^{\beta_1-1} I_0^{-(\beta_1-1)} (\delta\beta_1)^{-\beta_1} \\ A_2 &= 0 \\ x^* &= \frac{\beta_1}{\beta_1 - 1} \delta I_0. \quad \square \end{aligned}$$

**Lemma 1.** Suppose  $X_t$  follows a Brownian motion process with  $X_0 = 0$

$$dX_t = \bar{\mu}dt + \bar{\sigma}dz_t,$$

Define

$$M_t \equiv \sup\{X_s, 0 \leq s \leq t\},$$

and define the joint distribution function

$$F_t(x, y) \equiv P\{X_t \leq x, M_t \leq y\},$$

Then we have

$$F_t(x, y) = \Phi\left(\frac{x - \bar{\mu}t}{\bar{\sigma}\sqrt{t}}\right) - e^{2\bar{\mu}y/\bar{\sigma}^2} \Phi\left(\frac{x - 2y - \bar{\mu}t}{\bar{\sigma}\sqrt{t}}\right) \tag{23}$$

**Proof.** See Harrison (1985), pp. 12–13. □

**Proof of Proposition 2.2.** Applying Ito’s Lemma, we have

$$d \ln x_t = \left(\alpha - \frac{1}{2}\sigma^2\right)dt + \sigma dz_t.$$

Applying Lemma 1 by specifying the following parameters

$$X_t = \ln \frac{x_t}{x_0}, \quad \bar{\mu} = \alpha - \frac{1}{2}\sigma^2, \quad \bar{\sigma} = \sigma, \quad x = y = \ln \frac{x^*}{x_0}.$$

We get the probability of investing within a fixed time period T

$$\begin{aligned} P_{Inv} &= P(M_T \geq y) = 1 - P\{X_T \leq y, M_T \leq y\} = 1 - F_t(y, y) \\ &= 1 - \Phi\left(\frac{y - \bar{\mu}t}{\bar{\sigma}\sqrt{t}}\right) + e^{2\bar{\mu}y/\bar{\sigma}^2} \Phi\left(\frac{-y - \bar{\mu}t}{\bar{\sigma}\sqrt{t}}\right) \\ &= \Phi\left(\frac{-y + \bar{\mu}t}{\bar{\sigma}\sqrt{t}}\right) + e^{2\bar{\mu}y/\bar{\sigma}^2} \Phi\left(\frac{-y - \bar{\mu}t}{\bar{\sigma}\sqrt{t}}\right) \\ &= \Phi\left(\frac{-\ln \frac{x^*}{x_0} + (\alpha - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) - \left(\frac{x^*}{x_0}\right)^{\frac{2\alpha}{\sigma^2}-1} \Phi\left(\frac{-\ln \frac{x^*}{x_0} - (\alpha - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}\right) \quad \square \end{aligned}$$

**Appendix B. Definition of variables**

**Total volatility (TV), Idiosyncratic Volatility (IV), and Systematic Volatility (SV)** We decompose daily stock returns into systematic and idiosyncratic components by performing the following regression every year:

$$r_{itd} = a_{it} + b'_{it}f_{td} + \varepsilon_{itd},$$

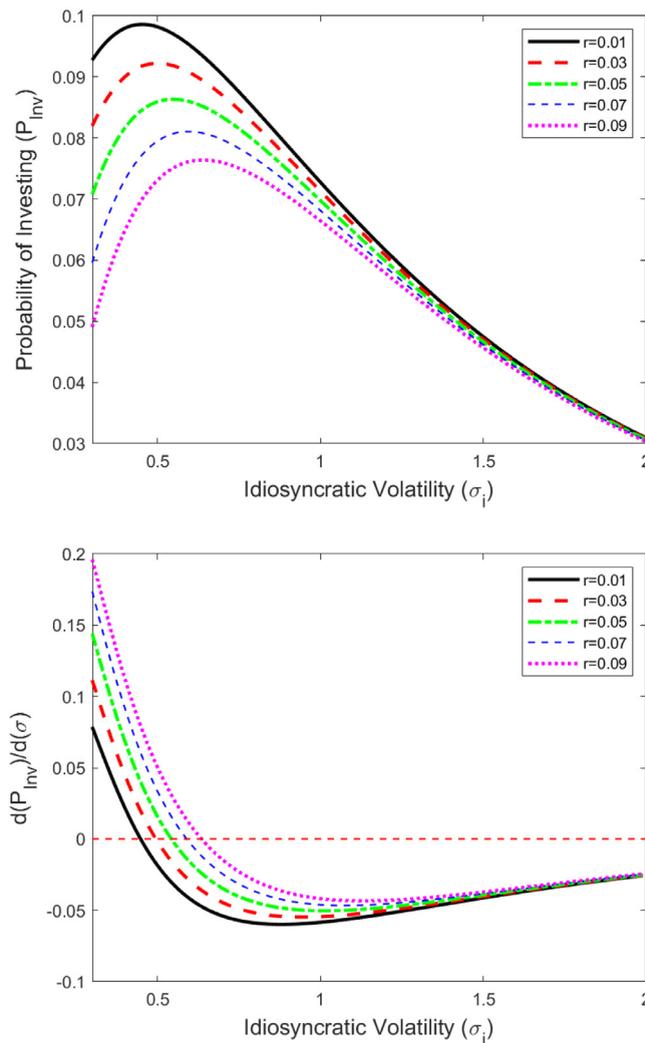
where  $r_{itd}$  is the stock return on trading day  $d$  in year  $t$  for firm  $i$  and  $f_{td}$  is the daily factor return on day  $d$  in year  $t$ . The total volatility, idiosyncratic volatility, and systematic volatility for stock  $i$  in year  $t$  are defined as:

$$\begin{aligned} TV_{it} &= \sqrt{\frac{252}{n} \sum_{d=1}^n r_{itd}^2}, \\ IV_{it} &= \sqrt{\frac{252}{n} \sum_{d=1}^n \varepsilon_{itd}^2}, \\ SV_{it} &= \sqrt{TV_{it}^2 - IV_{it}^2}, \end{aligned}$$

where  $n$  is the number of trading days in year  $t$  for firm  $i$  and all the volatility measures are annualized in order to facilitate comparison across different volatility measures. We require at least 100 daily stock return data points to calculate stock return volatility. Our main analysis is based on the benchmark market model, where the factor is the excess return on the value-weighted market portfolio.

**I/A** Investment-to-assets ratio or investment rate, defined as capital expenditure (item CAPX) scaled by lagged book assets (item AT).

**Tobin’s Q** (book assets – book value of equity + market value of equity) scaled by book assets. Book value of equity is defined as common equity (item CEQ) if available or total assets (item AT) minus liability (item LT), plus balance sheet deferred taxes



**Fig. A.1.** (a) Probability of investing as a function of idiosyncratic volatility for different value of  $r$  in (0.01, 0.03, 0.05, 0.07, 0.09); (b)  $dP/d\sigma$  as a function of idiosyncratic volatility for different value of  $r$  in (0.01, 0.03, 0.05, 0.07, 0.09). Other parameters are set as follows:  $\alpha = 0$ ;  $\mu = 0.15$ ;  $J_0 = 1$ ;  $x_0 = 0.07$ ;  $T = 5$ ;  $\lambda_m = 0.1$ ;  $\sigma_m = 0.2$ .

(item TXDB) if available and investment tax credits (item ITCI) if available, minus preferred stock liquidation value (item PSTKL) if available, or redemption value (item PSTKR) if available, or carrying value (item PSTK) if available. Market value of equity is defined as shares outstanding (CSHO) times share price at the fiscal year end (item PRCC\_f).

**CF** Cash flow-to-assets ratio, defined as (Income before extraordinary items (item IB) + depreciation (item DP)) scaled by lagged book assets.

**SIZE** Firm size, defined as the natural logarithm of book assets (in million US dollars) adjusted by annual CPI, normalized to 2010 dollar value.

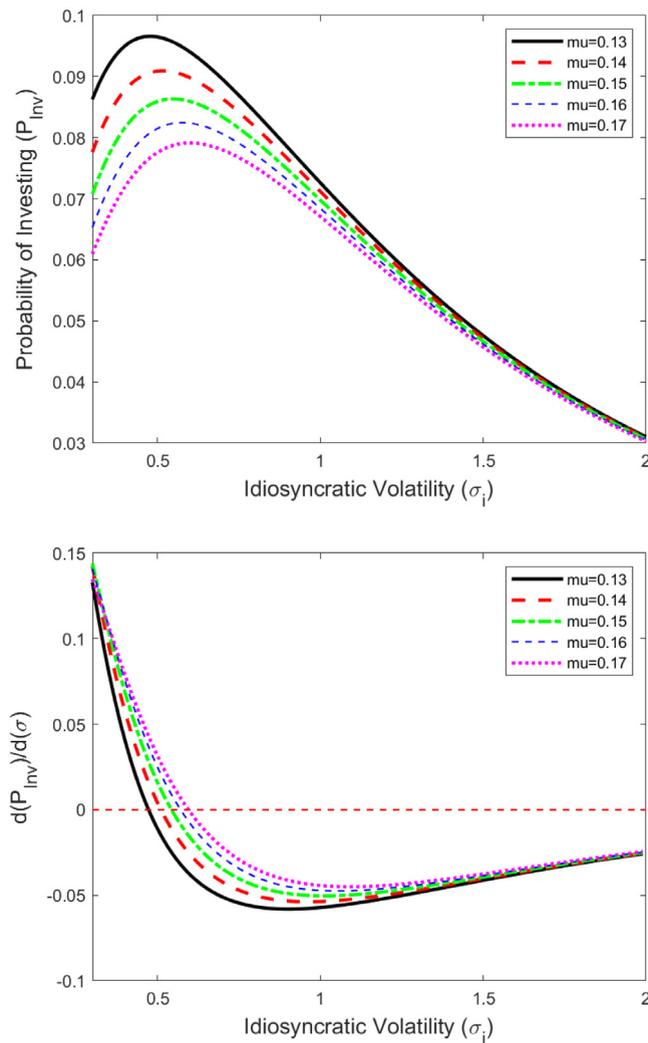
**LEV** Leverage, defined as (long-term debt (item DLTT) + debt in current liability (item DLT)) scaled by Book assets. DLTT and DLC are set to zero when missing.

**SG** Sales growth, defined as the annual growth rate of sales (item SALE).

**PG** Profit growth, defined as the annual growth rate of gross profit (item GP).

**RD/A** Research and development expense (item XRD) scaled by lagged book assets.

**Insider Ownership** Our first measure of insider ownership is from the Thomson Financial Institutional Holdings database of filings derived from Forms 3, 4, and 5. We calculate insider ownership in year  $t$  as the total number of firm shares held by firm officers



**Fig. A.2.** (a) Probability of investing as a function of idiosyncratic volatility for different value of  $\mu$  in (0.13, 0.14, 0.15, 0.16, 0.17); (b)  $dP/d\sigma$  as a function of idiosyncratic volatility for different value of  $\mu$  in (0.13, 0.14, 0.15, 0.16, 0.17). Other parameters are set as follows:  $\alpha = 0$ ;  $r = 0.05$ ;  $I_0 = 1$ ;  $x_0 = 0.07$ ;  $T = 5$ ;  $\lambda_m = 0.1$ ;  $\sigma_m = 0.2$ .

reported at the end of that year or at the latest filing date, as a fraction of the firm's shares outstanding. We include insiders with the following role classifications: O, OD, OE, OB, OP, OS, OT, OX, CEO, CFO, CI, CO, CT, H, GM, M, MD, P, EVP, VP, and SVP.

Our second insider ownership measure adjusts for option holdings by adding option deltas to stock holdings for each executive. We use data on executive option grants from Execucomp. Delta of stock options follow the [Black and Scholes \(1973\)](#) formula. The option holdings are divided into three categories: new grants, existing unexercisable grants, and existing exercisable grants. For new grants, the six parameters for the Black–Scholes formula are readily available. For existing grants, the time to maturity and exercise price are estimated following the methodology in [Core and Guay \(2002\)](#).

**Institutional Ownership** Institutional ownership is from Thomson Reuters 13F dataset.

**VEGA** VEGA of stock options is derived from the [Black and Scholes \(1973\)](#) formula. The option holdings are divided into three categories: new grants, existing unexercisable grants, and existing exercisable grants. For new grants, the six parameters for the Black–Scholes formula are readily available. For existing grants, the time to maturity and exercise price are estimated following the methodology in [Core and Guay \(2002\)](#). We construct the firm-level measures of VEGA by aggregating across executives in Execucomp.

## Appendix C. Numerical analysis with varying parameters

See [Figs. A.1–A.3](#).

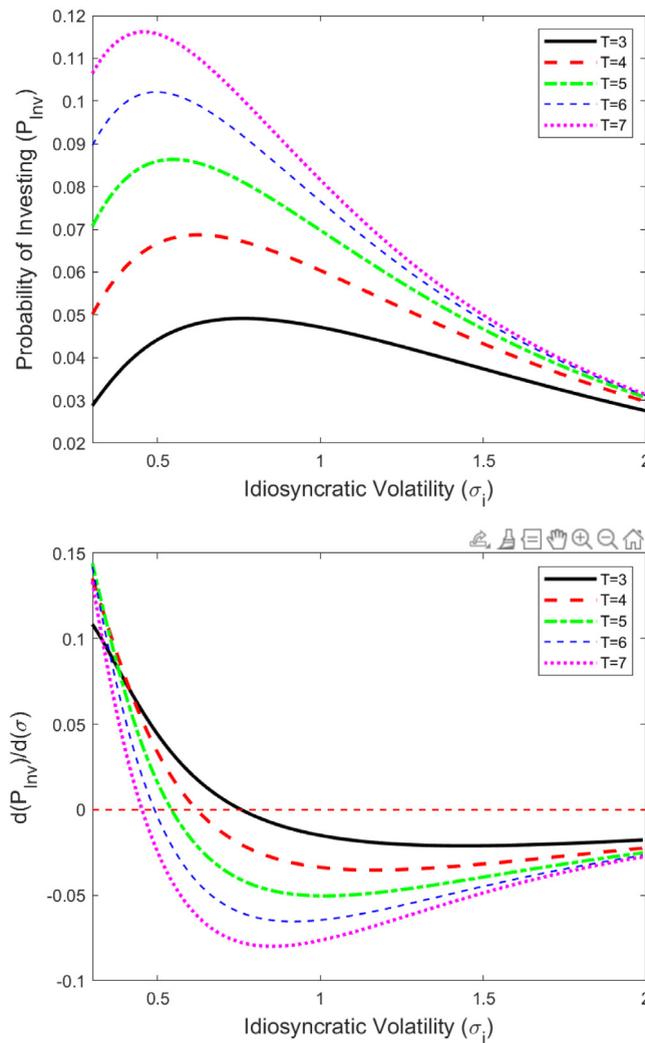


Fig. A.3. (a) Probability of investing as a function of idiosyncratic volatility for different value of  $T$  in (3, 4, 5, 6, 7); (b)  $dP/d\sigma$  as a function of idiosyncratic volatility for different value of  $T$  in (3, 4, 5, 6, 7). Other parameters are set as follows:  $\alpha = 0$ ;  $r = 0.05$ ;  $\mu = 0.15$ ;  $I_0 = 1$ ;  $x_0 = 0.07$ ;  $\lambda_m = 0.1$ ;  $\sigma_m = 0.2$ .

**Appendix D. Investment and idiosyncratic risk: Effect of insider ownership**

This table reports the results of the investment regressions, which include interactions between idiosyncratic volatility and insider ownership quintile dummies. The dependent variable is the investment rate at time  $t$ . We construct two measures of insider ownerships at time  $t - 1$ , one excluding stock options (columns 1–2) and the other including options (columns 3–4). Every year, we sort firms into quintiles based on insider ownership.  $INSD_L$  and  $INSD_H$  indicate the dummies for the first and fifth insider ownership quintile, respectively. Control variables are measured at  $t - 1$  and include systematic volatility ( $SV$ ), Tobin’s  $Q$  ( $Q$ ), cash flow-to-assets ratio ( $CF$ ), firm size ( $SIZE$ ) and leverage ( $LEV$ ). Estimated coefficients for control variables are not reported for brevity. Depending on the specification, we include firm (F) and year (Y) fixed effects and interact them with quintile dummies. The sample period is from 1986 to 2017. The standard errors are clustered at the firm-level, and  $t$ -statistics are reported in parenthesis.

$I(t)/K(t - 1)$	(1)	(2)	(3)	(4)
$INSD_L(t - 1) * IV(t - 1)$	-0.009 (-1.45)	-0.004 (-1.18)	-0.008 (-1.13)	-0.004 (-1.17)
$INSD_2(t - 1) * IV(t - 1)$	-0.014*** (-3.59)	-0.007* (-1.73)	-0.015*** (-3.65)	-0.007* (-1.79)
$INSD_3(t - 1) * IV(t - 1)$	-0.014*** (-3.78)	-0.010* (-1.87)	-0.015*** (-3.80)	-0.010* (-1.84)

$I(t)/K(t-1)$	(1)	(2)	(3)	(4)
$INSD_4(t-1) * IV(t-1)$	-0.013*** (-3.22)	-0.011** (-2.52)	-0.013*** (-2.87)	-0.011** (-2.52)
$INSD_H(t-1) * IV(t-1)$	-0.017*** (-5.25)	-0.019*** (-4.69)	-0.016*** (-4.74)	-0.019*** (-4.74)
$INSD_H - INSD_L$	-0.009	-0.015	-0.008	-0.015
p-value	0.046	0.007	0.066	0.007
Observations	74,628	74,628	74,628	74,628
Fixed effects	F	F, T	F	F, T
Controls	N	Y	N	Y
Option-adjusted ownership	N	N	Y	Y

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