



Management Science

Publication details, including instructions for authors and subscription information:
<http://pubsonline.informs.org>

Strategic Risk Shifting and the Idiosyncratic Volatility Puzzle: An Empirical Investigation

Zhiyao Chen,^a Ilya A. Strebulaev,^b Yuhang Xing,^{c,d} Xiaoyan Zhang^e

To cite this article:

Zhiyao Chen,^a Ilya A. Strebulaev,^b Yuhang Xing,^{c,d} Xiaoyan Zhang^e (2021) Strategic Risk Shifting and the Idiosyncratic Volatility Puzzle: An Empirical Investigation. *Management Science* 67(5):2751-2772. <https://doi.org/10.1287/mnsc.2020.3593>

Full terms and conditions of use: <https://pubsonline.informs.org/Publications/Librarians-Portal/PubsOnLine-Terms-and-Conditions>

This article may be used only for the purposes of research, teaching, and/or private study. Commercial use or systematic downloading (by robots or other automatic processes) is prohibited without explicit Publisher approval, unless otherwise noted. For more information, contact permissions@informs.org.

The Publisher does not warrant or guarantee the article's accuracy, completeness, merchantability, fitness for a particular purpose, or non-infringement. Descriptions of, or references to, products or publications, or inclusion of an advertisement in this article, neither constitutes nor implies a guarantee, endorsement, or support of claims made of that product, publication, or service.

Copyright © 2020, INFORMS

Please scroll down for article—it is on subsequent pages



With 12,500 members from nearly 90 countries, INFORMS is the largest international association of operations research (O.R.) and analytics professionals and students. INFORMS provides unique networking and learning opportunities for individual professionals, and organizations of all types and sizes, to better understand and use O.R. and analytics tools and methods to transform strategic visions and achieve better outcomes.

For more information on INFORMS, its publications, membership, or meetings visit <http://www.informs.org>

Strategic Risk Shifting and the Idiosyncratic Volatility Puzzle: An Empirical Investigation

 Zhiyao Chen,^a Ilya A. Strebulaev,^b Yuhang Xing,^{c,d} Xiaoyan Zhang^e

^a Chinese University of Hong Kong Business School, Chinese University of Hong Kong, Hong Kong; ^b Graduate School of Business, Stanford University, Stanford, California 94305; ^c National Bureau of Economic Research, Cambridge, Massachusetts 02138; ^d Jones Graduate School of Business, Rice University, Houston, Texas 77005; ^e PBC School of Finance, Tsinghua University, Beijing 100083, P.R. China

Contact: nicholaschen@baf.cuhk.edu.hk,  <https://orcid.org/0000-0001-8258-1459> (ZC); istrebulaev@stanford.edu,  <https://orcid.org/0000-0002-0897-0824> (IAS); yxing@rice.edu (YX); zhangxiaoyan@pbcfsf.tsinghua.edu.cn,  <https://orcid.org/0000-0001-6248-0669> (XZ)

Received: April 26, 2018
Revised: September 3, 2019
Accepted: December 1, 2019
Published Online in Articles in Advance: July 17, 2020

<https://doi.org/10.1287/mnsc.2020.3593>

Copyright: © 2020 INFORMS

Abstract. We find strong empirical support for the risk-shifting mechanism to account for the puzzling negative relation between idiosyncratic volatility and future stock returns. First, equity holders take on investments with high idiosyncratic risk when their firms are in distress and receive less monitoring from institutional holders as well as when the aggregate economy is in a bad state. Second, the strategically increased idiosyncratic volatility decreases equity betas, particularly in bad states when the market risk premium is high. The negative covariance between the equity beta and the market risk premium causes low and negative returns and alphas in firms with high idiosyncratic volatility.

History: Accepted by Tomasz Piskorski, finance.

Funding: X. Zhang acknowledges financial support from the National Natural Science Foundation of China [Grant 71790605].

Supplemental Material: The online appendix is available at <https://doi.org/10.1287/mnsc.2020.3593>.

Keywords: risk shifting • agency conflicts • idiosyncratic volatility puzzle

1. Introduction

Do agency conflicts affect stock prices? Our answer is yes. We demonstrate that the well-known risk-shifting problem between equity and debt holders helps to explain the idiosyncratic volatility puzzle. The puzzle refers to the puzzling negative relation between past idiosyncratic volatility and stock returns, documented by Ang et al. (2006, 2009).¹ We model the risk-shifting activities and offer a novel, risk-based explanation for this puzzling observation. First, equity holders undertake investments with high idiosyncratic risk when their firms are in distress or when the aggregate economy is in a bad state. Second, by shifting asset risk to debt holders, equity holders in distressed firms do not necessarily bear all the increased idiosyncratic risk. Instead, the idiosyncratic investments might lead the firms to move in a different direction than the sinking market, thereby reducing the exposure to the market risk (or equity beta), particularly in bad aggregate states in which the market risk premium is high. Consequently, the negative covariance between the market risk premium and the equity beta results in low stock returns and alphas in firms with a high level of idiosyncratic volatility.

Traditional asset pricing models typically exclude any role agents might play in determining stock returns and volatility dynamics. Nevertheless, agency conflicts between equity and debt holders could affect expected stock returns in a significant manner. We introduce the well-

known risk-shifting problem (Jensen and Meckling 1976) into asset/cash flow volatility dynamics and study its implications for the idiosyncratic volatility puzzle. Within this framework, equity is considered a call option on the underlying firm's assets (Merton 1974). Because of limited liability, equity holders do not have to pay anything out of their pockets at bankruptcy. Therefore, they have incentives to delay bankruptcy, which is an American put option according to put-call parity. This put option protects equity holders from downside risk and, therefore, makes them less sensitive to changes in asset values. To fully take advantage of this option, equity holders have incentives to strategically take on high-risk investments to increase the underlying assets' volatility when their firm's profitability is deteriorating. Because they do not have to bear the losses of new investments because of the limited liability, equity holders shift the risk to debt holders.

This risk-shifting mechanism connects a firm's profitability with its idiosyncratic risk as well as the expected return on stocks. To investigate how and to what extent this risk-shifting behavior helps to explain the negative relation between idiosyncratic volatility and subsequent stock returns, we test four predictions. The first two concern the conditions under which firms are likely to engage in risk-shifting activities. The first is conditional on the firm-specific financial status: when firms become distressed, equity holders choose to invest in projects with

high idiosyncratic volatility. The second is conditional on the state of the aggregate economy: equity holders prefer to make investments with high idiosyncratic risk when the market is in a bad aggregate state. Intuitively, equity holders of distressed firms do not want to sink with the market and want to strategically increase idiosyncratic risk in the hope that these “idiosyncratic investments” might generate *positive* cash flows to offset the large *negative* shocks from the market. In other words, equity holders increase idiosyncratic risk to hedge against a bad market and market risk.² Hence, when the market switches to the bad state, a greater increment in the idiosyncratic risk provides more protection for the equity holders and makes the equity less sensitive to the market risk.

Our third and fourth predictions relate to the implications of the risk-shifting behavior for the equity beta and the stock returns in the framework of the conditional capital asset pricing model (CAPM). Following our second prediction that equity holders increase idiosyncratic volatility to lower their equity beta in the bad states, our third prediction states that the firm’s strategic risk-shifting behavior leads to a negative relation between the equity beta and the market premium because the market risk premium is high in the bad states. Finally, in the conditional CAPM framework, because the negative covariance dominates the product of the expected equity beta and the market risk premium, our fourth prediction states that firms with a high level of idiosyncratic volatility receive low returns on average.

We find strong empirical support for the four predictions. Using firm-level panel regressions, we find that our profitability proxy, return on assets (RoA), is associated with the firm’s future risk taking. This negative association shows that equity holders increase their idiosyncratic risk taking when their firm’s profitability declines, providing support for the notion of risk shifting. To ensure that risk shifting is one of the important, sufficient conditions for the changes in idiosyncratic volatility,³ we use two composite indexes: the *o*-score (Ohlson 1980) and the default probability of Merton (1974). The first proxy relies on a historical estimation of the relative weights of other accounting variables, and the second is calculated from the option-based model. The second proxy is particularly suitable for our study because our theoretical predictions are developed from the option-based model as well. We demonstrate that equity holders are more likely to take on investments with high idiosyncratic risk when their firms are in distress. Moreover, we employ institutional holdings to proxy for the effective monitoring of management. Low institutional holdings imply less active monitoring and severer agency conflicts (Shleifer and Vishny 1986). We find that, when profitability declines, management

increases idiosyncratic risk more because it is monitored less by institutional block holders.

In examining our second prediction, that the idiosyncratic risk is higher in bad aggregate states, we use the National Bureau of Economic Research (NBER) recession dates to proxy for bad states. We find that idiosyncratic volatility increases during these times. This finding is in line with recent findings by Bloom (2009), Herskovic et al. (2015), and Bartram et al. (2016). Herskovic et al. (2015) find that the average volatilities of idiosyncratic *cash flow* and stock return residuals are high in recessions. Additionally, Bartram et al. (2016) have confirmed that both idiosyncratic cash flow volatility and return volatility increase with market risk when the market is in a bad state.

Our results still hold when we use different measures of idiosyncratic risk, such as idiosyncratic asset risk and cash flow risk, and use a different measure of operating performance, return on equity (RoE). To our knowledge, we are the first to demonstrate that the negative association between profitability and idiosyncratic risk is much more significant among distressed firms and during times of recession.

The strategically increased idiosyncratic volatility consequently affects the equity risk and returns. To verify our third and fourth predictions, we follow Lewellen and Nagel (2006), estimate the conditional monthly equity beta, and examine its covariance with the market risk premium. We empirically show that the time-varying equity beta is negatively correlated with the market return for the firms with high idiosyncratic volatility. The negative covariance among those firms dominates their levered equity beta, generating low stock returns and negative CAPM alphas for them. To our knowledge, our work is the first to provide a risk-based explanation for the low stock returns and CAPM alphas in firms with high idiosyncratic volatility.

Our paper relates to a few recent papers that examine the idiosyncratic cash flow risk, growth options, and stock returns.⁴ Among them, Babenko et al. (2016) provide a rationale for the idiosyncratic risk puzzle through the lens of a conditional one-factor model in which idiosyncratic risk affects equity betas. Complementing Babenko et al. (2016), our work shows how equity holders’ strategic actions generate the endogenous idiosyncratic risk over the business cycle. That is, the equity holders of a distressed firm increase the idiosyncratic cash flow risk to reduce the equity beta, particularly in recessions when the market risk premium is high.

The risk-shifting behavior of corporations has been studied extensively in previous research. A nonexclusive list includes Leland (1998), Ericsson (2000), Hennessy and Tserlukevich (2008), Cheng and Milbradt (2012), Favara et al. (2017), and Piskorski and Westerfield (2015). Empirically, Eisdorfer (2008) was the first to use a large sample of firms to identify

distressed firms' risk-shifting behavior. He identifies a positive relation between capital investments and uncertainty among distressed firms, which is empirically proxied by stock return volatility. Differently to Eisdorfer (2008), we emphasize the *idiosyncratic* risk taking in response to cross-sectional financial status and aggregate economic states in this paper.

Our paper belongs to an emerging literature that examines the implications of agency conflicts for asset prices. Davydenko and Strebulaev (2007) demonstrate that strategic default decisions made by equity holders have an adverse effect on bond prices. Albuquerque and Wang (2008) examine the impacts of corporate governance on stock valuation and show that firms in countries with weaker investor protection have more incentives to overinvest, lower Tobin's q values, and larger risk premia. Carlson and Lazrak (2010) show that managerial stock compensation induces risk-shifting behavior that helps explain the rates of credit default swaps and leverage choices. Huang et al. (2011) find that mutual funds that increase risk perform worse than funds with stable risk levels and conclude that agency issues might cause risk shifting by fund managers. Favara et al. (2012), Garlappi and Yan (2011), and Hackbarth et al. (2015) study the effect of equity holders' bargaining power at bankruptcy on stock returns. By studying another agency conflict, we demonstrate that the negative association between idiosyncratic volatility and the future stock return might be driven by strategic risk-shifting behavior.

Our paper is related to two contemporaneous papers that connect operating profitability with cross-sectional equity returns. Hou et al. (2015) show that an empirical q -factor model explains more than half of 80 anomalies, including the idiosyncratic volatility anomaly, but does not explicitly explain why their profitability factor determines the association between idiosyncratic volatility and future returns. Fama and French (2016, p. 92) propose a five-factor model to explain the idiosyncratic volatility puzzle and provide additional empirical evidence that "the returns of high volatility stocks behave like those of firms that are relatively unprofitable but nevertheless invest aggressively," which is indeed the manifestation of the standard risk-shifting problem whereby less profitable firms choose to invest more. Nevertheless, Fama and French (2016) do not provide an economic story to explain their finding either. We complement their study by providing a risk-shifting story to connect the aggressive investment behavior of unprofitable/distressed firms with their high-volatility but low stock returns.

The remainder of the paper proceeds as follows. We propose our four predictions in Section 2. Data and empirical measures are described in Section 3. Section 4 contains the empirical results. Section 5 concludes the

paper. We present a simple option-based model in the appendix and an extended model that incorporates the countercyclical market premium in the online appendix.

2. Empirical Predictions

We develop a simple model to generate four testable predictions on risk-shifting behavior and its effects on expected stock returns. We start by presenting the general idea of the cross-sectional risk-shifting behavior in prediction 1 and the time-series risk-shifting behavior in prediction 2. Then, we describe the implications for equity betas and returns in predictions 3 and 4 in the framework of the conditional CAPM. Moreover, we provide numerical illustrations of our four predictions in the appendix and further extend the simple model in the online appendix.

2.1. Strategic Risk Shifting

Risk shifting is different from risk taking.⁵ When investing in high-risk investments and suffering a loss, equity holders in a distressed firm do not have to pay back the debt holders from their own pockets because of their limited liability. Hence, equity holders in a distressed firm do not bear any losses themselves and, instead, shift the increased risk to the debt holders. As a result, high asset/cash flow risk does not guarantee high *equity* risk and, instead, might even lower the equity risk for distressed firms.

More importantly, asset and cash flow risk can be decomposed into systematic risk and idiosyncratic risk components. If the aggregate economy is deteriorating, equity holders in a distressed firm who anticipate a large, negative shock and do not want to sink with the market prefer to invest in projects that are different from, or idiosyncratic to, the market in the hope that these idiosyncratic investments might generate positive cash flows to offset the negative shocks from the deteriorating market. In other words, the action of taking on additional idiosyncratic asset risk is similar to a hedge against the market risk, reducing the equity holders' exposure to the market risk. Therefore, the increased idiosyncratic asset or cash flow risk implies a decrease in the *systematic equity* risk for distressed firms.

The simple model that captures the aforementioned risk-shifting notion can be described as follows. There are two levels of business risk. In the low risk level $l = L$, a firm invests in assets at time 0 and finances the investments with equity and debt. The installed investments produce cash flows X_t , which the firm uses to pay taxes at a rate τ to the government and coupon payments c to the debt holders. The dividend received by the equity holders is the entire cash flow X_t , net of the coupon payments c to the debt holders and net of the tax payments. If the cash flows X_t decline to a low threshold X_r , the firm chooses to invest in high-risk

assets and enters a high risk level, hoping that the increased cash flow volatility might lead to a cash flow windfall, which might save the firm. At the risk-shifting threshold X_r , given a proportional cost $\eta \geq 0$, the equity holders choose an *optimal* increment in cash flow volatility, ϵ^* , to maximize the equity value. If this corrective action does not save the firm, the equity holders decide to go into bankruptcy at the threshold X_d . Bankruptcy leads to immediate liquidation, in which equity holders receive nothing.

With the simple model, we have the following prediction 1 that states how equity holders determine the amount by which they increase the idiosyncratic asset risk in response to the operating performance of their own firm.

Prediction 1. *Equity holders in a distressed firm with a lower expected growth rate in cash flows or operating profits choose a greater increment in the idiosyncratic cash flow volatility.*

The amount of the increase in idiosyncratic risk chosen by the equity holders depends on the severity of the financial status. Equity holders who expect a lower growth rate choose to take on more idiosyncratic risk. Intuitively, the low expected cash flow growth implies a low likelihood of the firm surviving, inducing equity holders to gamble more. Hence, the lower the cash flow rate, the greater the taking of idiosyncratic risk will be. In Section A.5, we use a simple model to numerically illustrate that equity holders choose a *greater* optimal amount of idiosyncratic risk taking in response to a lower asset return.

Prediction 2 connects the status of the aggregate economy with corporate risk-shifting behavior.

Prediction 2. *Distressed firms strategically increase idiosyncratic risk more in bad aggregate states, when the market risk premium is high, than in good aggregate states, when the market risk premium is low.*

This prediction is similar to the first because bad aggregate states adversely affect firms' performance. Whereas the first prediction relies on the *cross-sectional* difference in the expected cash flow rate to generate cross-sectionally different risk-taking decisions, this prediction relies on the time-varying market risk premium to generate time-varying risk-shifting decisions over the business cycle. Intuitively, in addition to the already decreased cash flow level, the high market risk premium in the bad states increases the discount rate and further decreases the firm value, generating an even greater incentive to shift risk. Thus, the increased market risk premium in the bad states induces the distressed firms to take on more idiosyncratic investments, which, in turn, help equity holders to hedge against the market risk in the bad states. Our numerical example in the appendix confirms this insight.

2.2. Stock Returns in the Framework of the Conditional CAPM

The option-based framework and the CAPM are not mutually exclusive. Instead, they are connected. The earliest contingent-claims (or option-based) models can be dated back to the European options model of Merton (1974). By assuming the underlying asset value is driven by a single market factor, Galai and Masulis (1976) were the first to *theoretically* link the options model (Merton 1974) with the standard CAPM with a *constant* market risk premium.⁶ We extend the literature and study the effect of *strategic* risk shifting on the stock returns in the conditional CAPM.

Proposition 1. *When the firm is alive, its conditional excess return of equity r_t^{ex} is*

$$r_t^{ex} = \mathbf{E}_{t-1} \left[r_t^E \right] - rdt = \mathbf{E} \left[\gamma_{l,t} \beta \lambda dt \right] = \mathbf{E}_{t-1} \left[\beta_t^E \lambda_t dt \right], \quad (1)$$

where λ_t is the time-varying market risk premium and β_t^E is the time-varying equity beta

$$\beta_t^E = \gamma_l \beta = \frac{\partial E_{l,t} / E_{l,t}}{\partial X_t / X_t} \beta = \frac{\partial E_{l,t} / E_{l,t}}{\partial V_{l,t} / V_{l,t}} \beta, \quad (2)$$

where γ_l is the stock-cash flow elasticity, β is the cash flow or asset beta, and $V_{l,t}$ and $E_{l,t}$ are the asset value and equity value with a risk level l at time t .

Proof. See the appendix.

In the framework of conditional CAPM, Equation (1) states that the expected excess stock return is simply the market risk premium λ times the equity beta $\beta_{l,t}^E$ for the firms with a level of idiosyncratic risk l . We model the idiosyncratic volatility effect in the conditional CAPM that allows the time-varying levered beta and countercyclical market risk premium. The levered beta in our model effectively captures the size and value effects because Fama and French (1996) argue that size and value factors are indeed the conditioning variables in the conditional CAPM.⁷

Although the financial leverage and levered equity betas help to account for the size and value premia in the conditional CAPM, what is left unexplained is the idiosyncratic volatility effect. We introduce the strategic risk shifting into this framework and show that, because equity holders time the market to change the level of idiosyncratic risk, the equity betas and market risk premium negatively covary, which, in turn, generates the low returns in the firms with high idiosyncratic volatility.

Consider a special case in which the market risk premium is constant and *beta* = 1; we have the following proposition for levered equity betas.

Proposition 2. *When the firm is distressed and has a high level of idiosyncratic volatility, $l = H$ for $X_t > X_r$, and the firm’s equity beta is⁸*

$$\beta_{H,t}^E = 1 + \underbrace{\frac{c/r(1-\tau)}{E_{H,t}}}_{\text{Leverage}} - \underbrace{(1-\omega_{H,1}) \frac{(c/r - V_{H,d})}{E_{H,t}} \left(\frac{X_t}{X_d}\right)^{\omega_{H,1}} (1-\tau)}_{\text{American Put Option of Delaying Bankruptcy (+)}} \quad (3)$$

where r is the risk-free rate, $V_{H,d}$ is the asset value of the high-volatility firm at the bankruptcy threshold X_d , $E_{H,t}$ is equity value of the high-volatility firm, and $\omega_{H,1}$ is the negative root of a characteristic function. They are defined in the appendix.

When the firm is healthy and has a low level of idiosyncratic volatility, $l = L$ for $X_r > X_t > X_D$, and the firm’s equity beta is

$$\beta_{L,t}^E = 1 + \underbrace{\frac{c/r(1-\tau)}{E_{L,t}}}_{\text{Leverage}} + \underbrace{\frac{V_{L,r} - V_{H,r} + \eta\epsilon^2 V_{H,r}}{E_{L,t}} \left(\frac{X_t}{X_r}\right)^{\omega_{L,1}} (1-\tau)(1-\omega_{L,1})}_{\text{Option of Increasing Risk (+)}} - \underbrace{\frac{c/r - V_{H,d}}{E_{L,t}} \left(\frac{X_r}{X_d}\right)^{\omega_{H,1}} \left(\frac{X_t}{X_r}\right)^{\omega_{L,1}} (1-\tau)(1-\omega_{L,1})}_{\text{American Put Option of Delaying Bankruptcy (+)}} \quad (4)$$

where $V_{L,r}$ and $V_{H,r}$ are the asset values of the low- and high-volatility firms at the risk-shifting threshold X_r , $E_{H,t}$ is the equity value of the high-volatility firm, and $\omega_{L,1}$ and $\omega_{H,1}$ are the negative roots of a characteristic function. They are defined in the appendix.

Proof. See the appendix.

After the risk shifting, the equity beta in Equation (3) consists of three components. The first is normalized to one. The second is related to financial leverage as c/r can be regarded as risk-free equivalent debt. Not surprisingly, the equity beta is positively associated with the financial leverage.

The last component, the option of delaying bankruptcy, decreases the equity beta. The option of delaying bankruptcy, which is essentially an American put option, protects the equity holders from downside risk. Given limited liability, equity holders choose to go bankrupt only when the asset value $V_{H,d}$ falls below the risk-free equivalent debt c/r .⁹ Hence, $c/r - V_{H,d} > 0$. Moreover, the greater the cash flow volatility, the more opportunities equity holders have to receive a cash flow windfall. Therefore, equity holders of a firm with high idiosyncratic cash flow volatility v_H have more incentives to delay bankruptcy,

that is, $\partial V_{H,d} / \partial v_H < 0$. Everything else being equal, the payoff of the put option, $c/r - V_{H,d}$, increases with the idiosyncratic volatility $v_H = v_L + \epsilon^*$. Therefore, the increase in the value of the put option because of the strategically increased volatility, ϵ^* , decreases the equity beta. In short, the strategically increased idiosyncratic cash flow volatility, v_H , decreases the equity beta, $\gamma_{H,t}$, for firms with high idiosyncratic risk.

Prior to shifting the risk, the equity beta in Equation (4) has four elements for a preshifting firm, and the option to increase asset risk is a new element. The option to increase asset risk has a positive effect on the equity beta. Although the asset value decreases from $V_{L,r}$ to $V_{H,r}$ at X_r , the equity value increases from $E_{L,r}$ to $E_{H,r}$ because of the optimal increase in idiosyncratic risk ϵ^* . This contrast implies that the equity holders gain by taking on high-risk investments and transfer wealth from the debt holders to themselves. In contrast, the option to delay bankruptcy has a negative effect on the equity beta although it is slightly different from that in Equation (3). However, because the firm is still at the low level of risk, this out-of-the-money put option is less valuable to this healthy firm than it is to the underperforming firm at the high level of risk. Therefore, the option of increasing idiosyncratic risk dominates the option of delaying bankruptcy, and the potential increment of ϵ might positively affect the equity beta only among the preshifting firms.¹⁰

The following prediction 3 presents the relation between the equity betas and the market risk premium.

Prediction 3 states that the high equity betas of those distressed firms covary negatively with the market risk premium, λ_t , that is, $\text{cov}(\beta_t^E, \lambda_t) \leq 0$, because of the hedging effect from the increased idiosyncratic volatility.

The negative covariance between the equity betas and the market risk premium plays an important role in determining the expected stock returns for the firms with high idiosyncratic volatility. It is worth noting that equity holders do not necessarily bear the asset risk they have increased. Instead, they shift the increased risk to debt holders. Particularly, when the market switches from the good aggregate state to the bad state, the distressed firms strategically increase their idiosyncratic volatility and lower their exposure to the bad market (or equity beta). Because the market risk premium is high in the bad state, the equity beta and market risk premium are negatively correlated in the time series.

Moreover, the covariance between the market risk premium and the levered equity beta is independent of the level of the equity beta because of the definition of covariance. In other words, the high-volatility firms could have a high levered beta because of their depressed equity value and, thus, increase their

idiosyncratic risk more in response to the increased market risk premium.¹¹

Following prediction 3, the next prediction states the unconditional expected stock returns and CAPM alphas that are implied by the conditional CAPM.

Prediction 4 states that, for firms that have strategically increased their idiosyncratic volatility to a high level, if the negative covariance between the equity beta and the market risk premium dominates the product of the expected equity beta and the expected market risk premium, that is, $\mathbf{E}[\beta_t^E]\mathbf{E}[\lambda_t]dt + \mathbf{cov}(\beta_t^E, \lambda_t)dt < 0$, those firms are expected to earn low stock returns and CAPM alphas.

Lewellen and Nagel (2006) show that, if the conditional CAPM holds, the unconditional expected excess return is

$$\mathbf{E}[r_t^{ex}] = \mathbf{E}[\beta_t^E \lambda_t dt] = \mathbf{E}[\beta_t^E]\mathbf{E}[\lambda_t]dt + \mathbf{cov}(\beta_t^E, \lambda_t)dt, \quad (5)$$

and the unconditional alpha α^u is¹²

$$\alpha^u \approx \mathbf{cov}(\beta_t^E, \lambda_t)dt - \frac{\mathbf{E}[\lambda_t]}{(\mathbf{E}[\sigma_t^m])^2 \mathbf{cov}(\beta_t^E, (\sigma_t^m)^2)}, \quad (6)$$

where σ_t^m is the time-varying market volatility.

There are two components in Equation (5). The first component, $\mathbf{E}[\beta_t^E]\mathbf{E}[\lambda_t]dt$, increases with idiosyncratic volatility because of the leverage effect. The intuition is as follows. Because the distressed firms increase their idiosyncratic risk, the high idiosyncratic volatility implies low asset value and equity value, which, in turn, increase the leverage and the equity beta via the leverage effect. This leverage effect resulting from the depressed equity value can be seen by comparing the leverage component in Equation (4) with that in (3).

In contrast, the second component $\mathbf{cov}(\beta_t^E, \lambda_t)dt < 0$ from prediction 3.¹³ Therefore, the expected return $\mathbf{E}[r_t^{ex}]$ for high idiosyncratic volatility firms is low if the negative covariance $\mathbf{cov}(\beta_t^E, \lambda_t)$ dominates the positive $\mathbf{E}[\beta_t^E]\mathbf{E}[\lambda_t]$.

3. Data

We obtain stock returns from the Center for Research in Security Prices (CRSP) and accounting information from quarterly Compustat industrial data. To align with the availability of quarterly Compustat data, our sample period runs from January 1975 to December 2016. We restrict the sample to firm–quarter observations with nonmissing values for operating income and total assets and with positive total assets. We include common stocks listed on the NYSE, AMEX, and NASDAQ with CRSP share code 10 or 11. We exclude firms from the financial and utility sectors. The Fama–French factors and the risk-free rates are obtained from the website of Kenneth French. We merge the quarterly accounting data with the monthly stock return and idiosyncratic volatility. Following the

literature, to ensure the accounting variables are observable to investors, we lag them by two months.

3.1. Operating Performance Variables

There are different proxies for measuring a firm’s operating performance. We follow the literature (e.g., Hovakimian et al. 2001) and use RoA to proxy for profitability.¹⁴ To mitigate the potential seasonality problem resulting from the quarterly data, we use the average of operating income (OIBDPQ) from quarter t to $t - 3$ and then calculate the RoA by dividing the operating cash flow by the assets (ATQ) of quarter $t - 4$. Similarly, we construct an alternative proxy, RoE, as net income before depreciation (IBQ + DPQ) from quarters t to $t - 3$ divided by the assets (ATQ) of quarter $t - 4$. Our choice of RoE is motivated by Hou et al. (2015), who use the RoE to construct a profitability pricing factor in their q -factor model.

3.2. Idiosyncratic Risk Variables

Because the risk shifting is unobservable, we compute three proxies for subsequent idiosyncratic risk taking: namely the idiosyncratic volatility of stock returns, assets, and RoA.

The first measure is the idiosyncratic volatility of stock returns. The previous literature, including Eisdorfer (2008) and Hirshleifer et al. (2012), uses stock return volatility to proxy for the underlying cash flow volatility. Because our goal is to explain the idiosyncratic return volatility puzzle, we follow Ang et al. (2006) and estimate the idiosyncratic volatility of stock returns as the standard deviation of the residuals of daily stock returns.

We use daily stock returns to construct time series of idiosyncratic volatilities over one and three months. We first estimate the daily stock return residuals from the Fama–French (1993) three-factor model for quarter or month t as follows:

$$r_{i,d}^E = \alpha_{i,t} + \beta_{i,t}^{MKT} r_d^{MKT} + \beta_{i,t}^{SMB} r_d^{SMB} + \beta_{i,t}^{HML} r_d^{HML} + u_{i,d}, \quad (7)$$

where $r_{i,d}^E$ is the daily stock return for firm i at day d and r_d^{MKT} , r_d^{SMB} , and r_d^{HML} are the daily market, size, and value factors of Fama and French, respectively. To ensure an accurate estimate of idiosyncratic volatility, we require at least 50 daily return observations within one quarter for the three-month idiosyncratic volatility and at least 15 observations within one month for the one-month volatility. We then compute the stock return idiosyncratic volatility, $v_{i,t}^E$, as the standard deviation of daily residuals for each firm–quarter and each firm–month, respectively.

The asset volatility is suitable in our study because the dynamics of the asset values, V_t , share the growth rate and volatility of cash flows, X_t , that is, $dV_t/V_t = dX_t/X_t$ in our model. We remove the financial leverage effect from the idiosyncratic stock

return volatility. That is, $v_{i,t}^A = (1 - Lev_{i,t})v_{i,t}^E$, where $Lev_{i,t}$ is the financial leverage. This measure allows us to keep the advantage of high-frequency data and has frequently been used in the literature. For example, Schaefer and Strebulaev (2008) construct the asset volatility to study the predictability of the equity–debt hedging ratio. Choi (2013) constructs a similar measure to examine the value premium. Finally, Chen et al. (2013) create asset systematic risk to study its implications for debt maturity.

Our third measure of risk taking is the annualized standard deviation of 12 quarterly RoA residuals. As argued in Irvine and Pontiff (2009), increases in idiosyncratic return volatility can be attributed to increases in the idiosyncratic volatility of fundamental cash flows. To get rid of market-wide fluctuations in the RoA, we first obtain the firm-specific RoA, $u_{i,t}^{RoA}$, by regressing the firm-level RoA on the market-level RoA for the whole sample:

$$RoA_{i,t} = a_i + b_i RoA_{M,t} + u_{i,t}^{RoA}, \quad (8)$$

where $RoA_{M,t}$ is the market-level RoA proxied by the average of the RoA values, weighted with the book assets, across all firms at quarter t . We then compute $v_{i,t}^{RoA}$ as the standard deviation of the residual RoA from the next 12 future quarters.

3.3. Distress Indicators

Taking risk does not necessarily mean shifting risk—just doing so when firms are in distress as we discussed in relation to our first prediction. To ensure that the negative relation between the RoA and risk taking is driven by the risk-shifting mechanism, we use three conditional variables to proxy for the firm’s distress status. The three conditional variables indirectly indicate that the firms are more likely to shift risk. The use of indicator variables allows a better and clearer interpretation of the nonlinear risk-shifting effect.

The first condition is that firms have a high o -score, which is a composite index of a firm’s financial status, estimated and proposed by Ohlson (1980). We calculate the o -score as follows:

$$\begin{aligned} o\text{-score}_{i,t} = & -1.32 - 0.407 \ln(TA_{i,t}) + 6.03 \frac{TL_{i,t}}{TA_{i,t}} \\ & - 1.43 \frac{WC_{i,t}}{TA_{i,t}} + 0.076 \frac{CL_{i,t}}{CA_{i,t}} \\ & - 1.72I(TL_{i,t} > TA_{i,t}) - 2.37 \frac{NI_{i,t}}{TA_{i,t}} \\ & - 1.830.18 \frac{FFO_{i,t}}{TL_{i,t}} \\ & + 0.285I(\text{continuous two-quarter net loss}) \\ & - 0.521 \frac{NI_{i,t} - NI_{i,t-1}}{|NI_{i,t}| - |NI_{i,t-1}|}, \end{aligned}$$

where TA is total assets, TL is total liabilities, WC is working capital, CL is current liabilities, CA is current assets, the indicator $I(\cdot)$ equals one if the condition is met and zero otherwise, NI is net income, and FFO is funds from operations. The greater the o -score is, the more distressed the firm is. We sort all firms into terciles based on their o -score from the previous quarter and classify those in the top tercile as distressed.

The second conditional variable is the Merton (1974) default probability. This variable particularly suits our study because it is an option-based indicator. We follow Vassalou and Xing (2004) and calculate the objective default probability as $\pi = \mathbf{N}(-DD)$, where DD denotes the distance to default as follows:

$$DD = \frac{\log(V_t/F + (\mu - 0.5\sigma^2)T)}{\sigma\sqrt{T}}, \quad (9)$$

where $\mathbf{N}(\cdot)$ is the cumulative probability function of a standard normal distribution, μ is the growth rate of the asset value V_t , $T = 1$ is the time to maturity, σ is the volatility of the firm’s asset value, and F is the sum of half of the long- and short-term debt (annual Compustat item 1/2 DLTT + DLC). Both the asset value V_t and its annual volatility σ are estimated using the iteration method suggested by Vassalou and Xing (2004). We lag quarterly accounting variables to ensure that the accounting information is available to investors at the time of calculation. As with the o -score, we sort the firms into terciles based on their default probabilities in the last quarter and classify those in the top tercile as distressed.

Finally, to test our second prediction related to the business cycle, we use NBER recession dates to proxy for bad aggregate states.

3.4. Monitoring of Institutional Holders and Asset Sales

We use institutional equity holdings data to construct a proxy for the effectiveness of monitoring because Demsetz (1983) and Shleifer and Vishny (1986) argue that institutional holders of large blocks of shares have greater incentives to monitor managers. The lower the holdings by institutional investors, the less monitoring there is and, thus, the more severe the agency conflict or the risk shifting. Quarterly institutional holdings data from 1978 are obtained from the Thomson–Reuters Institutional Holdings (13F) Database.¹⁵ We calculate the fraction of institutional holdings as the shares of the top five block holders divided by the total shares outstanding. Similar to what we did with the distress indicators, we sort the firms into terciles based on the fraction of block holding of the last quarter and classify those in the bottom tercile as having low institutional holdings and monitoring.¹⁶

Arnold et al. (2017) find that distressed firms finance their investments with asset sales. Although Arnold et al. (2017) do not examine the resulting change in the riskiness after distressed asset sales, we complement their results and examine whether the idiosyncratic return volatility increases after “distressed” asset sales. Following their study, we calculate *AssetSale* as the sold assets (Item SPPE) divided by the assets in the last quarter (item PPENT).

3.5. Control Variables

When testing the first two predictions, we control for firm size, growth opportunities, and financial leverage. We use the logarithmic value of assets (Compustat item ATQ), $\log(BA)$, to proxy for the firm size; book-to-market assets, *MABA*, for the growth opportunities; and market leverage, *MktLev*, for the financial leverage. Market leverage, *MktLev*, is measured as the ratio of total debt to the total market asset value, which is the sum of total debt (item DLCQ plus item DLTTQ) and the market value of equity (PRCCQ times CSHOQ). In addition, although we assume managers act on behalf of equity holders and do not model their risk-taking incentives explicitly, we follow Hirshleifer et al. (2012) and control for managerial compensation because stock-based compensation has an effect on managerial risk taking. Using Standard and Poor’s ExecuComp database, we calculate delta and vega using the one-year approximation method of Core and Guay (1999) and take the natural logarithms of these two variables. *Delta* is defined as the dollar change in a CEO’s stock and option portfolio given a 1% change in stock price, which measures the managerial incentive to increase the stock price. *Vega* is the dollar change in a CEO’s option holdings in response to a 1% change in stock return volatility, which measures the risk-taking incentives generated by the managerial stock option holdings.

When testing the third prediction on the association between the equity beta and the market risk premium, we follow the literature and control for monthly contemporaneous factor loadings and lagged firm characteristics in our regressions. Firm characteristics include size (the natural logarithm of market equity, *ME*), book-to-market equity (*BE/ME*), market leverage (*MktLev*), and the previous six months’ cumulative stock return (*PreRets*).

Finally, we winsorize all the variables at the top and bottom 1% to reduce the impact of outliers and lessen the power of potential errors.

4. Empirical Results

In this section, we start by providing summary statistics. Then, we test the first two predictions on the risk-shifting behavior when the firm is in distress or

the aggregate economy is in a bad state. Lastly, we proceed to assess the next two predictions on the equity beta, returns and CAPM alphas.

4.1. Summary Statistics

Table 1 presents summary statistics for all the key and control variables we use in this study. We report the number of observations, minimums, 25th percentiles (P25), means, medians, 75th percentiles (P75), maximums, standard deviations, and the first autocorrelation coefficients (AR(1)).

On average, our sample includes 2,797 to 3,384 firms per quarter. The annualized RoA, our proxy for profitability, has a mean of 10.00% and a standard deviation of 20.87%. RoA is also highly persistent with an autocorrelation of 0.96. Similarly, the RoE, the alternative proxy for profitability, has a smaller mean of 4.36% and a standard deviation of 20.18%. For the three proxies for idiosyncratic risk, the mean of the annualized idiosyncratic return volatility $v_{i,t}^E$ computed over three months is 55.95%; that of idiosyncratic asset volatility $v_{i,t}^A$ is 41.64%; and the mean of the volatility of 12-quarter RoA, $v^{RoA}_{i,t}$, is 11.81%. All three proxies are highly persistent as indicated by their AR(1) coefficients, which are at least 0.69.

We have three firm-level conditioning variables. The *o-score*, our first proxy for financial status, ranges from -92.45 to 245.35 . The expected default probability (*DefProb*) of Merton (1974) is 6%, which is largely consistent with the actual default probability of 5% among U.S. firms. The third conditioning variable is the percentage of large institutional investors, which has a mean of 37% and a median of 31%. Moreover, asset sales (*AssetSale*), the potential cause of the idiosyncratic risk, are small and have a mean of 0.88%. As for the control variables, the average asset size is $139.77 (e^{4.94})$ million dollars. Market-to-book assets (*MABA*) and market leverage (*MktLev*) have means of 1.90 and 0.24, respectively, and are both highly persistent.

Panel B presents the monthly data we use to test the third prediction on the relation between the equity beta and the market risk premium. The annualized monthly stock return has an average of 15.57% and is slightly negatively serially correlated. The average annualized idiosyncratic volatility computed over one month has an average of 50.06%. The average size and book-to-market equity ratio in our monthly data are $76.71 (e^{4.34})$ million dollars and 0.89, respectively, both of which are about the same as those of a median firm in the U.S. stock markets. The average firm leverage ratio is 0.26. The average annualized lagged six-month cumulative return (*PreRets*) is 15.80% with a standard deviation of 87.39%. Overall, the statistics of our main variables are largely consistent with the empirical literature.

Table 1. Summary Statistics of Empirical Measures

Panel A: Quarterly data									
	Observations/Quarter	Minimums	P25	Mean	Median	P75	Maximums	Standard deviation	AR(1)
$RoA_{i,t}(\%)$	2,797	-165.31	4.36	10.00	12.89	20.65	62.51	20.87	0.86
$RoE_{i,t}(\%)$	3,182	-196.30	1.41	4.36	8.21	14.04	47.87	20.18	0.84
$v_{i,t}^E(\%)$	3,382	0.01	29.02	55.95	44.20	68.69	3635.91	44.58	0.69
$v_{i,t}^A(\%)$	3,356	0.00	19.33	41.64	31.73	52.97	3409.03	35.91	0.72
$v_{i,t}^{RoA}(\%)$	2,384	0.14	3.99	11.81	7.33	14.19	98.33	12.90	0.97
$o - score_{i,t}$	3,269	-92.45	-2.05	-0.71	-0.89	0.38	245.35	2.49	0.78
$DefProb_{i,t}$	3,382	0.00	0.00	0.06	0.00	0.00	1.00	0.18	0.82
$Inst_{i,t}$	3,004	0.00	0.10	0.37	0.31	0.59	8.88	0.30	0.93
$AssetSale_{i,t}$	3,323	-14.89	0.00	0.88	0.00	0.00	82.24	4.08	0.31
$\log(BA)_{i,t}$	3,365	-2.65	3.45	4.94	4.80	6.31	13.65	2.09	1.00
$MABA_{i,t}$	3,213	0.43	1.03	1.90	1.37	2.08	26.05	1.63	0.92
$MktLev_{i,t}$	3,327	0.00	0.02	0.24	0.17	0.38	1.00	0.24	0.96
$\log(1 + Delta)_{i,t}$	3,382	0.00	0.00	0.83	0.00	0.00	13.00	2.04	0.96
$\log(1 + Vega)_{i,t}$	3,382	0.00	0.00	0.59	0.00	0.00	9.25	1.51	0.97

Panel B: Monthly data									
	Observations/Month	Minimums	P25	Mean	Median	P75	Maximums	Standard deviation	AR(1)
$r_{i,t}^E(\%)$	3,482	-786.94	-81.59	15.57	0.31	89.42	2904.66	68.67	-0.05
$v_{i,t}^E(\%)$	3,482	2.76	26.80	50.06	40.53	61.44	662.60	36.42	0.59
$Size_{i,t}$	3,482	-0.86	2.96	4.34	4.19	5.56	11.50	0.66	1.00
$BE/ME_{i,t}$	2,661	0.04	0.41	0.89	0.68	1.08	30.99	0.51	0.95
$PreRets_{i,t}$	3,362	-166.58	-29.35	15.80	5.24	44.60	1302.37	87.39	0.80
$MktLev_{i,t}$	2,705	0.01	0.06	0.26	0.20	0.40	0.95	0.11	0.99

Notes. This table reports the number of observations, minimums, 25th percentiles (P25), means, medians, 75th percentiles (P75), maximums, standard deviations, and the first autocorrelation coefficients (AR(1)) for all the key and control variables. Panel A includes return on assets ($RoA_{i,t}$), return on equity ($RoE_{i,t}$), idiosyncratic stock return volatility ($v_{i,t}^E$), idiosyncratic volatility of assets ($v_{i,t}^A$), idiosyncratic volatility of 12-quarter RoA ($v_{i,t}^{RoA}$), o -score (Ohlson 1980), default probability ($DefProb$) of Merton (1974), fraction of institutional block holders ($Inst_{i,t}$), asset sales ($Assetsales_{i,t}$), natural logarithm of assets ($\log(BA)_{i,t}$), market-to-book assets ($MABA_{i,t}$), market leverage ($MktLev_{i,t}$), and natural logarithms of delta and vega of managerial stock options. All the variables are expressed as an annual percentage wherever possible. The monthly variables in panel B include the stock return ($r_{i,t}^E$), monthly idiosyncratic stock return volatility ($v_{i,t}^E$), logarithm of market capitalization ($Size_{i,t}$), book-to-market equity ($BE/ME_{i,t}$), cumulative six-month stock returns ($PreRets_{i,t}$), and market leverage ($MktLev_{i,t}$).

Table 2 summarizes the average returns of the value-weighted stock portfolios. Panel A shows that, although the difference in the stock returns for the firms in the lowest o -score tercile is -6.21% per year, the difference for the firms in the top tercile is -18.42% per year. The contrast suggests that the idiosyncratic volatility puzzle is stronger in distressed firms. Similarly, in panel B, where we use Merton’s default probability as the proxy for the distress status, the contrast between firms with low and high default probabilities is even stronger. Specifically, among the firms with the lowest default probabilities, the idiosyncratic volatility discount is only -6.37% per year and statistically insignificant with a t -statistic of -1.37 . In contrast, among those with the highest default probabilities, the volatility discount is -21.63% per year with a significant t -statistic of -5.13 .

When the conditional variable is the institutional holdings, in panel C, the difference in stock returns

is -15.09% per year (t -statistic = -2.37) among the firms with a low fraction of institutional holdings, and it is only -5.27% (t -statistic = -0.95) in the top tercile. This strong contrast implies that, when the management is subject to lower monitoring, the agency conflict and the idiosyncratic volatility effect are stronger.

In short, we find consistent evidence that the negative effects of idiosyncratic volatility on stock returns are stronger among firms that are distressed and whose management is subject to less monitoring from institutional holders.

4.2. Association Between Profitability and Risk Shifting

Our first prediction is that equity holders who expect low profitability take on investments with high idiosyncratic risk. We empirically test whether idiosyncratic risk significantly increases at quarter t given a decrease in $RoA_{i,t-1}$.

Table 2. Excess Returns

Panel A: Returns from double sort on <i>o</i> -score and idiosyncratic volatility						
	L(ow) IVol	2	3	4	H(igh) IVol	H – L
Low <i>o</i> -score	12.92	13.77	13.15	10.68	6.71	–6.21
(<i>t</i>)	(5.73)	(4.36)	(3.41)	(2.26)	(1.26)	(–1.42)
2	14.66	15.83	14.88	11.96	2.66	–12.00
(<i>t</i>)	(6.05)	(5.24)	(3.86)	(2.60)	(0.51)	(–2.86)
High <i>o</i> -score	14.94	15.02	11.78	8.46	–3.48	–18.42
(<i>t</i>)	(5.85)	(4.38)	(2.80)	(1.70)	(–0.60)	(–3.77)
Panel B: Returns from double sort on default probability and idiosyncratic volatility						
	L(ow) IVol	2	3	4	H(igh) IVol	H – L
Low DefProb	12.83	15.25	15.93	13.60	6.46	–6.37
(<i>t</i>)	(5.95)	(5.40)	(4.46)	(2.93)	(1.20)	(–1.37)
2	12.63	12.02	12.58	9.02	–2.10	–14.73
(<i>t</i>)	(4.44)	(3.66)	(3.28)	(2.01)	(–0.42)	(–3.75)
High DefProb	16.97	11.16	7.73	3.13	–4.66	–21.63
(<i>t</i>)	(4.53)	(2.72)	(1.65)	(0.60)	(–0.81)	(–5.13)
Panel C: Returns from double sort on institutional holdings and idiosyncratic volatility						
	L(ow) IVol	2	3	4	H(igh) IVol	H – L
Low Inst	12.69	7.73	11.26	5.05	–2.40	–15.09
(<i>t</i>)	(4.74)	(1.91)	(2.16)	(0.80)	(–0.33)	(–2.37)
2	12.63	13.40	10.73	7.65	1.82	–10.81
(<i>t</i>)	(4.68)	(3.81)	(2.32)	(1.28)	(0.26)	(–1.87)
High Inst	13.21	13.37	13.87	11.12	7.94	–5.27
(<i>t</i>)	(4.54)	(3.53)	(2.87)	(1.81)	(1.15)	(–0.95)

Notes. This table reports the averages of the value-weighted excess returns of double-sorted portfolios. At the beginning of a month, firms are first sorted into terciles based on their *o*-score (panel A), default probability (panel B), and institutional holdings (panel C) and then sorted based on the idiosyncratic volatility of equity returns in the last month. The idiosyncratic volatility of equity returns is the standard deviation of the residuals of daily stock returns in each month. We lag the *o*-score, the default probability, and the institutional holdings by two months to ensure the information would be observable to investors. The average equity returns are computed over the next month, and the portfolios are rebalanced each month. The *t*-statistics in parentheses are calculated based on the heteroskedasticity-consistent standard errors of Newey and West (1987).

To examine the firms' risk-taking policy in response to changing asset values, we perform the standard panel regressions at the firm level, as follows:¹⁷

$$y_{i,t} = a + bRoA_{i,t-1} + cD(.)RoA_{i,t-1} + dD(.) + f Control_{i,t-1} + e_{i,t}, \quad (10)$$

where the dependent variable $y_{i,t}$ is the proxy of risk taking, the idiosyncratic return volatility over the next three months, $v_{i,t}^E$. To examine the asymmetric association between profitability and idiosyncratic risk taking, we include a dummy variable, $D(.)$, to identify the scenarios in which the firms are in distress or their management is subject to less monitoring from institutional holders. The indicator takes a value of one if the *o*-score of the last quarter is classified into the top tercile ($OS = 3$), if the default probability of Merton is classified into the top tercile ($DefProb = 3$), if the economy of the previous month falls into an NBER recession period, or if the institutional holdings of the

top five block holders are classified into the bottom tercile ($inst = 1$). Although b_t measures idiosyncratic risk taking in response to the RoA, regardless of the likelihood of risk shifting, c_t measures the additional effect when risk shifting is highly likely to occur. That is, $b_t + c_t$ captures the effect of $RoA_{i,t-1}D(.)$ on the future idiosyncratic volatility when risk shifting is more likely, that is, $D(.) = 1$. Finally, we include a vector of various control variables, $Control_{i,t-1}$ described in the previous section, such as the logarithmic value of assets, book-to-market equity, the market leverage ratio, delta and vega of managerial stock options, and standardized unexpected earnings. We include fixed firm and time effects. The estimated coefficients of $D(.)$ are absorbed by the firm fixed effect.

We report the regression results in Table 3. We consider two alternative specifications for each scenario: namely Reg I and II. Reg I does not include control variables, and Reg II does. The first two columns are the baseline case with no interaction with

Table 3. Profitability and Subsequent Idiosyncratic Risk Taking

	Baseline		OS = 3		Def Prob = 3		Recession = 1		inst = 1	
	Reg I	Reg II	Reg I	Reg II	Reg I	Reg II	Reg I	Reg II	Reg I	Reg II
Intercept	60.25	87.06	50.01	78.04	54.87	84.29	58.93	80.58	55.78	85.87
(<i>t</i>)	(308.01)	(36.25)	(171.76)	(36.47)	(261.65)	(34.89)	(537.27)	(63.33)	(221.46)	(34.83)
RoA _{<i>i,t-1</i>}	-0.40	-0.25	-0.18	-0.12	-0.27	-0.16	-0.41	-0.28	-0.34	-0.19
(<i>t</i>)	(-19.11)	(-19.97)	(-14.83)	(-9.93)	(-16.85)	(-13.38)	(-39.36)	(-28.77)	(-16.24)	(-15.13)
D(.)			24.49	17.90	13.02	4.56	14.41	12.24	11.53	4.39
(<i>t</i>)			(27.77)	(25.92)	(25.63)	(14.90)	(36.91)	(32.12)	(21.42)	(10.93)
D(.)RoA _{<i>i,t-1</i>}			-0.43	-0.39	-0.16	-0.19	-0.21	-0.20	-0.10	-0.12
(<i>t</i>)			(-17.21)	(-17.99)	(-10.23)	(-10.64)	(-11.66)	(-11.01)	(-6.62)	(-8.34)
log(BA) _{<i>i,t-1</i>}		-9.32		-8.32		-8.87		-7.15		-9.06
(<i>t</i>)		(-22.35)		(-21.91)		(-21.07)		(-36.32)		(-21.29)
MABA _{<i>i,t-1</i>}		-1.49		-0.73		-1.75		-0.87		-1.33
(<i>t</i>)		(-4.44)		(-2.45)		(-4.70)		(-7.53)		(-3.96)
MktLev _{<i>i,t-1</i>}		48.89		31.46		45.91		53.40		50.87
(<i>t</i>)		(23.73)		(16.82)		(22.36)		(47.46)		(24.11)
log(1 + Delta) _{<i>i,t-1</i>}		1.77		1.56		1.66		1.61		1.67
(<i>t</i>)		(10.15)		(9.59)		(9.71)		(10.62)		(9.82)
log(1 + Vega) _{<i>i,t-1</i>}		-0.19		-0.14		-0.13		-0.71		-0.23
(<i>t</i>)		(-0.81)		(-0.61)		(-0.56)		(-5.17)		(-0.97)
1 _{MissingExec}		9.47		8.74		9.07		0.93		8.05
(<i>t</i>)		(8.63)		(8.87)		(8.31)		(1.12)		(7.86)
SUE _{<i>i,t-1</i>}		0.22		0.21		0.24		0.08		0.20
(<i>t</i>)		(2.91)		(3.14)		(3.26)		(1.33)		(2.60)
Adj.R ²	0.50	0.55	0.54	0.58	0.51	0.55	0.45	0.50	0.51	0.55
Total number of observations	429,966	411,416	371,056	353,284	418,065	400,101	430,078	411,509	393,650	376,919

Notes. This table reports results from firm-level panel regressions with fixed firm and time effects. We regress quarterly idiosyncratic stock return volatility $v_{i,t}^E$ on a constant, the lagged quarterly RoA, and lagged firm characteristics, as follows: $v_{i,t}^E = a + bRoA_{i,t-1} + cD(.)RoA_{i,t-1} + dD(.) + f Control_{i,t-1} + e_{i,t}$, where $D(.)$ is an indicator that identifies a situation in which a firm is more likely to shift risk. The indicator takes a value of one if the o -score of the last quarter is classified into the top tercile ($OS = 3$), if the default probability of Merton is classified into the top tercile ($Def Prob = 3$), if the economy of the previous month is identified in the NBER recession dates, or if the fraction of institutional holdings is classified into the bottom tercile ($Inst = 1$). The past firm characteristics include the natural logarithm of assets $log(BA)_{i,t-1}$, market-to-book assets $MABA_{i,t-1}$, market leverage $MktLev_{i,t-1}$, and standardized unexpected earnings $SUE_{i,t-1}$ as well as the natural logarithms of the delta and vega of managerial stock options. If the delta and vega are missing from ExecuComp, they are replaced with zero, and the indicator $I_{MissingExec}$ is set to one. The standard errors are clustered by firm. Adjusted R^2 is the adjusted R^2 s.

the indicator variables. The coefficient on $RoA_{i,t-1}$ is -0.40 (t -statistic = -19.11) in Reg I and becomes -0.25 (t -statistic = -19.97) in Reg II. This implies that a decline of one standard deviation in $RoA_{i,t-1}$ (0.21) is associated with an increase of 0.05 (0.21×0.25) in the idiosyncratic volatility, which is about 11% of its sample median of 0.44.

We next examine whether firms take on more investments with high idiosyncratic risk when they are in distress. When the o -score is the proxy for financial distress, the estimated coefficients of $RoA_{i,t-1}$ and $RoA_{i,t-1}D(OS = 3)$ are -0.12 (t -statistic = -9.93) and -0.39 (t -statistic = -17.99), respectively, in Reg II. That is, among the firms with a high o -score, in response to a decrease of one standard deviation in RoA (0.21), $v_{i,t}^E$ increases significantly by 0.11 (i.e., $0.21 \times (0.12 + 0.39)$), which is about 25% of the sample median of idiosyncratic volatility. Moreover, this

increase of 0.11 among the distressed firms doubles the increase of 0.05 among all the firms. When we use an alternative firm-level distress indicator, the probability of default, we obtain similar results. In Reg II, the coefficient on $RoA_{i,t-1}$ is -0.16 (t -statistic = -13.38), and the coefficient on $RoA_{i,t-1}D(Def Prob = 3)$ is -0.19 (t -statistic = -10.64).

Next, to test the risk-shifting behavior over the business cycle mentioned in prediction 2, we use the NBER recession dates to identify the aggregate distress status. That is, if the economy of the previous month falls within an NBER recession, $D(recess) = 1$. The coefficient on $RoA_{i,t-1}$ is -0.28 (t -statistic = -28.77), and the coefficient on $RoA_{i,t-1}I(recess = 1)$ is -0.20 (t -statistic = -11.01). That is, in response to a decrease of one standard deviation in RoA (0.21) in the recessions, the idiosyncratic volatility increases by 0.10 ($0.21 \times (0.28 + 0.20)$), which is 67.7% ($(0.10 - 0.06)/0.06$) more

than the increase of 0.06 in the expansions, confirming that the aggregate distress status induces the asymmetric response as well. This confirms that, similar to the firm-specific distress condition, the bad aggregate states cause the firms to take on more idiosyncratic risk than do the good aggregate states. Moreover, the increase in the idiosyncratic risk, to 0.10 during the recession, in the data are also largely consistent with the optimal increment of 0.1231 in the bad state in the calibrated model as shown in panel A of Table OA2 in the online appendix.

Finally, we examine whether the risk-shifting problem is more severe when the monitoring of management by institutional holders is low. In Reg II, the coefficient on $RoA_{i,t-1}$ is -0.19 (t -statistic = -15.13), and the coefficient on $RoA_{i,t-1}D(Inst=1)$ is -0.12 (t -statistic = -8.34), confirming a stronger negative association between profitability and risk taking in the presence of low monitoring from institutional holders.

In short, we empirically confirm our first two predictions of a negative relation between profitability and future idiosyncratic risk taking, particularly in firms in distress, during economic downturns. We also show that firms subject to less active monitoring have a severer agency conflict problem.

To ensure the robustness of our results, we conduct additional tests with alternative proxies for profitability and idiosyncratic volatility as well as alternative specifications. We report the results in Section B of the online appendix. To summarize, we first use an alternative measure for profitability, $RoE_{i,t}$, and replace the current independent variable RoA_{t-1} in Equation (10). Then, to mitigate the potential bias from the persistence of the variables, we test the association between the changes in the idiosyncratic return volatility and the changes in RoA. Finally, because the idiosyncratic risk is unobservable, we replace idiosyncratic return volatility with idiosyncratic asset volatility and idiosyncratic cash flow volatility. Overall, we find consistent support for a negative relation between profitability and future idiosyncratic risk taking.

4.3. Empirical Tests of the Conditional CAPM

We now test predictions 3 and 4, which are concerned with the covariance between the equity beta and market return and the stock returns, in the framework of the conditional CAPM. Following Lewellen and Nagel (2006), we use the excess stock market return r_t^m to proxy for the market risk premium λ_t and the monthly CAPM beta to proxy for the time-varying market beta β_t^E . The monthly CAPM beta is obtained by regressing daily returns on daily excess market returns. We also use the procedure of Dimson (1979)

to mitigate microstructure noise. Empirically, the unconditional expected stock excess return is

$$\begin{aligned} E[r_t^{ex}] &= E[r_t^E] - rdt = E[\beta_t^E r_t^m] \\ &= E[\beta_t^E] E[r_t^m] dt + \text{cov}(\beta_t^E, r_t^m) dt, \end{aligned} \tag{11}$$

and the unconditional CAPM alpha is

$$\alpha^u \approx \text{cov}(\beta_t^E, r_t^m) dt - \frac{E[r_t^m]}{(E[\sigma_t^m])^2} \text{cov}(\beta_t^E, (\sigma_t^m)^2). \tag{12}$$

4.3.1. Equity Beta and Market Risk Premium. The mechanism in our model is that the negative covariance, $\text{cov}(\beta_t^E, r_t^m)$, causes the low returns and negative alphas in high-volatility firms. A simple calculation of the covariance of $\text{cov}(\beta_t^E, r_t^m)$ does not allow us to test whether the covariance is potentially driven by the risk-shifting mechanism. To examine the role of the risk shifting, we use panel regressions and introduce into them the interaction term between the market risk premium and the risk-shifting conditioning variables. Additionally, the panel regressions allow us to control for other firm characteristics as well as firm and time fixed effects.

We regress the monthly equity beta, $\beta_{i,t}^E$, on r_t^m as follows:

$$\begin{aligned} \beta_{i,t}^E &= a_i + gD(\cdot) + \sum_{j=1}^J I_{i,t}(j) \{a_j + b_j r_t^m + c_j r_t^m D(\cdot) \\ &\quad + d_j D(\cdot) + f_j \text{Control}_{i,t-1}\} + e_{i,t}, \end{aligned} \tag{13}$$

where $I_{i,t}(j)$ is an indicator function that takes a value of one if firm i is in quintile j and $D(\cdot)$ is an indicator that identifies a high likelihood of financial distress or low institutional holdings by the top five block holders. We classify the firms into quintiles j based on their idiosyncratic return volatility in the last month and into terciles based on their o -score, Merton’s default probability and institutional holdings. $D(\cdot) = 1$ if the firm is in the top tercile for the o -score or default probability or in the bottom tercile for institutional holdings. We include control variables, such as market capitalization ($size_{i,t-1}$), book-to-market equity ($BE/ME_{i,t-1}$), market leverage ($MktLev_{i,t-1}$), six-month cumulative stock returns ($PreRets_{i,t-1}$), and standardized unexpected earnings $SUE_{i,t-1}$ as well as firm and time fixed effects. The estimated coefficients for $D(\cdot)$ are absorbed by the firm fixed effect, a_i . The standard errors are clustered by firm and time.

In our specification, the relation between $\beta_{i,t}^E$ and the market risk premium r_t^m of a firm in quintile j is measured by b_j . According to our third prediction, we expect $b_j < 0$ and to be increasingly negative as

idiosyncratic volatility increases, particularly in distressed and less monitored firms. Thus, we are interested in the coefficient $b_j + c_j D(\cdot)$ because it allows us to investigate whether the risk-shifting incentives drive the negative covariance between the market risk premium and levered beta.

Table 4 reports the estimated coefficients b_j for the key variable $I_{i,t}(j)r_t^m$. Note that a higher value of $j = 1, \dots, 5$ indicates that the firm is in a group with a higher idiosyncratic volatility. For brevity, we report the estimates of b_j and c_j . Panel A shows that the equity beta is negatively correlated with the market returns in all quintiles and becomes increasingly

negative as idiosyncratic volatility rises. For the firms in the top quintile, the estimated coefficient is -2.09 (t -statistic = 17.44), which is much greater than the -0.77 (t -statistic = -6.54) for firms in the bottom quintile.

To examine whether the correlation is stronger in firms with a high likelihood of shifting risk, we add the interaction term in the next three panels. In panel B, where we use the o -score to proxy for distress status, the estimated coefficients of c_j are all negative and increase in absolute terms from -0.55 (t -statistic = -5.26) to -1.28 (t -statistic = -4.06). More importantly, $b_j + c_j D(OS = 3)$ increases *monotonically* in absolute terms from -1.28 to -3.18 , confirming that distressed firms

Table 4. Equity Betas and Excess Market Returns

Panel A: Firm-level betas versus market returns					
	L(ow)	2	3	4	H(igh)
$b(j)$	-0.77	-1.09	-1.53	-1.92	-2.09
(t)	(-6.54)	(-9.26)	(-12.98)	(-16.25)	(-17.44)
Panel B: Conditional on o -score					
	L(ow)	2	3	4	H(igh)
$b(j)$	-0.73	-0.87	-1.09	-1.33	-1.89
(t)	(-5.23)	(-6.40)	(-8.00)	(-9.63)	(-12.91)
$c(j)$	-0.55	-1.47	-1.49	-1.41	-1.28
(t)	(-2.03)	(-5.71)	(-5.66)	(-5.11)	(-4.06)
$b(j) + c(j)D(OS = 3)$	-1.28	-2.34	-2.58	-2.74	-3.18
(t)	(-5.80)	(-10.55)	(-11.67)	(-12.18)	(-11.57)
Panel C: Conditional on default probability					
	L(ow)	2	3	4	H(igh)
$b(j)$	-0.92	-1.02	-1.46	-1.90	-2.06
(t)	(-6.17)	(-7.13)	(-10.17)	(-13.04)	(-13.22)
$c(j)$	0.08	-0.67	-0.49	-0.36	-1.28
(t)	(0.31)	(-2.59)	(-1.83)	(-1.27)	(-3.95)
$b(j) + c(j)D(Def Prob = 3)$	-0.84	-1.69	-1.95	-2.26	-3.34
(t)	(-3.61)	(-7.15)	(-7.97)	(-8.52)	(-12.52)
Panel D: Conditional on institutional holdings					
	L(ow)	2	3	4	H(igh)
$b(j)$	-0.51	-0.55	-0.98	-0.74	-0.91
(t)	(-4.10)	(-4.58)	(-8.11)	(-6.04)	(-6.95)
$c(j)$	-0.59	-1.31	-1.41	-1.51	-1.50
(t)	(-2.47)	(-5.71)	(-6.14)	(-6.26)	(-5.38)
$b(j) + c(j)D(Inst = 1)$	-1.10	-1.86	-2.40	-2.25	-2.41
(t)	(-5.27)	(-9.58)	(-12.3)	(-10.67)	(-9.82)

Notes. This table reports the estimates from panel regressions of firm-level equity betas on excess market returns with firm fixed effects across five groups of firms. A firm i is sorted into group j based on the idiosyncratic volatility of its equity returns in the last month at time t , and the indicator for this classification is $I_{i,t}(j)$. In baseline regressions, we regress monthly equity betas, $\beta_{i,t}^E$, on excess market returns r_t^m , the interaction between the excess market returns r_t^m , and an indicator of financial distress or low institutional holdings, and lagged firm characteristics as follows: $\beta_{i,t}^E = a_i + gD(\cdot) + \sum_{j=1}^5 I_{i,t}(j)\{a_j + b_j r_t^m + c_j r_t^m D(\cdot) + d_j D(\cdot) + f_j Control_{i,t-1}\} + e_{i,t}$, where $D(\cdot)$ is the indicator that identifies a high likelihood of financial distress or low institutional holdings of the top five block holders. This indicator takes a value of one if the o -score for the last quarter is classified into the top tercile ($OS = 3$), if the default probability of Merton is classified into the top tercile ($Def Prob = 3$), or if the fraction of institutional holdings is classified into the bottom tercile ($inst = 1$). The past firm characteristics include market capitalization ($size_{i,t-1}$), book-to-market equity ($BE/ME_{i,t-1}$), market leverage ($Mkt Lev_{i,t-1}$), six-month cumulative stock returns ($PreRets_{i,t-1}$), and standardized unexpected earnings $SUE_{i,t-1}$. The standard errors are clustered by firm. Adjusted R^2 is the adjusted R^2 s. To save space, we report the estimates of b_j and c_j .

are more negatively correlated with market returns. Similarly, in panel C, where we use the default probability of Merton, all the estimated c_j values decrease from 0.08 to -1.28 , and $b_j + c_j D(\text{Def Prob} = 3)$ increases monotonically in absolute terms from -0.84 to -3.34 . Therefore, we confirm consistently that the correlation between the equity beta and excess market returns becomes increasingly negative as the idiosyncratic volatility rises, particularly in the distressed firms.

Finally, we investigate the effect of monitoring on the equity beta. In panel D, where the indicator $D(\text{Inst} = 1)$ is for firms with low institutional holdings and monitoring, the estimated c_j increases in absolute terms from -0.59 (t -statistic = -2.47) to -1.50 (t -statistic = -5.38), and $b_j + c_j D(\text{Inst} = 1)$ increases in absolute terms from -1.10 to -2.41 . This implies that, when the firms are subject to relatively less monitoring from institutional holders, the management is more likely to shift risk and take on investments with high idiosyncratic risk.

In short, we demonstrate empirically that the increased idiosyncratic volatility helps equity holders to reduce their exposure to market risk, the equity beta, in the bad states when the market risk premium is high. The negative relation is particularly strong for firms in distress or with low monitoring of management.

4.3.2. Implying Unconditional Excess Returns and Alphas. Having demonstrated the significant, negative correlation between equity betas and excess market returns, we turn to testing the fourth prediction in the framework of the conditional CAPM. Because the conditional betas are high in the firms with high idiosyncratic volatility, it is important for us to show that the negative covariance dominates the leverage effect, generating the low excess return among those firms.

Table 5 reports the results. Panel A presents the average excess return in percentage, $r_{i,t}^E$, and the unconditional CAPM alpha, α^u , for the value-weighted portfolios, sorted on the idiosyncratic stock return volatility ($v_{i,t}^E$) of the last month. The excess return first increases from 6.34% to 8.61% per year for the fourth decile portfolio and then decreases to -6.89% for the 10th one. The difference is -13.23% with a t -statistic of -3.21 . It is worth noting that the negative average returns are in the top two deciles, consistent with the finding of Ang et al. (2006) that the negative average stock return only features in the top quintile.

Next, we turn to the conditional CAPM. We first report the value-weighted average of the conditional market equity betas, $\beta_{i,t}^E$, in the first row of panel B. The difference between the equity betas of the portfolios with high and low volatility is 0.29 because the high-volatility firms are distressed and have high

financial leverage. The small decreases in the top two decile portfolios are likely attributable to strategic risk-shifting behavior whereby the distressed firms increase their idiosyncratic volatility to decrease their equity beta.

Using the $\beta_{i,t}^E$ from the data, we calculate the excess return as in Equation (11) and the unconditional alpha based on Equation (12). The second and third rows of panel B report the two components of the excess return, $\mathbf{E}[\beta_{i,t}^E] \mathbf{E}[r_t^m]$ and $\mathbf{cov}(\beta_{i,t}^E, r_t^m)$, respectively. Given the market risk premium of 6.2% per year from 1963 to 2016, the spread in $\mathbf{E}[\beta_{i,t}^E] \mathbf{E}[r_t^m]$ is 1.81% ($0.29 \times 6.2\%$) per year. Hence, the equity beta *alone*—or the unconditional CAPM—is not able to explain the idiosyncratic volatility when we assume the *true* market return to be observable.

The covariance, $\mathbf{cov}(\beta_{i,t}^E, r_t^m)$, in the third row decreases monotonically from 0.81% to -6.61% , a total fall of 7.42%. Even though the equity beta, $\beta_{i,t}^E$, implies a small positive spread, the covariance term dominates the equity beta effect and gives a return spread of -5.61% , which explains about 42% of the 13.23% in the data. The last row shows that the model-implied unconditional alpha, α^u , decreases from 1.09% to -7.83% , a drop of -8.91% , which is about 50% of the 17.94% in the data. It is worth noting that the monotonic decrease in α^u is mainly a result of $\mathbf{cov}(\beta_{i,t}^E, r_t^m)$ because the second item in the α^u formula is very small. Therefore, we have demonstrated that the spreads in the model-implied excess returns and unconditional alphas account for about 42%–50% of their empirical counterparts, largely confirming our prediction 4.

We conclude that the unconditional CAPM alone is not able to explain the idiosyncratic volatility puzzle empirically because of the high equity beta in the firms with high idiosyncratic volatility. However, when the covariance between the equity beta and the market risk premium dominates the high beta effect in the conditional CAPM, it helps explain the low excess returns and negative CAPM alphas among the firms with high idiosyncratic volatility. Our work is not the only mechanism that exclusively explains the idiosyncratic volatility. Various economic mechanisms have been proposed to explain the idiosyncratic volatility puzzle, but our work is the first to our knowledge to provide a risk-based story that explains 42% of the stock return spread and about 50% of the CAPM alpha spread.¹⁸

5. Concluding Remarks

We examine a prominent agency conflict problem, the risk-shifting behavior of equity holders, and its implications for the negative relation between idiosyncratic volatility and future stock returns. We build a simple risk-shifting model based on Leland (1998) and deliver four testable predictions.

Table 5. Implied Excess Returns and Alphas

Panel A: Excess returns and alphas from the data											
	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H – L
$r_{i,t}^{ex}$	6.34	7.25	7.76	8.61	7.70	7.23	4.62	2.06	-2.03	-6.89	-13.23
(t)	(3.45)	(3.40)	(3.28)	(3.11)	(2.47)	(2.04)	(1.17)	(0.48)	(-0.44)	(-1.40)	(-3.21)
α^u	1.38	1.46	1.02	1.10	-0.39	-1.50	-4.75	-7.81	-11.97	-16.57	-17.94
(t)	(1.91)	(2.28)	(1.45)	(1.24)	(-0.32)	(-0.92)	(-2.36)	(-3.55)	(-4.65)	(-5.23)	(-5.01)
Panel B: Model-implied excess returns and alphas											
	L(ow)	2	3	4	5	6	7	8	9	H(igh)	H – L
$\beta_{i,t}^E$	0.85	0.99	1.11	1.21	1.27	1.36	1.37	1.37	1.33	1.14	0.29
$E(\beta_{i,t}^E)E(r_t^m)$	5.22	6.09	6.84	7.46	7.86	8.38	8.46	8.45	8.20	7.03	1.81
$\text{cov}(\beta_{i,t}^E, r_t^m)$	0.81	0.75	0.12	-1.12	-2.12	-2.19	-3.44	-4.90	-6.73	-6.61	-7.42
$r_{i,t}^{ex}$	6.03	6.85	6.96	6.33	5.74	6.19	5.02	3.55	1.47	0.42	-5.61
α^u	1.09	0.93	0.28	-1.10	-2.19	-2.44	-4.00	-5.81	-7.83	-7.83	-8.91

Notes. This table reports the annualized excess stock returns and unconditional alphas from the data and those implied by the conditional model from 1963 to 2016. Panel A presents the value-weighted averages (%) of annualized excess returns, $r_{i,t}^{ex}$, and unconditional CAPM alpha, α^u , for stock portfolios sorted on the idiosyncratic stock return volatility ($v_{i,t}^E$) of the last month. Panel B reports the model-implied unconditional expected excess stock return, $r_{i,t}^{ex}$, and unconditional alpha, α^u , from the unconditional CAPM. The first row is the value-weighted average of the conditional beta, $\beta_{i,t}^E$, from the data. Those monthly firm-level market CAPM betas are obtained from the regression of daily returns on the daily excess market returns, month by month, and are adjusted using the procedure of Dimson (1979). Using the monthly conditional beta, we follow Jagannathan and Wang (1996) and calculate the unconditional expected excess return as $r_{i,t}^{ex} = E[\beta_{i,t}^E]E[r_t^m] + \text{cov}(\beta_{i,t}^E, r_t^m)$ and the unconditional CAPM alpha as $\alpha^u \approx \text{cov}(\beta_{i,t}^E, r_t^m) - \frac{E[r_t^m]}{E[\sigma_{i,t}^E]^2} \text{cov}(\beta_{i,t}^E, (\sigma_{i,t}^E)^2)$. We also report the two main components, $E[\beta_{i,t}^E]E[r_t^m]$ and $\text{cov}(\beta_{i,t}^E, r_t^m)$.

We conduct extensive tests of the four predictions in the cross-section and in time series. We confirm that, when firms are in distress and when the aggregate economy is in a bad state, equity holders take on more investments with high idiosyncratic risk. We also demonstrate that the increased idiosyncratic risk decreases the equity beta, particularly in the bad states in which the market risk premium is high. More importantly, we find a strong negative covariance between the time-varying equity betas and the market risk premium at the firm level and at the portfolio level among the firms with high idiosyncratic volatility. The negative covariance between the lowered equity beta and the increased market risk premium dominates the leverage effect and generates low excess stock returns and unconditional alpha in the conditional CAPM for the firms with high idiosyncratic volatility. Thus, we deliver a risk-based explanation for the idiosyncratic volatility puzzle in the conditional CAPM instead of the traditional unconditional CAPM.

Although our study assumes the manager is acting on behalf of the equity holders and focuses on the agency conflict between equity and debt holders, it would be fruitful to incorporate managerial incentives in our modeling framework although we do control for managerial compensation in our empirical investigation.

Acknowledgments

The authors acknowledge detailed and constructive comments from Tomasz Piskorski and two anonymous referees as

well as suggestions and discussions from Yakov Amihud, Kerry Back, Ilan Cooper, Lorenzo Garlappi, Jarrad Harford, Avi Kamara, Gi Kim, Gang Li, Ravindra Sastry, Stephan Siegel, Neng Wang, Lance Young, and Fernando Zapatero as well as seminar participants at Australian National University, City University of Hong Kong, City University of New York (Queens), Erasmus University, Maastricht University, Manchester Business School, Purdue University, Tilburg University, University of Connecticut, University of Illinois at Urbana-Champaign, University of Massachusetts, University of New South Wales, University of Hong Kong, University of Technology Sydney, University of Washington, the 2015 Annual Meetings of Western Finance Association (Seattle), the 2015 Annual Meetings of European Finance Association (Vienna), the 2014 North American Summer Meeting of the Econometric Society (Minnesota), the 2014 Jerusalem Finance Conference (Hebrew University), the 2014 Frontiers of Finance Conference (Warwick Business School), and the 2014 China International Conference in Finance. They thank Yuxin Luo for excellent research assistance.

Appendix A. Model, Valuations, and Stock Returns

We develop a simple model based on Leland (1998) to motivate our predictions in the main text. Instead of further complicating the model, we use the comparative statics analysis to show the implication of the time-varying market premium by varying the market risk premium in the simple model. However, we formally introduce the countercyclical market risk premium into the simple model in the online appendix in an effort to confirm our findings from the simple model.

We start with presenting the model setup and general asset valuation framework and then provide the closed-form solutions for equity values and returns for firms after risk shifting and for those prior to risk shifting.

A.1. Model Setup

The model is partial equilibrium with a pricing kernel, m_t , as follows:¹⁹

$$\frac{dm_t}{m_t} = -rdt - \theta d\hat{W}_t^m, \tag{A.1}$$

where r is the constant risk-free rate, θ is the price of the risk, and \hat{W}_t^m is a standard Brownian motion.

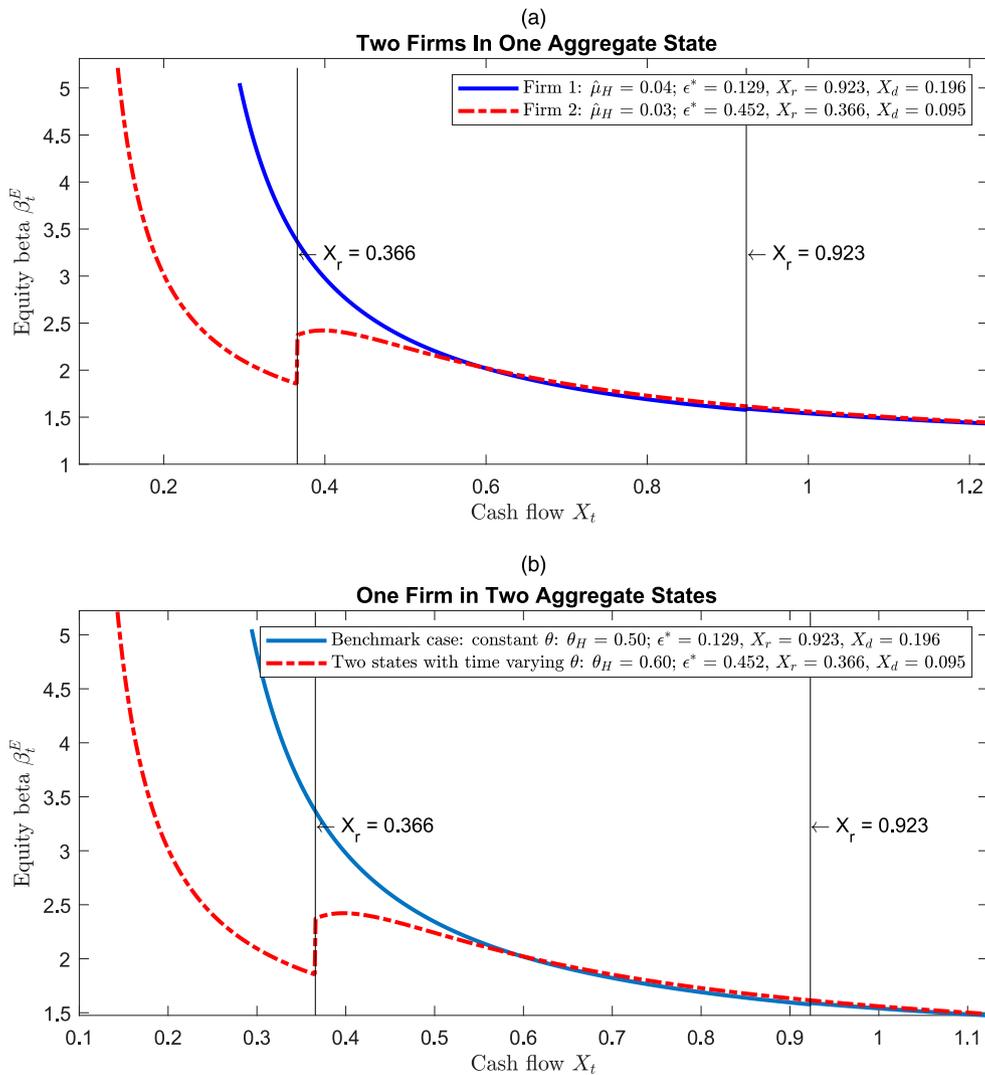
The economy consists of a large number of firms. Consider a representative firm that operates in two levels of risk, that is, a high and a low risk level. That is, the level, l , can take two values, H (high) or L (low). Before the firm goes

bankrupt, the firm’s assets produce instantaneous cash flows X_t over the two levels, governed by the following stochastic process:

$$\frac{dX_t}{X_t} = \hat{\mu}_l dt + \beta \sigma^m d\hat{W}_t^m + v_l d\hat{W}_t^i, \tag{A.2}$$

where $\hat{\mu}_l$ is the expected growth rate of the cash flow, β is the asset exposure to the market risk, σ^m is the constant market volatility, v_l is the idiosyncratic volatility of the cash flow growth rate, and \hat{W}_t^i is a standard Brownian motion. The total volatility of the cash flow growth rate is $\sigma_l = \sqrt{(\beta\sigma^m)^2 + v_l^2}$.

Figure A.1. (Color online) Optimal Risk Increment and Equity Beta



Notes. This figure plots the equity beta, $\beta_{l,t}^E$, against cash flows, X_t , with optimal policies as shown in the legend. In panel (a), we consider two firms, firms 1 and 2, for which the aggregate state does not change. They start with the same $\hat{\mu}_L = 0.04$ with a low level of idiosyncratic volatility $\hat{v}_L = 0.1$ at $X_0 = 1$. When their conditions deteriorate, these firms have different expected cash flow growth rates, $\hat{\mu}_H = 0.03, 0.04$, respectively. Given the $\hat{\mu}_H$ s, they choose different optimal increments in idiosyncratic volatility, ϵ^* ; different optimal thresholds of risk shifting, X_r ; and different optimal default thresholds, X_d . In panel (b), we use as the benchmark firm 1 from panel (a) and compare its equity beta against that in a setting with changing risk premiums in two aggregate states. In the second setting, firm 1 has a constant cash flow growth rate in both the bad and good aggregate states, that is, $\hat{\mu}_L = \hat{\mu}_H = 0.04$. However, the market price of risk increases from θ_L (0.5) in the good state to θ_H (0.6) in the bad state at the threshold $X_r = 0.366$. We calculate $\beta_{l,t}^E$ according to Equation (A.22) for $X_t < X_r$ and Equation (A.30) for $X_t \geq X_r$.

Table A.1. Parameter Values

Parameter	Symbol	Value
Risk-free rate	r	0.05
Effective tax rate	τ	0.15
Market return volatility	σ_m	0.1
Market price of risk	θ	0.5
Initial output	X_0	1
Initial asset value	$V_{L,0}$	$X_0/(r_f - \mu_L)$
Coupon	c	0.3
Physical growth rate	$\hat{\mu}_L$	0.04
Physical growth rate	$\hat{\mu}_H$	0.01, 0.04
Cash flow beta (both levels)	β	1
Idiosyncratic volatility (low-level risk)	v_L	0.1
Total volatility (low-level risk)	σ_L	0.2059
Cost of excess volatility	η	0.3

Notes. This table presents the parameter values for the model. The economy-wide and firm-specific parameters of the model are obtained from the extant literature except for the cost of excess volatility η . Following the literature, we set the nominal risk-free rate r to 5% and effective tax rate to 15%. We set σ^m to 0.1, β to one, and v_L to 0.1 based on the empirical evidence provided by Schaefer and Strebulaev (2008), who delever the equity return volatility and estimate the underlying cash flow volatility. The market price of risk is the Sharpe ratio of 0.5, which is standard in the literature. Given the cash flow beta and market price of risk, we have a cash flow risk premium of 0.05. We also set the coupon rate to 0.35. The cash flow growth rate of the low-risk, healthy firms is set to 0.5 (Leland 1994).

According to Gordon’s growth model under the risk-neutral measure Q , the asset value is as follows:

$$V_{l,t} \equiv V(l, X_t) = \mathbf{E}^Q \left[\int_t^\infty X_t e^{-r\tau} d\tau \right] = \frac{X_t}{r - \mu_l}. \quad (\text{A.3})$$

Here, $\mu_l = \hat{\mu}_l - \beta\lambda$ is the risk-neutral counterpart of $\hat{\mu}_l$ and $\lambda = (\theta\sigma^m)$ is the constant market risk premium. Note that this partial equilibrium model is silent on the systematic structure of the risk premium λ .

Because $V_{l,t}$ is linear in X_t in each level, it follows that

$$\frac{dV_{l,t}}{V_{l,t}} = \hat{\mu}_l dt + \beta\sigma^m d\hat{W}_t^m + v_l d\hat{W}_{l,t}. \quad (\text{A.4})$$

Hence, the assets and their generated cash flows share the same dynamics in each level. To be consistent, we refer to $\hat{\mu}_l$ as the expected cash flow growth rate (or asset growth), λ as the cash flow risk premium, β as the cash flow beta, and v_l as idiosyncratic cash flow growth volatility throughout the rest of the paper.

To focus on the idiosyncratic volatility puzzle, we assume that, after a firm with a low expected rate of cash flow growth has entered the high risk level, the equity holders only increase the idiosyncratic volatility irreversibly (instead of systematic volatility σ^m) from v_L to v_H by $\epsilon = \sqrt{v_H^2 - v_L^2} \geq 0$. The intuition for this is twofold. First, given that an increase in the cash flow beta (β) reduces the risk-adjusted (risk-neutral) expected growth rate, that is, $\mu_l = \hat{\mu}_l - \beta(\theta\sigma^m)$, therefore, the asset value as in Equation (A.3), equity holders have more incentives to increase idiosyncratic volatility than total (or systematic) volatility. Second, the equity holders have no incentives to ride on the market if the firm’s declining performance is due to the contracting economy.²⁰

Meanwhile, we assume that the total lump sum cost for risk shifting is $\eta e^2 V_{H,r}(1 - \tau)$, proportional to the idiosyncratic volatility e^2 , where $V_{H,r}$ is the asset value at X_r .²¹ The proportional adjustment cost is intuitive. First, the cost to search capable workers with certain special expertise for idiosyncratic investments is higher than those for common projects. Second, firms with a lower asset value $V_{H,r}$ have less cash to spend on job advertisements. Given the proportional cost, equity holders choose an optimal increment of cash flow idiosyncratic volatility.

Compared with the original Leland’s model that assumes an exogenous increase in the total volatility, our model endogenously determines the optimal amount of excess risk taking. Meanwhile, we make two simplifications by assuming exogenous debt financing and irreversible risk-shifting decisions, which allow us to obtain closed-form solutions for stock returns.

To summarize, the expected cash flow growth rate $\hat{\mu}_l$ and idiosyncratic cash flow growth volatility v_l are constant within each level but differ across the two levels. We have $\hat{\mu}_H \leq \hat{\mu}_L$ and $v_H \geq v_L$ because equity holders increase idiosyncratic volatility from v_L to v_H given the decrease in cash flow return from $\hat{\mu}_L$ to $\hat{\mu}_H$. We assume that $\hat{\mu}_H$, $\hat{\mu}_L$, and v_L are public information and are exogenously given, and v_H is controlled by the owners of the firm–equity holders.

A.2. Asset Valuation Framework

Under the risk-neutral measure, the Bellman equation describes the valuation of any claim $G(l, X_t)$ on operating cash flows X_t in the volatility level, l , as follows:

$$G(l, X_t) = H_t dt + e^{-rdt} \mathbf{E}^Q(G(l, X_t + dX_t)), \quad (\text{A.5})$$

where H_t denotes the cash flows accruing to claim holders. Standard dynamic programming suggests that $G(l, X_t)$ must satisfy the ordinary differential equation

$$\mu_l X G'_{l,t} + \frac{\sigma_l^2}{2} X^2 G''_{l,t} - r G_{l,t} + H_{l,t} = 0, \quad (\text{A.6})$$

where $G_{l,t} \equiv G(l, X_t)$; $G'_{l,t}$ and $G''_{l,t}$ denote the first- and second-order derivatives of $G_{l,t}$ with respect to X_t , respectively.

Because the cash flow generated by the assets is $H_t = X_t$, the value of assets in place, $V_{l,t}$, under the risk-neutral measure Q , is

$$V_{l,t} \equiv V(l, X_t) = \frac{X_t}{r - \mu_l}. \quad (\text{A.7})$$

Given the cash flows $H_t = (X_t - c)(1 - \tau)$, the value function of equity is

$$E(l, X_t) = (1 - \tau) \left(\frac{X_t}{r - \mu_l} - \frac{c}{r} \right) + e_{l,1} X_t^{\omega_{l,1}} + e_{l,2} X_t^{\omega_{l,2}}, \quad (\text{A.8})$$

$$= (1 - \tau) \left(V_{l,t} - \frac{c}{r} \right) + e_{l,1} X_t^{\omega_{l,1}} + e_{l,2} X_t^{\omega_{l,2}}, \quad (\text{A.9})$$

where $\omega_{l,1} < 0$ and $\omega_{l,2} > 1$ are the two roots of the characteristic equation in level l :

$$\frac{1}{2} \sigma_l^2 \omega_l (\omega_l - 1) + \mu_l \omega_l - r = 0. \quad (\text{A.10})$$

It's lemma implies that the equity value $E(l, X_t) \equiv E_{l,t}$ satisfies

$$\frac{dE_{l,t}}{E_{l,t}} = \frac{1}{E_{l,t}} \left(\frac{\partial E_{l,t}}{\partial t} + \hat{\mu}_l X_t \frac{\partial E_{l,t}}{\partial X_t} + \frac{\sigma_l}{2} X_t^2 \frac{\partial^2 E_{l,t}}{\partial X_t^2} \right) dt + \frac{1}{E_{l,t}} X_t \sigma_l \frac{\partial E_{l,t}}{\partial X_t}. \tag{A.11}$$

The standard no-arbitrage argument gives us the following partial differential equation:

$$\frac{\partial E_{l,t}}{\partial t} + \mu_s X_t \frac{\partial E_{l,t}}{\partial X_t} + \frac{\sigma_s^2}{2} X_t^2 \frac{\partial^2 E_{l,t}}{\partial X_t^2} - rE_{l,t} + D_t = 0. \tag{A.12}$$

Substituting this equation into Equation (A.11), we obtain

$$\begin{aligned} \frac{dE_{l,t}}{E_{l,t}} &= \frac{1}{E_{l,t}} \left[(\hat{\mu}_l - \mu_l) X_t \frac{\partial E_{l,t}}{\partial X_t} + rE_{l,t} - D_t \right] dt \\ &\quad + \frac{1}{E_{l,t}} X_t \sigma_l \frac{\partial E_{l,t}}{\partial X_t} d\hat{W}. \end{aligned} \tag{A.13}$$

Simple algebraic manipulation yields

$$\frac{dE_{l,t} + D_t dt}{E_{l,t}} - r dt = \frac{1}{E_{l,t}} (\hat{\mu}_l - \mu_l) X_t \frac{\partial E_{l,t}}{\partial X_t} dt + \frac{1}{E_{l,t}} X_t \sigma_l \frac{\partial E_{l,t}}{\partial X_t} d\hat{W}. \tag{A.14}$$

Denoting $(dE_{l,t} + D_t dt)/E_{l,t}$ by $r_{l,t}^E$ and $(X_t \partial E_{l,t})/(E_{l,t} \partial X_t)$ by $\gamma_{l,t}$, we have

$$r_{l,t}^E - r dt = \gamma_{l,t} (\hat{\mu}_l dt + \sigma_{l,t} d\hat{W} - \mu_{l,t} dt), \tag{A.15}$$

$$= \gamma_{l,t} \left(\frac{dV_t}{V_t} - \mu_{l,t} dt \right). \tag{A.16}$$

Because $\hat{\mu}_l dt - \mu_{l,t} dt = \beta \lambda dt$, taking expectations on both sides yields

$$\mathbf{E} \left[r_{l,t}^E \right] = r dt + \mathbf{E} [\gamma_{l,t} (\beta \lambda) dt]. \tag{A.17}$$

The elasticity of the stock to the underlying cash flows $\gamma_{l,t}$ is

$$\begin{aligned} \gamma_{l,t} &= \frac{X_t \partial E_{l,t}}{E_{l,t} \partial X_t} = \frac{V_{l,t} \partial E_{l,t}}{E_{l,t} \partial V_{l,t}} \\ &= \frac{1}{E_{l,t}} \left(X_t (1 - \tau) + e_{l,1} \omega_{l,1} X_t^{\omega_{l,1}} + e_{l,2} \omega_{l,2} X_t^{\omega_{l,2}} \right) \\ &= \frac{1}{E_{l,t}} \left(E_{l,t} + \frac{c(1 - \tau)}{r} - e_{l,1} X_t^{\omega_{l,1}} + e_{l,1} \omega_{l,1} X_t^{\omega_{l,1}} \right. \\ &\quad \left. - e_{l,2} X_t^{\omega_{l,2}} + e_{l,2} \omega_{l,2} X_t^{\omega_{l,2}} \right) \\ &= 1 + \frac{c(1 - \tau)}{r E_{l,t}} + \frac{(\omega_{l,1} - 1)}{E_{l,t}} e_{l,1} X_t^{\omega_{l,1}} + \frac{(\omega_{l,2} - 1)}{E_{l,t}} e_{l,2} X_t^{\omega_{l,2}}. \end{aligned} \tag{A.18}$$

Because we solve the model by backward induction, we first show how a firm determines its optimal timing of bankruptcy after risk shifting and then present the optimal risk-shifting policies for the same firm before it increases its idiosyncratic risk. We apply the general value function of equity of (A.9) and equity return of (A.17) to studying the preshifting and postshifting firms.

A.3. The Firm After Risk Shifting

Equity holders choose the optimal default threshold X_d to maximize their own equity value $E_{l,t} \equiv E(l, X_t)$. The two standard conditions are as follows:

$$E(l = H, X_t = X_d) = 0; \tag{A.19}$$

$$E'(l = H, X_t = X_d) = 0, \tag{A.20}$$

where $E'(l, X_t)$ denotes the first-order partial derivative of the equity value function $E(l, X_t)$ with respect to X_t in level l . Equation (A.19) is the value-matching condition, which states that equity holders receive nothing at bankruptcy.²² Equation (A.20) is the smooth-pasting condition that allows equity holders to choose their optimal bankruptcy threshold by facing a trade-off between the costs of keeping the firm alive and the benefits from future tax shelter (Leland 1994).

The following proposition states the expected stock return of postshifting firms, $\mathbf{E}[r_{H,t}^E]$, and the default threshold X_d .

Proposition A.1. *When the firm is in the high risk level but has not yet entered bankruptcy, $X_d \leq X_t < X_r$, the expected instantaneous stock return $\mathbf{E}[r_{H,t}^E]$ is*

$$\mathbf{E} \left[r_{H,t}^E \right] = r dt + \mathbf{E} [\gamma_{H,t} \beta \lambda dt] = r dt + \mathbf{E} \left[\beta \frac{E_{H,t}^E}{E_{H,t}} \lambda dt \right], \tag{A.21}$$

where the elasticity of stocks to cash flow values, $\gamma_{H,t}$, is

$$\gamma_{H,t} = 1 + \underbrace{\frac{c/r(1 - \tau)}{E_{H,t}}}_{\text{Leverage}} - \underbrace{(1 - \omega_{H,1}) \frac{(c/r - V_{H,d})}{E_{H,t}} \left(\frac{X_t}{X_d} \right)^{\omega_{H,1}} (1 - \tau)}_{\text{American Put Option of Delaying Bankruptcy (+)}}. \tag{A.22}$$

The optimal default threshold X_d is

$$X_d = \frac{c(r - \mu_H) \omega_{H,1}}{r(\omega_{H,1} - 1)}, \tag{A.23}$$

and equity value $E_{H,t}$ is by

$$E_{H,t} = \left[\underbrace{\left(V_{H,t} - \frac{c}{r} \right)}_{\text{Equity-in-Place}} + \underbrace{\left(\frac{c}{r} - V_{H,d} \right) \left(\frac{X_t}{X_d} \right)^{\omega_{H,1}}}_{\text{Option of Delaying Bankruptcy}} \right] (1 - \tau). \tag{A.24}$$

Proof. The no-bubble condition implies $e_{H,2} = 0$, and the value-matching condition of Equation (A.19) gives $e_{H,1} = -(V_H - c/r)(1 - \tau)/X_d^{\omega_{H,1}}$. Simply substituting $e_{H,1}$ and $e_{H,2}$ into Equations (A.18) and (A.9), we obtain Equations (A.22) and (A.24), respectively. Q.E.D.

A.4. The Firm Prior to Risk Shifting

In the low risk level, the firm chooses to invest in assets that generate cash flows, characterized by the growth rate and volatility pair $(\hat{\mu}_L, \sigma_L)$. Equity holders choose the optimal risk-shifting threshold X_r at which they optimally switch to a higher risk strategy as well as the optimal excess idiosyncratic cash flow growth volatility $\epsilon^* \in [0, +\infty)$.

The following two boundary conditions determine the threshold X_r :

$$E_{L,r} = E_{H,r} - \eta\epsilon^2 V_{H,r}(1 - \tau), \quad (\text{A.25})$$

$$E'_{L,r} = E'_{H,r} - \eta\epsilon^2(1 - \tau)/(r - \mu_H). \quad (\text{A.26})$$

The value-matching condition in Equation (A.25) is the no-arbitrage condition at X_r . Although the asset value decreases from $V_{L,t}$ to $V_{H,t}$ because $\mu_H < \mu_L$, equity holders are able to increase their own wealth to $E_{H,r} \equiv E(l = H, X_t = X_r)$ by increasing the idiosyncratic cash flow growth volatility from v_L to v_H at a cost of $\eta\epsilon^2 V_{H,r}(1 - \tau)$. Equation (A.26) is the smooth-pasting condition that determines the optimal risk-shifting threshold X_r .

In response to the expected decline from $\hat{\mu}_L$ to $\hat{\mu}_H$, equity holders strategically increase idiosyncratic volatility by ϵ^* . Unlike the exogenous risk increment in Leland (1998), we allow equity holders to choose the optimal increment ϵ^* to maximize the equity value $E_{H,r}$ at X_r after debt is in place:²³

$$\epsilon^* = \underset{\epsilon}{\operatorname{argmax}} E_{H,r} - \eta\epsilon^2 V_{H,r}(1 - \tau). \quad (\text{A.27})$$

On the one hand, the excess risk ϵ increases the equity value because of the option-like feature of equity. On the other hand, excess risk taking means greater proportional adjustment costs. Hence, equity holders make a cost-benefit trade-off and determine the optimal excess risk-taking ϵ^* so as to maximize their own wealth at X_r . After obtaining a semiclosed-form solution for X_r as a function of ϵ^* , we solve for ϵ^* and X_r jointly.

The next proposition gives the expected stock return of the preshifting firms, $\mathbf{E}[r_{L,t}^E]$, and the optimal risk-shifting threshold, X_r .

Proposition A.2. *When the firm is in the low risk level, $X_t \geq X_r$, the expected instantaneous stock return $\mathbf{E}[r_{L,t}^E]$ is*

$$\mathbf{E}[r_{L,t}^E] = rdt + \mathbf{E}[\gamma_{L,t}\beta\lambda dt] = rdt + \mathbf{E}[\beta_{L,t}^E\lambda dt], \quad (\text{A.28})$$

where the elasticity of stock to cash flows, $\gamma_{L,t}$, is

$$\begin{aligned} \gamma_{L,t} &= \frac{\partial E_{L,t}/E_{L,t}}{\partial V_{L,t}/V_{L,t}}, \quad (\text{A.29}) \\ &= 1 + \underbrace{\frac{c/r(1-\tau)}{E_{L,t}}}_{\text{Leverage}} + \underbrace{\frac{V_{L,r} - V_{H,r} + \eta\epsilon^2 V_{H,r} \left(\frac{X_t}{X_r}\right)^{\omega_{L,1}}}{E_{L,t}}}_{\text{Option of increasing risk (+)}} (1-\tau)(1-\omega_{L,1}) \\ &\quad - \underbrace{\frac{c/r - V_{H,d} \left(\frac{X_r}{X_d}\right)^{\omega_{H,1}}}{E_{L,t}}}_{\text{Option of delaying bankruptcy (+)}} \left(\frac{X_t}{X_r}\right)^{\omega_{L,1}} (1-\tau)(1-\omega_{L,1}). \end{aligned} \quad (\text{A.30})$$

The optimal risk-shifting threshold X_r is

$$X_r = \left[\frac{(c/r - V_{H,d})(\omega_{H,1} - \omega_{L,1})}{X_d^{\omega_{H,1}} \left(\frac{1}{r-\mu_L} - \frac{1-\eta\epsilon^2}{r-\mu_H}\right)(1-\omega_{L,1})} \right]^{\frac{1}{1-\omega_{H,1}}}, \quad (\text{A.31})$$

and equity value $E_{L,t}$ is given by

$$\begin{aligned} E_{L,t} &= \left[\left(V_{L,t} - \frac{c}{r} \right) + (V_{H,r}(1 - \eta\epsilon^2) - V_{L,r}) \left(\frac{X_t}{X_r} \right)^{\omega_{L,1}} \right. \\ &\quad \left. + \left(\frac{c}{r} - V_{H,d} \right) \left(\frac{X_r}{X_d} \right)^{\omega_{H,1}} \left(\frac{X_t}{X_r} \right)^{\omega_{L,1}} \right] (1 - \tau). \end{aligned} \quad (\text{A.32})$$

Proof. The no-bubble condition implies that $e_{L,2} = 0$ for Equation (A.9), and the value-matching condition of Equation (A.25) suggests

$$\begin{aligned} &\left(V_{L,r} - \frac{c}{r} \right) (1 - \tau) + e_{L,1} X_r^{\omega_{L,1}} \\ &= \left(V_{H,r} - \frac{c}{r} \right) (1 - \tau) + \left(\frac{c}{r} - V_{H,d} \right) \left(\frac{X_r}{X_d} \right)^{\omega_{H,1}} (1 - \tau) - \eta\epsilon^2 V_{H,r}(1 - \tau). \end{aligned} \quad (\text{A.33})$$

Hence,

$$e_{L,1} = \frac{(1 - \tau)}{X_r^{\omega_{L,1}}} \left[\left(V_{H,r}(1 - \eta\epsilon^2) - V_{L,r} \right) + \left(\frac{c}{r} - V_{H,d} \right) \left(\frac{X_r}{X_d} \right)^{\omega_{H,1}} \right]. \quad (\text{A.34})$$

Substituting $e_{L,1}$ and $e_{L,2}$ into Equations (A.18) and (A.9), we obtain Equations (A.30) and (A.32), respectively. Using the smooth-pasting condition in Equation (A.26), we obtain the optimal risk-shifting threshold X_r after some algebraic manipulation. Q.E.D.

A.5. Numerical Example

We use a numerical example to qualitatively examine the cross-sectional and time-series risk-shifting behavior and its implications for stock returns.

We obtain the parameter values from extant works, such as Carlson et al. (2004) and Strebulaev (2007). Following the literature, we set the nominal risk-free rate r to 5% and effective tax rate to 15%. We set the market volatility σ^m to 0.1, cash flow beta β to one, and cash flow idiosyncratic volatility v_L to 0.1 based on the empirical evidence provided by Schaefer and Strebulaev (2008). They deleverage the equity return volatility and estimate the underlying cash flow volatility. The market price of risk is the Sharpe ratio of 0.5, which is standard in the literature. Given the cash flow beta and the market price of risk, we have a cash flow risk premium of 0.05. We also set the coupon rate to 0.3. The cash flow growth rate of the low-risk, healthy firms is set to 0.04 (Leland 1994). For the proportional cost of excess risk η , we choose a value of 0.80 so as to produce a reasonable value of v_H . The specific choice of η has no material impact on the qualitative implications of the model. The parameter values are listed in Table A.1.

In panel (a) of Figure A.1, we perform comparative statics analysis across firms. Suppose there are two identical firms that start with the same cash flow growth rate $\hat{\mu}_L = 0.04$ and with $\hat{v}_L = 0.1$ at $X_0 = 1$ at the low risk level but with different $\hat{\mu}_H$ equal to 0.03 and 0.04, respectively. We are interested in the optimal increment in idiosyncratic volatility, ϵ^* , and the subsequent impacts of ϵ^* on expected stock returns after X_r .

In panel (b), we allow the market risk premium θ to change over time. For simplicity, we consider one firm, which is firm 1 in panel (a), that experiences the aggregate good and bad states. We use the constant market risk premium as our benchmark and compare the risk-shifting behavior and the equity beta from the benchmark model with those generated from the case with a time-varying market premium. In the case of state-varying market risk premium, we keep the cash flow growth rate constant, that is, $\hat{\mu}_L = \hat{\mu}_H = 0.04$, between the bad and good states and only allow the market risk premium θ to change. This is to ensure that any difference in risk-shifting behavior generated from this example is entirely because of the increase in the market risk premium. Without losing generality, we assume that the firms increase their idiosyncratic risk taking at the threshold X_r , which is at the same time when the economy enters a bad state and the market risk premium increases from θ_L (0.5) in the good state to θ_H (0.6) in the bad state.

A.5.1. Prediction 1. We list the optimal policies, such as the optimal increment ϵ^* , X_r , and X_d , in the legend of panel (a) of Figure A.1. Although the optimal increment ϵ^* is 0.129 for the firm with a high $\hat{\mu}_H$ of 0.04, it becomes 0.452 for the firm with a low $\hat{\mu}_H$ of 0.03. The contrast confirms our intuition that equity holders in the *distressed* firm with a low expected cash flow growth rate would choose investments with high idiosyncratic cash flow growth volatility and thereby illustrates the prominent risk-shifting problem.

A.5.2. Prediction 2. As shown in the legend of panel (b) of Figure A.1, all the optimal policies for the benchmark case are the same as those for firm 1 in panel (a) because the firm we study is exactly firm 1 in the benchmark case. In the second case, with a stochastic market price of risk, when the economy is entering the bad state and the market price of risk θ increases to 0.6, the optimal increment in the idiosyncratic risk, ϵ^* , increases from 0.129 to 0.452, confirming our second prediction that firms would increase their idiosyncratic risk taking more in the bad states than in the good states. Thus, the larger increase in the idiosyncratic volatility is completely a result of the increased market risk premium in the bad states because everything else, including $\hat{\mu}_H$, is the same for both cases.

A.5.3. Prediction 3. We are interested in the cross-sectional equity beta $\beta_{i,t}^E$, which varies across firms with different levels of increased idiosyncratic volatility ν_i . We plot the equity beta $\beta_{i,t}^E$ against X_i . To emphasize the negative impact of the increased idiosyncratic volatility on stock returns, our discussion focuses on the elasticity, $\beta_{H,t}^E$, for high idiosyncratic volatility firms after risk shifting.

In panel (a), both firms have already increased their idiosyncratic risk by ϵ^* for $X_t < 0.366$. It is evident that, given a certain level of cash flows X_t , the firm that chooses a low increment ϵ^* has a high $\beta_{H,t}^E$. In short, consistent with our closed-form solutions for equity returns, panel (a) shows that only the idiosyncratic risk strategically increased by equity holders has a negative impact on equity beta.

What is more important is the negative covariance between equity beta (elasticity) and time-varying market risk

premium. As shown in panel (b), the increase in the idiosyncratic volatility decreases the equity beta at $X_r = 0.366$ immediately. The difference in the equity beta between the benchmark case and the time-varying risk premium case is about 0.5. As discussed for prediction 2, equity holders increase the idiosyncratic volatility to reduce their exposure to the market risk (levered beta) because of the increased market risk premium and discount rate in the bad states. Therefore, the levered equity beta is negatively covarying with the market risk premium.

A.5.4. Prediction 4. The simple model in this appendix is to deliver the closed-form expressions for the equity betas for developing intuition. The comparative statics for this simple model implicitly assume a permanent switch from the good to bad state. To assess our fourth prediction on equity returns quantitatively, we develop and simulate a fully fledged model in the online appendix. In the fully fledged model, we allow the good and bad states to switch between each other following a Markov chain, which is more realistic. Additionally, the firms are allowed to change the risk taking after (not at the same time when) the onset of the bad state.

Endnotes

¹They find that firms with low idiosyncratic stock volatility outperform firms with high volatility by 1.06% per month in both domestic and international stock markets.

²Chen and Petkova (2012) made a similar point that idiosyncratic volatility could hedge against the systematic volatility related to real (growth) options. The hedging mechanism of idiosyncratic volatility works mainly via the put option of *strategically* increasing idiosyncratic risk instead of the call of investments (or real options). This different option is particularly important because the literature and we have shown that the idiosyncratic volatility puzzle mainly manifests among distressed firms instead of healthy firms that are going to exercise their growth options.

³Risk taking does not necessarily mean risk shifting. Only when firms are distressed do they shift the risk to debt holders because taking more risks does not necessarily put debt holders in danger when a firm is healthy, and the equity holders, therefore, pay for any losses themselves.

⁴Galai and Masulis (1976), Johnson (2004), and Bhamra and Shim (2013) link asset growth volatility with the idiosyncratic volatility puzzle. These studies model growth options and do not consider the options of strategically increasing idiosyncratic volatility and going into bankruptcy.

⁵Risk taking means that equity holders have to bear any losses themselves if they take on more risk. When their firm is healthy, equity holders who take on risky investments are still able to pay the debt holders back from their own pockets and, thus, bear any losses themselves. Because equity holders bear the asset risk they take on, equity risk increases with asset risk. Risk shifting is different. Let us consider an extreme case in which the asset value is already below the debt value and the equity value is zero. In the bad scenario, regardless of how much of a loss the new, high-risk investments cause, the equity value is always zero and the debt holders bear all of the new losses. In the good scenario, a positive cash windfall generated by the new investments may drag the firm out of distress and turn the equity value positive. Therefore, the greater asset risk induces a greater *expected* equity value. To take advantage of the high risk, equity holders strategically increase the business risk.

⁶Recent applications of the option pricing framework include Berk et al. (1999), Carlson et al. (2004), and others.

⁷Additionally, Ferguson and Shockley (2003) show that the financial leverage can help explain the size and value premia. Choi (2013) and Obreja (2013) show that the financial leverage drives the value premium.

⁸We follow Garlappi and Yan (2011) and assume the cash flow beta $\beta = 1$.

⁹Empirically, Davydenko (2008) documents that the majority of firms with negative net worth do not default for at least a year and that the mean (median) of the market value of assets at default is only 66% (61.6%) of the face value of debt. This finding shows the importance of the option to delay bankruptcy.

¹⁰These two mechanisms have opposite effects on the equity beta. Their relative effects depend not only on their payoffs, but also on the probability that they will be exercised. First, the potential increment in idiosyncratic volatility ϵ has a positive impact on the payoff from the option of increasing volatility. As shown in Equation (4), given the constant cost η , the greater the risk increment ϵ , the greater the payoff ($V_{L,r} - V_{H,r} + \eta\epsilon^2 V_{H,r}$). Second, before the risk is shifted, the likelihood of going bankrupt and the expected value of the option of going bankrupt are small because the firm is still at a low level of risk. Mathematically, the probability of exercising those two options can be approximated by the distance of X_t from their exercising thresholds. When the firm is approaching the high level of risk, $X_t \rightarrow X_r$, the risk-neutral probability $(X_t/X_r)^{\omega_{L,1}} \rightarrow 1$ for the option of increasing asset risk, and the risk-neutral probability $(X_r/X_d)^{\omega_{H,1}}$ (X_t/X_r) $^{\omega_{L,1}} \rightarrow (X_r/X_d)^{\omega_{H,1}} \leq 1$ for the option of delaying bankruptcy.

¹¹Consider the firm in panel (b) of Figure A.1. Its equity betas (dotted line) decrease after the firm increases its idiosyncratic risk at $X_r = 0.446$, where the market risk premium increases. Hence, the betas negatively covary with the market risk premium.

¹²They demonstrate that the third item $\frac{E[r^m]}{(E[\sigma^m])^2 \text{cov}(\beta_i^m, r^m - E[r^m])^2}$ is trivial.

¹³Consider the numerical example in the appendix. In panel (b) of Figure A.1, the equity betas (shown by the dotted line) in the distressed area where $X_t < X_r = 0.446$ are greater than those in the healthy area where $X_t > 0.8$.

¹⁴The changes in asset values, V_t , are not a result of the investments, but entirely driven by the cash flow shock, X_t . As shown in Equations (A.2) and (A.4), the asset growth dV_t/V_t is exactly the same as the growth of cash flow dX_t/X_t , that is, $dV_t/V_t = dX_t/X_t$. We assume that the change $dV_t = X_t$, and therefore, $dV_t/V_t = X_t/V_t$, which is profitability.

¹⁵Under the 1978 amendment to the Securities and Exchange Act of 1934, all institutional investors managing a portfolio with an investment value of \$100 million or more are required to file quarterly 13F reports to the SEC, listing their equity positions greater than 10,000 shares or \$200,000 in market value as of the last date of each quarter.

¹⁶In unreported tables, we sort the firm into quintiles based on size and find that the institutional holdings of the top five investors increase from 0.10 to 0.22 with the firm size, implying that small firms are more vulnerable to agency problems because of a lack of monitoring.

¹⁷Our results were very similar when we used a Fama–MacBeth regression in an early version of this research.

¹⁸In recent work, Hou and Loh (2015) conduct a comprehensive comparison of explanations of the puzzle and conclude that most of them account for less than 10% of the puzzle. Even when all the explanations are combined, only 29% to 54% of the puzzle is explained. Barberis and Huang (2008) discuss the lottery preferences of investors, and Boyer et al. (2010) provide empirical evidence that this behavioral theory may indeed explain the idiosyncratic volatility puzzle. A few papers focus on the relation between idiosyncratic

volatility and firms' operating performance. Jiang et al. (2009) show that idiosyncratic volatility contains information about future earnings. Avramov et al. (2013) use credit ratings to classify firms' financial status and provide evidence that the idiosyncratic volatility puzzle exists only in distressed firms. Market frictions, such as the one-month-return reversal effect (Fu 2009, Huang et al. 2010), illiquidity (Han and Lesmond 2011), price delay (Hou and Moskowitz 2005), short-sale constraints (Boehme et al. 2009), and limits to arbitrage (Stambaugh et al. 2015), are also examined as potential reasons for the idiosyncratic volatility puzzle. Galai and Masulis (1976), Johnson (2004), Bhamra and Shim (2013), and Babenko et al. (2016) link asset growth volatility with the idiosyncratic volatility puzzle. These studies model growth options and do not consider the options of strategically increasing idiosyncratic volatility and going into bankruptcy.

¹⁹Similar pricing kernels are used in Berk et al. (1999) and Carlson et al. (2004).

²⁰An asset is more idiosyncratic if it cannot be easily redeployed by other firms for common operations. For example, R&D investment is generally regarded as less redeployable (Titman 1984). Practically, a firm can invest more in R&D projects to increase its idiosyncratic risk taking. For example, Research in Motion, the manufacturer of Blackberry smartphones, has increased its R&D expenditure more than fourfold since 2008 although its annual revenue growth rate has declined from 100% to -34%.

²¹Because of costly monitoring, debt holders might not be able to prevent the risk-shifting behavior. For example, Piskorski and Westerfield (2015) study the role of costly monitoring in the moral hazard problem.

²²It is simple to introduce a Nash bargaining game at default as in Fan and Sundaresan (2000) and Garlappi and Yan (2011). However, the qualitative results remain unchanged.

²³It makes no difference if we maximize $E_{L,r}$ because it equals $E_{H,r} - \eta\epsilon^2 V_{H,r}(1 - \tau)$ according to the value-matching condition in Equation (A.25).

References

- Albuquerque R, Wang N (2008) Agency conflicts, investment, and asset pricing. *J. Finance* 63(1):1–40.
- Ang A, Hodrick RJ, Xing Y, Zhang X (2006) The cross-section of volatility and expected returns. *J. Finance* 61(1):259–299.
- Ang A, Hodrick RJ, Xing Y, Zhang X (2009) High idiosyncratic volatility and low returns: International and further U.S. evidence. *J. Financial Econom.* 91(1):1–23.
- Arnold M, Hackbarth D, Xenia Puhan T (2017) Financing asset sales and business cycles. *Rev. Finance* 22(1):243–277.
- Avramov D, Chordia T, Jostova G, Philipov A (2013) Anomalies and financial distress. *J. Financial Econom.* 108(1):139–159.
- Babenko I, Boguth O, Tserlukevich Y (2016) Idiosyncratic cash flows and systematic risk. *J. Finance* 71(1):425–456.
- Barberis N, Huang M (2008) Stocks as lotteries: The implications of probability weighting for security prices. *Amer. Econom. Rev.* 98(5):2066–2100.
- Bartram SM, Brown G, Stulz RM (2016) Why does idiosyncratic risk increase with market risk? Working paper, Warwick Business School, Warwick, UK.
- Berk JB, Green RC, Naik V (1999) Optimal investment, growth options and security returns. *J. Finance* 54(5):1553–1607.
- Bhamra H, Shim K (2017) Stochastic idiosyncratic cash flow risk and real options: Implications for stock returns. *J. Econom. Theory* 168:400–431.
- Bloom N (2009) The impact of uncertainty shocks. *Econometrica* 77(3): 623–685.
- Boehme RD, Danielsen BR, Kumar P, Sorescu SM (2009) Idiosyncratic risk and the cross-section of stock returns: Merton (1987) meets Miller (1977). *J. Financial Markets* 12(3):438–468.

- Boyer B, Mitton T, Vorkink K (2010) Expected idiosyncratic skewness. *Rev. Financial Stud.* 23(1):169–202.
- Carlson M, Lazrak A (2010) Leverage choice and credit spreads when managers risk shift. *J. Finance* 65(6):2323–2362.
- Carlson M, Fisher A, Giammarino R (2004) Corporate investment and asset price dynamics: Implications for the cross-section of returns. *J. Finance* 59(6):2577–2603.
- Chen H, Xu Y, Yang J (2013) Systematic risk, debt maturity, and the term structure of credit spreads. *J. Financial Econom.* Forthcoming.
- Chen Z, Petkova R (2012) Does idiosyncratic volatility proxy for risk exposure? *Rev. Financial Stud.* 25(9):2745–2787.
- Cheng IH, Milbradt K (2012) The hazards of debt: Rollover freezes, incentives, and bailouts. *Rev. Financial Stud.* 25(4):1070–1110.
- Choi J (2013) What drives the value premium? The role of asset risk and leverage. *Rev. Financial Stud.* 26(11):2845–2875.
- Core J, Guay W (1999) The use of equity grants to manage optimal equity incentive levels. *J. Accounting Econom.* 28(2):151–184.
- Davydenko SA (2008) When do firms default? A study of the default boundary. Working paper, University of Toronto, Toronto.
- Davydenko SA, Strebulaev IA (2007) Strategic actions and credit spreads: An empirical investigation. *J. Finance* 62(6):2633–2671.
- Demsetz H (1983) The structure of ownership and the theory of the firm. *J. Law Econom.* 26(2):375–390.
- Dimson E (1979) Risk measurement when shares are subject to infrequent trading. *J. Financial Econom.* 7(2):197–226.
- Eisdorfer A (2008) Empirical evidence of risk shifting in financially distressed firms. *J. Finance* 63(2):609–637.
- Ericsson J (2000) Asset substitution, debt pricing, optimal leverage and optimal maturity. Working paper, McGill University, Montreal.
- Fama EF, French KR (1993) Common risk factors in the returns on stocks and bonds. *J. Financial Econom.* 33(1):3–56.
- Fama EF, French KR (1996) Multifactor explanations of asset pricing anomalies. *J. Finance* 51(1):55–84.
- Fama EF, French KR (2016) Dissecting anomalies with a five-factor model. *Rev. Financial Stud.* 29(1):69–103.
- Fan H, Sundaresan SM (2000) Debt valuation, renegotiation, and optimal dividend policy. *Rev. Financial Stud.* 13(4):1057–1099.
- Favara G, Morellec E, Schroth E, Valta P (2017) Debt enforcement, investment, and risk taking across countries. *J. Financial Econom.* 123(1):22–41.
- Favara G, Schroth E, Valta P (2012) Strategic default and equity risk across countries. *J. Finance* 67(6):2051–2095.
- Ferguson MF, Shockley RL (2003) Equilibrium “anomalies.” *J. Finance* 58(6):2549–2580.
- Fu F (2009) Idiosyncratic risk and the cross-section of expected stock returns. *J. Financial Econom.* 91(1):24–37.
- Galai D, Masulis RW (1976) The option pricing model and the risk factor of stock. *J. Financial Econom.* 3(1/2):53–81.
- Garlappi L, Yan H (2011) Financial distress and the cross section of equity returns. *J. Finance* 66(3):789–822.
- Hackbarth D, Haselmann R, Schoenherr D (2015) Financial distress, stock returns, and the 1978 bankruptcy reform act. *Rev. Financial Stud.* 28(6):1810–1847.
- Han Y, Lesmond D (2011) Liquidity biases and the pricing of cross-sectional idiosyncratic volatility. *Rev. Financial Stud.* 24(5):1590–1629.
- Hennessy CA, Tserlukevich Y (2008) Taxation, agency conflicts, and the choice between callable and convertible debt. *J. Econom. Theory* 143(1):374–404.
- Herskovic B, Kelly BT, Lustig H, Van Nieuwerburgh S (2016) The common factor in idiosyncratic volatility: Quantitative asset pricing implications. *J. Financial Econom.* 119(2):249–283.
- Hirshleifer D, Low A, Teoh SH (2012) Are overconfident CEOs better innovators? *J. Finance* 67(4):1457–1498.
- Hou K, Loh R (2016) Have we solved the idiosyncratic volatility puzzle? *J. Financial Econom.* 121(1):167–194.
- Hou K, Moskowitz TJ (2005) Market frictions, price delay, and the cross-section of expected returns. *Rev. Financial Stud.* 18(3):981–1020.
- Hou K, Xue C, Zhang L (2015) Digesting anomalies: An investment approach. *Rev. Financial Stud.* 28(3):650–705.
- Hovakimian A, Opler T, Titman S (2001) The debt-equity choice. *J. Financial Quant. Anal.* 36(1):1–24.
- Huang J, Sialm C, Zhang H (2011) Risk shifting and mutual fund performance. *Rev. Financial Stud.* 24(8):2575–2616.
- Huang W, Liu Q, Rhee SG, Zhang L (2010) Return reversals, idiosyncratic risk, and expected returns. *Rev. Financial Stud.* 23(1):147–168.
- Irvine PJ, Pontiff J (2009) Idiosyncratic return volatility, cash flows, and product market competition. *Rev. Financial Stud.* 22(3):1149–1177.
- Jagannathan R, Wang Z (1996) The conditional CAPM and the cross-section of expected returns. *J. Finance* 51(1):3–53.
- Jensen MC, Meckling WH (1976) Theory of the firm: Managerial behavior, agency costs and ownership structure. *J. Financial Econom.* 3(4):305–360.
- Jiang GJ, Xu D, Yao T (2009) The information content of idiosyncratic volatility. *J. Financial Quant. Anal.* 44(1):1–28.
- Johnson TC (2004) Forecast dispersion and the cross section of expected returns. *J. Finance* 59(5):1957–1978.
- Leland HE (1994) Corporate debt value, bond covenants, and optimal capital structure. *J. Finance* 49(4):1213–1252.
- Leland HE (1998) Agency costs, risk management, and capital structure. *J. Finance* 53(4):1213–1243.
- Lewellen J, Nagel S (2006) The conditional CAPM does not explain asset-pricing anomalies. *J. Financial Econom.* 82(2):289–314.
- Merton RC (1974) On the pricing of corporate debt: The risk structure of interest rates. *J. Finance* 29(2):449–470.
- Newey WK, West KD (1987) Hypothesis testing with efficient method of moments estimation. *Internat. Econom. Rev.* 28(3):777–787.
- Obreja I (2013) Book-to-market equity, financial leverage, and the cross-section of stock returns. *Rev. Financial Stud.* 26(5):1146–1189.
- Ohlson JA (1980) Financial ratios and the probabilistic prediction of bankruptcy. *J. Accounting Res.* 18(1):109–131.
- Piskorski T, Westerfield MM (2016) Optimal dynamic contracts with moral hazard and costly monitoring. *J. Econom. Theory* 166:242–281.
- Schaefer SM, Strebulaev IA (2008) Structural models of credit risk are useful: Evidence from hedge ratios on corporate bonds. *J. Financial Econom.* 90(1):1–19.
- Shleifer A, Vishny RW (1986) Large shareholders and corporate control. *J. Political Econom.* 94(3, part 1):461–488.
- Stambaugh RF, Yu J, Yuan Y (2015) Arbitrage asymmetry and the idiosyncratic volatility puzzle. *J. Finance* 70(5):1903–1948.
- Strebulaev IA (2007) Do tests of capital structure theory mean what they say? *J. Finance* 62(4):1747–1787.
- Titman S (1984) The effect of capital structure on a firm’s liquidation decision. *J. Financial Econom.* 13(1):137–151.
- Vassalou M, Xing Y (2004) Default risk in equity returns. *J. Finance* 59(2):831–868.