Review of Economic Dynamics 38 (2020) 127–153

Contents lists available at ScienceDirect

Review of Economic Dynamics

www.elsevier.com/locate/red

The welfare and distributional effects of fiscal volatility: A quantitative evaluation ☆

Rüdiger Bachmann [a], Jinhui H. Bai [b,c,*], Minjoon Lee [d], Fudong Zhang [e]

[a] University of Notre Dame, CEPR, CESifo and ifo, United States of America
[b] Washington State University, United States of America
[c] Peking University HSBC Business School, China
[d] Carleton University, Canada
[e] Tsinghua University, China

A R T I C L E   I N F O

Article history:
Received 18 May 2018
Received in revised form 20 March 2020
Available online 16 April 2020

JEL classification:
E30
E32
E60
E62
H30

Keywords:
Fiscal volatility
Welfare costs
Distributional effects
Labor income risk
Wealth inequality
Transition path

A B S T R A C T

This study explores the welfare and distributional effects of fiscal volatility using a neoclassical stochastic growth model with incomplete markets. In our model, households face uninsurable idiosyncratic risks in their labor income and discount factor processes, and we allow aggregate uncertainty to arise from both productivity and government purchases shocks. We calibrate our model to key features of the U.S. economy, before eliminating government purchases shocks. We then evaluate the distributional consequences of the elimination of fiscal volatility and find that, in our baseline case, welfare gains increase with private wealth holdings.

1. Introduction

One consequence of the financial crisis followed by political turmoil has been the perception of high volatility in government policies in both the U.S. and in Europe.[1] In this paper, we study, from the viewpoint of the household, the welfare

☆ We thank seminar and conference participants at Korea University, Peking University, Renmin University of China, Shanghai Jiao Tong University, Shanghai University of Finance and Economics, the Kansas City Fed, the Philadelphia Fed, the St. Louis Fed, Tsinghua University, University of Bonn, University of Connecticut, University of Hawaii, University of Hong Kong, University of Illinois at Urbana-Champaign, University of Missouri at Columbia, University of Southampton, University of Southern California, University of Tokyo, Washington State University, as well as the Midwest Macro Fall Meeting 2013, the SED 2014 conference, the CEF 2015 conference, the Fudan Economic Dynamics Workshop 2015, the SITE Summer Workshop 2016, the CCER Summer Institute 2017, and the CICM 2018.

* Corresponding author.
E-mail addresses: rbachmann@nd.edu (R. Bachmann), jinhui.bai@wsu.edu (J.H. Bai), minjoon.lee@carleton.ca (M. Lee), zhangfd@pbcsf.tsinghua.edu.cn (F. Zhang).

[1] This paper focuses on "fiscal volatility." However, in describing the related literature, we follow the widespread use in the recent literature and treat "fiscal uncertainty" and "fiscal volatility" as synonymous.

https://doi.org/10.1016/j.red.2020.04.001
1094-2025/

costs of the volatility of government purchases, both in the aggregate and across different wealth holdings. We do so in a neoclassical model with incomplete markets and a richly specified government sector, where we eliminate the volatility of government purchases once and for all.

Most of the existing research on the consequences of fiscal volatility has focused on the aggregate effects of short-run volatility fluctuations on various macroeconomic variables. In one study, Baker et al. (2016) analyze Internet news and find a (causal) relationship between high policy uncertainty and subdued aggregate economic activity. In another study, based on a New Keynesian DSGE model, Fernández-Villaverde et al. (2015) find large contractionary effects of fiscal volatility on economic activity accompanied by inflationary pressure, especially when the nominal interest rate is at the zero lower bound. By contrast, we study the effects of permanently eliminating fiscal volatility on household welfare with a particular emphasis on distributional aspects.[2] In studying the welfare effect of permanent changes in fiscal policy, we take a similar approach to McKay and Reis (2016). They focus on permanent changes in the automatic stabilizer role of fiscal policy. Our study complements theirs through its focus on government purchases rather than transfers (see below for a more detailed discussion of the literature).

To quantify the welfare costs of fluctuations in government purchases for households, we follow the approach of Krusell and Smith (1998) and use an incomplete market model where heterogeneous households face uninsurable idiosyncratic risks in their labor income and discount factor processes. We then calibrate this model with U.S. data, in particular data on U.S. wealth inequality. Our model has aggregate uncertainty arising from both productivity and government purchases shocks. We thus specify government purchases shocks as the only fundamental source of fiscal volatility. In line with the data, we further assume that government purchases shocks are independent of aggregate productivity and employment conditions.[3] Government purchases enter the utility function of the households as separable goods.[4] We also employ an empirical aggregate tax revenue response rule, which includes government debt and is estimated from U.S. data.

Because the government partially funds its expenditures through taxation, purchases fluctuations generate volatile household-specific tax rates. To capture the distributional effects of fiscal shocks through taxation, we model key features of the progressive U.S. income tax system. Importantly, even though all the households face the same progressive tax schedule, depending on where they belong in the income distribution, their household-specific tax rate risks are differentially impacted by the aggregate fiscal risk. In U.S. data, we indeed find that higher tax revenues are associated with more progressive income taxes, rather than uniform shifts up in the tax schedule. This fact calls for capturing realistic household heterogeneity in our model.

To eliminate fiscal volatility, following Krusell and Smith (1999) and Krusell et al. (2009), we start from a stochastic steady state of the economy with both productivity and government purchases shocks, and remove the fiscal shocks at a given point in time by replacing them with their conditional expectations, while retaining the aggregate productivity process. We then compute the transition path towards the new stochastic steady state in full general equilibrium. Based on the quantitative solution for this transition path, we then compare the welfare of various household groups in the transition-path equilibrium to their welfare level with both aggregate shocks in place.

Our results show that the aggregate welfare costs from fiscal shocks are fairly small. The effect of removing fiscal volatility is equivalent to a 0.03% increase in the lifetime consumption on average. This is comparable to the welfare costs of business cycle fluctuations reported in Lucas (1987, 2003), as well as to those from a representative-agent version of our model, even though in our model aggregate (spending) fluctuations lead to differential impacts on household-specific tax rate risks in addition to the before-tax factor prices volatility, so that they, a priori, may lead to larger welfare costs than in Lucas (1987, 2003) (see Krusell et al., 2009).

By contrast, our results reveal interesting variations in the welfare costs of fiscal volatility along the wealth distribution. The welfare gains of eliminating fiscal volatility are increasing in household wealth according to the baseline specification, where the implementation of the progressive U.S. federal income tax system and the aggregate tax revenue response rule is modeled to best match the cyclicality of important moments of the U.S. tax system.

Since volatile tax rates pre-multiply labor income levels, they generate – loosely speaking – multiplicative after-tax labor income risk.[5] Just as with the additive labor endowment risk in early incomplete market models, this after-tax labor income risk leads households to self-insure through precautionary saving. Wealth-rich households can thus achieve a higher degree of self-insurance relative to wealth-poor households. Consequently, from a precautionary saving perspective, wealth-poor households should gain more when fiscal volatility is eliminated.

However, due to the multiplicative nature of the after-tax capital income risk, the tax-rate uncertainty induced by government purchases fluctuations also creates a rate-of-return risk to savings, which in turn, impacts the quality of capital and

[2] There are a few exceptions in an older literature with either no or rather limited heterogeneity: Bizer and Judd (1989), Chun (2001), and Skinner (1988).

[3] This might seem like an extreme assumption. It might be interesting to explore an alternative environment where government purchases are at least partly endogenously determined (see, e.g., Bachmann and Bai, 2013a,b). However, this assumption makes the implementation and interpretation of the thought experiment of eliminating fiscal volatility clean and transparent, and is akin to the original thought experiment about the elimination of business cycles in Lucas (1987, 2003).

[4] We consider other utility specifications with complementary and substitutable private-public good relationships, respectively, in extensions to the baseline calibration.

[5] This is cleanest in a linear tax system. However, even in a progressive tax system, the fluctuating average tax rates work like multiplicative after-tax income risk.

bonds as saving vehicles.[6] In a realistic incomplete asset market model where the after-tax return of all the financial assets is subject to tax rate uncertainty, wealth-rich households have much larger exposure to such a rate-of-return risk. As a result, from the rate-of-return risk perspective, wealth-rich households should gain more when fiscal volatility is eliminated.

Finally, the distributional effects of eliminating fiscal volatility can depend on its effect on the average factor prices.[7] The precautionary saving and rate-of-return risk effects lead to endogenous responses of the aggregate capital stock, changing both the pre-tax capital rate-of-return and real wages. In our baseline specification, the aggregate capital stock first declines and then increases after the elimination of fiscal volatility, causing a higher interest rate and lower wage rate in the early transition periods followed by a reversal later on.

Whether the combination of these three effects favors the wealth-rich or the wealth-poor households depends *in principle* – as we will show – on the details of the implementation of the progressive tax system and the aggregate tax revenue rule. Under the baseline specification, which is calibrated to best mimic the cyclical behavior of key moments of the U.S. tax system, the wealth-rich households are significantly exposed to the rate-of-return risk caused by tax-rate uncertainty, and they also benefit from changes in average factor prices. As a result, we find that the welfare gains are increasing in household wealth. The first contribution of the paper is thus to provide a calibration strategy that allows us to quantify the net effect of the precautionary saving, the rate-of-return risk, and the average factor price effects.

In addition to our baseline, we consider alternative implementations of how the progressive tax system and the aggregate tax revenue rule interplay. The distributional effects of fiscal volatility vary in these exercises, and thus, despite their counterfactual implications, help us uncover the mechanisms through which fiscal volatility influences economic welfare. A second contribution of the paper is thus to map out the relationship between tax instruments in a progressive tax system used to obtain the cyclical adjustment of the government budget and the distributional effects of fiscal volatility.

We also consider several alternative fiscal regimes: for example, a balanced budget regime with a progressive tax system, a linear tax system, and a lump-sum tax system, with the latter two again allowing for government debt. The welfare results under those three regimes are all in line with our baseline. In another variation, we show that when private and public consumption are complements, the overall welfare gains from eliminating government purchases fluctuations are higher, because a higher government purchases level leads to a higher marginal utility of private consumption when taxes are high (because government purchases are large). In addition, we extend our baseline model to allow for a positive fiscal impact multiplier consistent with the data and find similar distributional effects. Finally, motivated by recent policy discussions of the possible permanence of heightened fiscal volatility, we examine the welfare consequences of doubling the historical government purchases volatility level. Our results suggest that the welfare effects of fiscal volatility are symmetric between zero and twice the pre-crisis volatility of government purchases.

In addition to its substantive contributions, our study also makes a technical contribution to the literature. Specifically, we merge the algorithm for computing the *deterministic* transition path in heterogeneous-agent economies from Huggett (1997) and the algorithm for computing a *stochastic* recursive equilibrium in Krusell and Smith (1998) to show that an approximation of the wealth distribution and its law of motion by a finite number of moments can also be applied to a stochastic transition path analysis. Recall that after fiscal volatility is eliminated, our economy is still subject to aggregate productivity shocks. This solution method should prove useful for other quantitative studies of stochastic transition-path equilibria.

Related Literature

Besides the general link to the literature on incomplete markets and wealth inequality (see Heathcote et al., 2009 for an overview), our study is most closely related to three strands of literature.

First, our paper contributes to research on the welfare costs of aggregate fluctuations (see Lucas, 2003 for a comprehensive discussion). As in Krusell and Smith (1999), Mukoyama and Sahin (2006) and Krusell et al. (2009), we quantify the welfare and distributional consequences of eliminating macroeconomic fluctuations. However, while these studies focus on TFP fluctuations, we examine the welfare consequences of eliminating fluctuations in government purchases. Our study complements theirs by examining fluctuations due to fiscal policy, arguably a more plausible candidate fluctuation to be (fully) eliminated by a policy maker – they are, after all, the result of a policy decision.

Second, our paper relates to the recent literature about the effects of economic uncertainty on aggregate economic activity. Most of the research in this stream of literature has focused on the amplification and propagation mechanisms for persistent, but temporary volatility shocks, which are typically modeled and measured as changes to the conditional variance of traditional economic shocks. These uncertainty shocks include second-moment shocks to aggregate productivity, and policy and financial variables, which are often propagated through physical production factor adjustment costs, sticky prices, or financial frictions (see e.g., Arellano et al., 2019, Bachmann and Bayer, 2013, 2014, Baker et al., 2016, Basu and Bundick, 2017, Bloom, 2009, Bloom et al., 2018, Born and Pfeifer, 2014, Croce et al., 2012, Fernández-Villaverde et al., 2015, Gilchrist et al., 2014, Kelly et al., 2016, Mumtaz and Surico, 2018, Mumtaz and Zanetti, 2013, Nodari, 2014, Pastor and Veronesi, 2012, 2013, and Stokey, 2016). Other studies investigate the effects of uncertainty in the (time-varying) parameters

6 Angeletos and Calvet (2006), in a seminal contribution on risk in incomplete markets, discuss this tension between labor endowment risk and rate-of-return risk.

7 There is also a direct utility effect because households are risk averse with respect to government purchases fluctuations.

of monetary or fiscal feedback rules (Bi et al., 2013, Davig and Leeper, 2011, and Richter and Throckmorton, 2015), or in the bargaining power parameter of search and matching models (Drautzburg et al., 2017). Our study complements this literature by focusing on the welfare and distributional effects of a permanent change in fiscal volatility.

Finally, our work contributes to the growing body of literature on macroeconomic policy in heterogeneous-agent environments (Auclert, 2019, Bachmann and Bai, 2013a, Bhandari et al., 2017b,a, 2018, Böhm, 2015, Brinca et al., 2016, Dyrda and Pedroni, 2017, Ferriere and Navarro, 2017, Gornemann et al., 2016, Gomes et al., 2013, Hagedorn et al., 2019, Heathcote, 2005, Hedlund et al., 2016, Kaplan and Violante, 2014; Kaplan et al., 2018, Li, 2013, McKay and Reis, 2016, and Röhrs and Winter, 2017). In particular, Heathcote (2005) provided the first quantitative investigation into aggregate and distributional effects of exogenously varying fiscal policy in a heterogeneous-agent incomplete market model à la Krusell and Smith (1998). In Heathcote (2005), the source of aggregate fiscal risks consists in temporary changes in the level of proportional income tax rates. We provide a complementary analysis by focusing on the effects of a permanent change in the volatility of government purchases in a progressive income tax system. There is also a budding empirical literature on the distributional consequences of policy actions: see Coibion et al. (2017) for the case of monetary policy and Giorgi and Gambetti (2012) for the case of fiscal policy.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 discusses its calibration. Section 4 describes our solution method. Section 5 presents the baseline findings on the welfare and distributional effects of eliminating government purchases fluctuations, while Section 6 investigates these welfare and distributional effects in alternative model specifications. We close in Section 7 with final comments and relegate the details of the quantitative procedure to various appendices.

2. Model

Following Aiyagari (1994) and Huggett (1993), we model an incomplete market setting where a continuum of infinitely-lived heterogeneous households face uninsurable idiosyncratic risks in their labor efficiency processes. We also include aggregate productivity shocks as well as shocks to a household's discount factor, as in Krusell and Smith (1998). We then add aggregate uncertainty from government purchases shocks. In our model exposition, we focus our discussion on the fiscal elements.

2.1. The private sector

Our households are *ex-ante* identical, with preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta_t u\left(c_t, G_t\right), \tag{2.1}$$

where β_t denotes the cumulative discount factor between period 0 and period t. In particular, $\beta_t = \tilde{\beta}\beta_{t-1}$, where $\tilde{\beta}$ is an idiosyncratic shock following a three-state, first-order Markov process. Furthermore, c_t denotes private consumption, and G_t the public good provided by the government (government purchases).

The strictly concave flow utility function has constant relative risk aversion (CRRA) with respect to a constant-elasticity-of-substitution (CES) aggregate of c and G,

$$u\left(c_t, G_t\right) = \frac{\left(\theta c_t^{1-\rho} + (1-\theta) G_t^{1-\rho}\right)^{\frac{1-\gamma}{1-\rho}} - 1}{1-\gamma}, \tag{2.2}$$

where γ is the risk aversion parameter and $1/\rho$ is the elasticity of substitution between c and G. We discuss the details of the G_t-process in the next subsection.

Our households also face idiosyncratic employment shocks. We denote the employment process by ε, which follows a first-order Markov process with two states $\{0, 1\}$. $\varepsilon = 1$ denotes that the household is employed, providing a fixed amount of labor $\tilde{l}$ to the market, and is paid the market wage, w. $\varepsilon = 0$ represents the unemployed state of a household who receives an unemployment insurance payment that equals a fraction ω of the current wage income of an employed household.

We represent the aggregate production technology as a Cobb-Douglas function:

$$Y_t = z_t F(K_t, L_t) = z_t K_t^{\alpha} L_t^{1-\alpha}, \tag{2.3}$$

where K_t is aggregate capital, L_t is aggregate labor efficiency input, and z_t is the aggregate productivity level. z_t follows a two-state (z_g, z_b) first-order Markov process, where z_g and z_b denote aggregate productivity in good and bad times, respectively. Note that, because of the law of large numbers, L_t equals $(1-u_t)\tilde{l}$, where u_t is the unemployment rate. We also allow the unemployment rate to take one of two values: u_g in good times and u_b in bad times. In this way, u_t and z_t move perfectly together.

We now specify the standard aggregate resource constraint:

$$C_t + K_{t+1} + G_t = Y_t + (1-\delta)K_t, \tag{2.4}$$

where C_t represents aggregate consumption, and δ the depreciation rate.

The markets in our model are perfectly competitive. Labor and capital services are traded on spot markets each period, at factor prices $r(K_t, L_t, z_t) = \alpha z_t K_t^{\alpha-1} L_t^{1-\alpha} - \delta$ and $w(K_t, L_t, z_t) = (1-\alpha) z_t K_t^{\alpha} L_t^{-\alpha}$. In addition, we assume that the households can trade one-period government bonds on the asset market in each period t. For computational tractability, we follow Heathcote (2005) and assume that government bonds pay the same rate-of-return as physical capital in all future states in $t+1$. Because of the assumed perfect substitutability between capital and bonds, each household has access to effectively only one asset in self-insuring against stochastic shocks. We use a to denote a household's total asset holdings, i.e., the sum of physical capital and government bonds.

2.2. Fiscal volatility and the government budget

Our model has three government spending components: government purchases, G_t, aggregate unemployment insurance payments, Tr_t, and aggregate debt repayments, $(1+r_t)B_t$. Government purchases are the only fundamental source of fiscal volatility. They follow an AR(1) process in logarithms:

$$\log(G_{t+1}) = \left(1-\rho_g\right)\overline{\log(G)} + \rho_g \log(G_t) + (1-\rho_g^2)^{\frac{1}{2}} \sigma_g \epsilon_{g,t+1}, \tag{2.5}$$

where ρ_g is a persistence parameter, $\overline{\log(G)}$ is the unconditional mean of $\log(G_t)$, $\epsilon_{g,t+1}$ is an innovation term which is normally distributed with mean zero and variance one, and σ_g is the unconditional standard deviation of $\log(G_t)$. Note that the government purchases process is independent of the process for aggregate productivity. As is well known and as we show below, government purchases are roughly acyclical in U.S. quarterly data.

The aggregate unemployment insurance payment, $Tr_t = u_t \omega w_t \tilde{l}$, depends on both the unemployment rate, u_t, and the size of the unemployment insurance payment for each household, $\omega w_t \tilde{l}$.

We assume that government spending at time t is financed through a combination of aggregate tax revenue, T_t, and new government debt, B_{t+1}. As in Bohn (1998) and Davig and Leeper (2011), we model the aggregate tax revenue net of transfers (as a fraction of GDP) as an (increasing) function of the debt-to-GDP ratio, making the debt-to-GDP ratio stationary. We can thus specify the following tax revenue response rule for determining tax revenue:

$$\frac{T_t - Tr_t}{Y_t} = \rho_{T,0} + \rho_{T,Y} log(\frac{Y_t}{\overline{Y}}) + \rho_{T,B}\frac{B_t}{Y_t} + \rho_{T,G}\frac{G_t}{Y_t}, \tag{2.6}$$

where $(\rho_{T,0}, \rho_{T,Y}, \rho_{T,B}, \rho_{T,G})$ is a vector of positive coefficients and $\overline{Y}$ is a constant number equal to the unconditional mean of GDP in the ergodic distribution.[8] Furthermore, $\rho_{T,Y}$ captures the automatic stabilizer role of the U.S. tax system when $\rho_{T,Y} > 0$, and $\rho_{T,B}$ and $\rho_{T,G}$ reflect the capability of the endogenous revenue adjustment system in maintaining long-run fiscal sustainability. Note that our tax revenue response rule implies that the government purchases level (relative to GDP) and the GDP gap are the main non-debt determinants of the primary surplus.

Given the total tax revenue in (2.6), we can use the government budget constraint to determine the dynamics of aggregate government debt B_{t+1}:

$$B_{t+1} = (1+r_t)B_t + (G_t + Tr_t - T_t). \tag{2.7}$$

2.3. The progressive tax system

Because the distribution of the tax burden across households is important for quantifying the distributional effects of fiscal policies, we model the tax system to approximate the current U.S. tax regime as realistically as possible while maintaining a certain tractability. Specifically, the government uses a flat-rate consumption tax and a progressive income tax to raise the aggregate tax revenue T_t. The consumption tax is given by:

$$\tau^c(c_t) = \tau_c c_t. \tag{2.8}$$

This specification allows the model to capture sources of tax revenue other than income taxes, which in turn provides a total income tax burden that is in line with the data.

---

[8] $\overline{Y}$ serves as a normalization to make the coefficients of the tax revenue response rule scale-free (we obtain $\overline{Y}$ through a fixed-point iteration procedure as it endogenously affects the average income of the economy through the tax revenue response rule). Also, while $\rho_{T,B} > 0$ is necessary for the debt-to-GDP ratio to be stationary, this condition is not imposed. Instead, all coefficients in equation (2.6) are estimated from the data, and this estimated $\rho_{T,B}$ just turns out to be positive.

Following Castañeda et al. (2003), we specify the progressive income tax function as:

$$\tau^y(y_t) = \begin{cases} \tau_1\left[y_t - \left(y_t^{-\tau_2} + \tau_3\right)^{-\frac{1}{\tau_2}}\right] + \tau_0 y_t & \text{if } y_t > 0 \\ 0 & \text{if } y_t \leq 0, \end{cases} \tag{2.9}$$

where $(\tau_0, \tau_1, \tau_2, \tau_3)$ is a vector of tax coefficients and y_t is taxable household income; or $y_t = r_t a_t + w_t \varepsilon_t \tilde{l}$.[9] The first term in the above equation is based on Gouveia and Strauss' (1994) characterization of the effective federal income tax burden of U.S. households.[10] The federal income tax accounts for about 40% of federal government revenue and is the main driver of progressivity in the U.S. tax system (Piketty and Saez, 2007). The linear term, $\tau_0 y_t$, is used to capture any remaining tax revenue, including state income taxes, property taxes and excise taxes.

With these tax specifications, a household's budget constraint can be written as:

$$(1+\tau_c)c_t + a_{t+1} = a_t + y_t - \tau^y(y_t) + (1-\varepsilon_t)\omega w(K_t, L_t, z_t)\tilde{l}. \tag{2.10}$$

Note that equation (2.6) specifies a tax revenue response rule to calculate the aggregate government tax revenue. Equations (2.8) and (2.9), on the other hand, model the concrete tax instruments with which the government collects tax revenue. These two sets of equations are compatible only if we treat one of the parameters in equation (2.9) as an endogenous tax instrument, to be determined in equilibrium, rather than a fixed tax parameter. We choose, in the baseline specification, τ_1 for this endogenous parameter, $\tau_{1,t}$, and denote the resulting tax function by $\tau^y(y_t; \tau_{1,t})$. Adjusting τ_1 means that the top marginal (average) tax rates, $\tau_0 + \tau_1$, are the main instruments for the required tax schedule adjustments.[11] As we will show in Section 3, choosing τ_1 to be the endogenous tax instrument best matches certain time series evidence on the progressivity measures of the federal income tax code documented in Gouveia and Strauss (1994). This adjustment can satisfy the empirical tax revenue response rule that describes aggregate U.S. tax adjustments well, and, more importantly, ensures the stationarity of the debt-to-GDP ratio. Consequently, we take the empirical tax revenue response rule as given and endogenously adjust one aspect of the tax system to make the two sets of equations compatible, as in Davig and Leeper (2011) and Fernández-Villaverde et al. (2015).

Given our tax function specification, we can now specify total tax revenue as follows:

$$T_t = \tau_c C_t + \int_0^1 \left[\tau_0 y_{i,t} + \tau_{1,t}\left(y_{i,t} - \left(y_{i,t}^{-\tau_2} + \tau_3\right)^{-\frac{1}{\tau_2}}\right)\right] \times \mathbb{1}(y_{i,t} > 0)di. \tag{2.11}$$

Equation (2.11) defines an implicit function of $\tau_{1,t}$. Recall that T_t is governed by G_t, Y_t, B_t, and Tr_t through the tax revenue response rule specified in equation (2.6). This means that, for a given inherited level of bond holdings, B_t, $\tau_{1,t}$ fluctuates in response to changes in both G_t and the income distribution. As a result, in our baseline model the *aggregate* volatility in G_t translates into *idiosyncratic* tax rate uncertainty.

2.4. The household's decision problem and the competitive equilibrium

In this subsection, we discuss the household's dynamic decision problem, which is determined by both the idiosyncratic state vector $(a, \varepsilon, \tilde{\beta})$ and the aggregate state vector (Γ, B, z, G), where Γ denotes the measure of households over $(a, \varepsilon, \tilde{\beta})$. We begin by letting H_Γ denote the equilibrium transition function for Γ[12]:

$$\Gamma' = H_\Gamma(\Gamma, B, z, G, z'). \tag{2.12}$$

We next let H_B denote the (exogenous) transition function for B, as described in equation (2.7):

$$B' = H_B(\Gamma, B, z, G). \tag{2.13}$$

Finally, we let Θ denote the equilibrium function for the endogenous tax parameter τ_1, which is implicitly determined in equation (2.11):

[9] Unlike in Castañeda et al. (2003), where households cannot borrow and thus cannot have negative income, y_t can be negative in our model in rare cases, so that we have to specify the tax function also for the case of $y_t < 0$.

[10] The definition of income in Gouveia and Strauss (1994) is total taxable income including capital income, which is consistent with our treatment in the model.

[11] Both derivatives of equation (2.9) and equation (2.9) divided by y_t converge to $\tau_0 + \tau_1$ for large y_t. In Section 6, we examine three alternative specifications, where we let τ_0, τ_2, and τ_3, respectively, be the tax instruments that adjust endogenously.

[12] Note that z', but not G', is an argument of H_Γ. This is because, in our setting, which reflects the setting in Krusell and Smith (1998), the future z affects the employment transition process, while the G-process is independent of other processes. Note that we also leave time subscripts and switch into recursive notation now.

$$\tau_1 = \Theta(\Gamma, B, z, G). \tag{2.14}$$

The dynamic programming problem faced by a household can now be written as follows:

$$V(a, \varepsilon, \tilde{\beta}, \Gamma, B, z, G; H_\Gamma, \Theta) = \max_{c,a'}\{u(c, G) + \tilde{\beta}E[V(a', \varepsilon', \tilde{\beta}', \Gamma', B', z', G'; H_\Gamma, \Theta)|\varepsilon, \tilde{\beta}, z, G]\}$$

$$\begin{aligned}\text{subject to: } (1+\tau_c)c + a' &= a + y - \tau^y(y; \tau_1) + (1-\varepsilon)\omega w(K, L, z)\tilde{l}\\ y &= r(K, L, z)a + w(K, L, z)\varepsilon\tilde{l},\\ a' &\geq \underline{a},\\ \Gamma' &= H_\Gamma(\Gamma, B, z, G, z'),\\ B' &= H_B(\Gamma, B, z, G),\\ \tau_1 &= \Theta(\Gamma, B, z, G),\end{aligned}$$

where ε and $\tilde{\beta}$ follow the processes specified in Section 2.1, G follows the process specified in equation (2.5), and $\underline{a}$ is an exogenously set borrowing constraint. Finally, we can summarize the optimal saving decision for households in the following policy function:

$$a' = h(a, \varepsilon, \tilde{\beta}, \Gamma, B, z, G; H_\Gamma, \Theta). \tag{2.15}$$

Our recursive competitive equilibrium is then defined as: the law of motion H_Γ,[13] individual value and policy functions $\{V, h\}$, pricing functions $\{r, w\}$, and the Θ-function for the endogenous parameter τ_1, such that:

1. $\{V, h\}$ solve the household's problem.
2. $\{r, w\}$ are competitively determined.
3. Θ satisfies equation (2.11) with the tax revenue response rule (2.6) replacing T_t.
4. H_Γ is generated by h.[14]

The economy without a fluctuating G_t is identical, except for the deterministic G_t-process.

3. Calibration

In this section, we discuss our model calibration beginning with basic parameters. The frequency of our model economy is quarterly. We parameterize the model to match important aggregate and cross-sectional statistics of the U.S. economy (Table 1).

3.1. Basic parameters

We set the relative risk aversion parameter $\gamma = 1$, and the elasticity of substitution between private consumption and the public good $1/\rho = 1$. To calibrate the weight of private consumption in the utility function, θ, we assume that the Lindahl-Samuelson condition holds for our economy in the long-run. This means that there is efficient provision of public goods, i.e., there are equalized marginal utilities from private and public goods. Mathematically, this is represented as $\int_0^1 \frac{(1-\theta)/G_t}{\theta/c_{it}} di = 1$, on average over many time periods. With this procedure, θ is calibrated to 0.722.

We take other parameter values directly from Krusell and Smith (1998) that are based on U.S. data in the 1980s and 1990s, roughly the middle segment of the sample period used to calibrate the fiscal parameters (1960Q1-2007Q4; see Section 3.2): the depreciation rate is $\delta = 0.025$, the capital elasticity of output in the production function is $\alpha = 0.36$, and labor supply is normalized to $\tilde{l} = 0.3271$. We allow our aggregate productivity process, z_t, to take on two values, $z_g = 1.01$ and $z_b = 0.99$, with unemployment rates of $u_g = 0.04$ and $u_b = 0.1$, respectively. The transition matrix for z_t is as follows:

$$\begin{bmatrix} 0.875 & 0.125 \\ 0.125 & 0.875 \end{bmatrix},$$

where rows represent the current state and columns represent the next period's state. The first row and column correspond to z_g. The transition matrix for the employment status, ε, is a function of both the current aggregate state (z) and the future

[13] Note that since H_B is exogenously determined by equation (2.7), it is not an equilibrium object.

[14] Note that, aggregate asset holdings in this economy equal $K + B$ (capital and bonds are perfectly substitutable for households). Therefore, H_Γ and H_B determine the evolution of the supply of physical capital, K. The competitively determined r then clears the capital market.

Table 1
Summary of parameters.

Parameter	Value	Description	Source / Target
Taken from the literature			
$1/\rho$	1.00	Elasticity of substitution between c and G	Standard value
γ	1.00	Relative risk aversion	Standard value
α	0.36	Capital share	Standard value
δ	0.025	Depreciation rate	Standard value
$\tilde{l}$	0.3271	Hours of labor supply of employed	Normalization
(z_l, z_h)	(0.99, 1.01)	Support of aggr. productivity process	Krusell and Smith (1998)
$\Pi_{z,z'}$	See text	Transition matrix of aggr. productivity process	Krusell and Smith (1998)
(u_g, u_b)	(4%, 10%)	Possible unemployment rates	Krusell and Smith (1998)
$\Pi_{\varepsilon\varepsilon'\|zz'}$	See text	Transition matrix of employment process	Krusell and Smith (1998)
ω	0.10	Replacement rate	Krusell and Smith (1998)
τ_2	0.768	Parameter in the progressive tax function	Gouveia and Strauss (1994)
Estimated from the data			
τ_0	5.25%	Income tax parameter	
τ_c	8.14%	Consumption tax rate	
$\rho_{T,B}$	0.0173	Debt coefficient of fiscal rule	
$\rho_{T,Y}$	0.2820	Output coefficient of fiscal rule	
$\rho_{T,G}$	0.4835	Government purchases coefficient of fiscal rule	
$(G_l/G_m, G_h/G_m)$	(0.951, 1.049)	Size of the G-shock	
$\Pi_{G,G'}$	See text	Transition matrix of the G-process	
Calibrated in the model			
θ	0.7221	Weight on private consumption in utility	Lindahl-Samuelson condition
G_m	0.2318	Value of the middle grid of the G-process	Mean G/Y (20.86%)
$\underline{a}$	-4.15	Borrowing constraint	Negative wealth share (11%)
$\rho_{T,0}$	0.1007	Intercept of tax revenue rule	Average annualized B/Y (30%)
$\tilde{\beta}_m$	0.9919	Medium value of discount factor	Average annualized K/Y (2.5)
$\tilde{\beta}_h - \tilde{\beta}_m, \tilde{\beta}_m - \tilde{\beta}_l$	0.0046	Size of discount factor variation	Gini coeff. (0.79)
$\Pi_{\tilde{\beta},\tilde{\beta}'}$	See text	Transition matrix of discount factor	Top 1% wealth share (30%)
τ_3	1.776	Parameter in the progressive tax function	Mean of τ_1 (25.8%)

aggregate state (z'). There are thus four possible cases, (z_g, z_g), (z_g, z_b), (z_b, z_g), and (z_b, z_b), corresponding to the following employment status transition matrices[15]:

$$\begin{bmatrix} 0.33 & 0.67 \\ 0.03 & 0.97 \end{bmatrix}, \begin{bmatrix} 0.75 & 0.25 \\ 0.07 & 0.93 \end{bmatrix}, \begin{bmatrix} 0.25 & 0.75 \\ 0.02 & 0.98 \end{bmatrix}, \begin{bmatrix} 0.60 & 0.40 \\ 0.04 & 0.96 \end{bmatrix},$$

where the first row and column correspond to $\varepsilon = 0$ (unemployed).

We calibrate the borrowing constraint and the idiosyncratic time preference process to match key features of the overall wealth distribution in the U.S. The borrowing constraint is set to $\underline{a} = -4.15$ to match the fraction of U.S. households with negative wealth holdings, 11%.[16]

$\tilde{\beta}$ takes on values from a symmetric grid, ($\tilde{\beta}_l = 0.9873, \tilde{\beta}_m = 0.9919, \tilde{\beta}_h = 0.9965$). In the invariant distribution, 96.5% of the population is in the middle state, and 1.75% is distributed across either of the extreme points. The expected duration of the extreme discount factors is set at 50 years, to capture a dynastic element in the evolution of time preferences (Krusell and Smith, 1998). In addition, transitions occur only across adjacent values, where the transition probability from either extreme value to the middle grid is 1/200, and the transition probability from the middle grid to either extreme value is 7/77200. This Markov chain for $\tilde{\beta}$ allows our model to generate a long-run U.S. capital-output ratio of 2.5, and a Gini coefficient for the U.S. wealth distribution of 0.79. It also allows our model to match the wealth share of the top 1% (Krusell and Smith, 1998). An accurate calibration of this moment is important because, as we will show, the welfare effects of fiscal volatility for top wealth holders, characterized by high levels of buffer-stock savings and high capital income, can be quantitatively rather different from those for other households.

[15] The numbers are rounded to the second decimal point.

[16] We check that the total resources available to a household, taking into account unemployment insurance benefits and the borrowing limit, are never negative under this calibration. Under our baseline calibration, the average quarterly output turns out to be close to one (1.11), so the wealth levels can roughly be considered ratios to quarterly gross income per household. Therefore, households are allowed to borrow up to about the average annual gross income per household (1.1 × 4).

3.2. Fiscal parameters

3.2.1. Fiscal volatility and tax revenue rule

To estimate the parameters related to fiscal volatility and the aggregate tax revenue rule, we use U.S. quarterly data from the first quarter of 1960 to the last quarter of 2007. We restrict the data window up to 2007IV because, arguably, fiscal policy was special during and after the Great Recession and for calibration purposes we want to focus on "normal" times. Our model is stationary—that is, our paper is not about long-run trend or medium-run regime changes in the U.S. fiscal system[17]—so we use detrended data to empirically discipline the fiscal parameters. We provide the details of our fiscal parameter estimation in Appendix A. Here we briefly outline the general procedure.

For the government purchases process (equation (2.5)), we use the Rouwenhorst method (Rouwenhorst, 1995) to construct a three-state first-order Markov chain approximation to the AR(1) process of the linearly detrended $\log(G)$ series.[18] The middle grid point of the G-process, G_m, is calibrated using the average G/Y-ratio in the data; see Appendix A.1 for the details.

To determine the parameters of our tax revenue rule (equation (2.6)), we first estimate the federal revenue rule as in Bohn (1998) and Davig and Leeper (2011), and the state and local rule without debt. We then take the weighted average of the federal rule and the state and local rule to get the general government tax revenue function, the empirical counterpart of our model. We describe the details of this procedure in Appendix A.2.

TR in equation (2.6) is aggregate unemployment insurance payments. We set the unemployment insurance replacement rate, ω, to 10% of the current market wage income, in line with the data. From Stone and Chen (2014) we know that the overall replacement rate from unemployment insurance is about 46% of a worker's wage, and its average pre-2008 benefits duration is 15 weeks. This translates to about 53% of a worker's quarterly wage. In our case, since we spread the unemployment benefits through the agent's whole unemployment period and the average duration of unemployment in the model is about 2 quarters, this translates to about 27% of the quarterly wage level. Moreover, from Auray et al. (2019) we know that about 60% out of all the unemployed workers were eligible for unemployment benefits from 1989 to 2012, and that about 75% of those eligible for benefits actually collected them. Thus, we set our unemployment insurance payment to be 10% of the market wage.[19]

3.2.2. Tax instruments

Recall that to satisfy the tax revenue rule (equation (2.6)) we need to treat one of the tax parameters in the income tax function as an endogenous equilibrium object:

$$\tau_1 \left[y - \left(y^{-\tau_2} + \tau_3 \right)^{-\frac{1}{\tau_2}} \right] + \tau_0 y. \tag{3.1}$$

Which tax parameter we choose to be an endogenous variable then influences how the distribution of the tax burden across income changes over the business cycles. We thus run the model with each of τ_0, τ_1, τ_2, and τ_3 as the endogenous variable one by one, and examine the cyclicality of the tax system in each case. We then select the case where the cyclicalities of both the tax parameters and the (average) residual income elasticity (RIE, defined in equation (3.2)) of the federal income tax part in equation (3.1), a classical (inverse) summary measure of tax progressivity in the public finance literature (see Musgrave and Thin, 1948), best match the data. RIE is the elasticity of after-tax income to pre-tax income. It is a decreasing function of tax progressivity, because the more progressive the tax system is, the smaller the proportional increase in the after-tax income, compared to that in the before-tax income.[20]

$$RIE = \int_0^1 \frac{\partial \left(y_i - \tau^y \left(y_i \right) \right) / \partial y_i}{\left(y_i - \tau^y \left(y_i \right) \right) / y_i} di = \int_0^1 \frac{1 - \tau_1 + \tau_1 \left(1 + \tau_3 y_i^{\tau_2} \right)^{-\frac{1}{\tau_2} - 1}}{1 - \tau_1 + \tau_1 \left(1 + \tau_3 y_i^{\tau_2} \right)^{-\frac{1}{\tau_2}}} di. \tag{3.2}$$

We focus on matching the cyclicality of tax progressivity because, as we show later, this turns out to be the main determinant of the distributional effects of fiscal volatility. Gouveia and Strauss (1999) provide U.S. time series data for the federal income tax system not only on RIE, but also on their estimates of τ_1, τ_2 and τ_3. According to this data, the RIE correlates

[17] Studying long-run trend or medium-run regime changes in the U.S. fiscal system is of independent interest, see Richter and Throckmorton (2015), though outside the scope of this paper.

[18] Kopecky and Suen (2010) show that the Rouwenhorst method has an exact fit in terms of five important statistical properties: unconditional mean, unconditional variance, correlation, conditional mean and conditional variance. The last two properties are important for our elimination of fiscal volatility, where both the conditional mean and variance matter for the transition-path equilibrium.

[19] Our calibration also matches the aggregate data on unemployment insurance well: 0.0049 for the average unemployment insurance to output ratio (0.0041 in the data), and 0.0021 for its standard deviation, after removing a linear trend (0.0019 in the data). In both the model and the data, the unemployment-insurance-to-output ratio is countercyclical. Also note that in Krusell and Smith (1998), the unemployment insurance is treated as a fixed amount, ψ, and calibrated to be about 10% of the long-run quarterly wage.

[20] In equation (3.2), τ^y refers, with a slight abuse of notation, only to the federal income tax part in equation (3.1), because our data on RIE are from the federal tax system.

Table 2
Moments for tax instrument choice.

A: Data (1966 - 1989)						
	ρ(RIE, Y)	ρ(RIE, T-Tr)	$\rho(\tau_0$,T-Tr)	$\rho(\tau_1$,T-Tr)	$\rho(\tau_2$,T-Tr)	$\rho(\tau_3$,T-Tr)
	-0.3353	-0.3652	-0.1865	0.3235	-0.2184	-0.0344
B: Model simulation						
	ρ(RIE, Y)	ρ(RIE, T-Tr)	$\rho(\tau_0$,T-Tr)	$\rho(\tau_1$,T-Tr)	$\rho(\tau_2$,T-Tr)	$\rho(\tau_3$,T-Tr)
τ_0-adjustment	-0.2978	-0.3108	0.3986	-	-	-
	(0.2689)	(0.2675)	(0.3056)	-	-	-
τ_1-adjustment	-0.2900	-0.3803	-	0.2999	-	-
	(0.3187)	(0.3077)	-	(0.2788)	-	-
τ_2-adjustment	0.2887	0.3744	-	-	-0.3333	-
	(0.2105)	(0.1951)	-	-	(0.2320)	-
τ_3-adjustment	0.1615	0.2478	-	-	-	0.3261
	(0.2886)	(0.2880)	-	-	-	(0.2199)

Notes: In Panel A, Y and $T - Tr$ are HP-filtered (with a smoothing parameter of 6.25) real log series of output and tax revenue net of transfers, respectively. τ_0, τ_1, τ_2 and τ_3 are linearly-detrended tax parameters, where τ_0 is estimated by the authors (see Appendix A.3) and τ_1, τ_2 and τ_3 are from Gouveia and Strauss (1999). RIE is the quadratic-detrended residual income elasticity from Gouveia and Strauss (1999). In Panel B, all variables are defined and filtered the same way as those in Panel A. The reported numbers are the average values from 2,000 independent simulations of the same length as the data (24 years), where quarterly data are converted to annual data to match the data frequency in Panel A. We show the standard deviations across these simulations in parentheses.

negatively with output and tax revenue net of transfers; see the first two columns of Panel A, Table 2. And the first two columns of Panel B, Table 2, show that our model can obtain the right cyclicality of RIE only when we use either τ_0 or τ_1 as the tax instrument to cyclically adjust the government budget. The intuition for this result is: to have a negative correlation between the RIE and tax revenue (a positive correlation between tax progressivity and tax revenue), the tax burden on income-rich individuals from the federal income tax must increase with tax revenue. This means, given the specification of our federal income tax function, that τ_1 has to adjust instead of τ_2 or τ_3, because adjustments in τ_1 lead to differential changes in individual marginal tax rates proportional to the existing progressive rates. In contrast, adjustments in τ_2 or τ_3 affect the poor- and medium-income households more than the high income group, since they leave the highest marginal tax rate unaffected.[21]

To make the further choice between τ_0- and τ_1-adjustments, we examine how τ_0 and τ_1 themselves are correlated with tax revenue net of transfers.[22] The third and fourth columns of Table 2 report these two correlations in the data (Panel A), negative for τ_0 and positive for τ_1, whereas the model implies positive correlations for both cases (Panel B), and hence τ_1 appears to be the driver for the empirical cyclicality of RIE. Therefore, we choose τ_1 as the endogenous equilibrium object in the baseline model. We thus show that time series data on the progressivity of the U.S. tax system are informative of which tax instruments are likely to be used for cyclical government budget adjustment. It is top marginal tax rates, which is also consistent with evidence documented in Mertens and Montiel Olea (2018). They report the time series of the average marginal tax rates of various income groups in the U.S. We calculate the difference in the average marginal tax rates between the top 1% and bottom 90% income groups as a measure of the progressivity of the income tax schedule. We find that this measure is positively correlated with output and tax revenue net of transfers, consistent with Table 2 (columns 1 and 2, Panel A; recall that the RIE is inversely related to tax progressivity).[23]

We then calibrate the remaining tax parameters in the progressive part of the income tax function based on the values estimated by Gouveia and Strauss (1994) for U.S. data from 1989 (see Castañeda et al., 2003 and Conesa and Krüger, 2006), the last year in their sample and close to the midpoint of the sample period in this paper. Note that equation (3.1) is linearly homogeneous in y, if τ_3 is readjusted appropriately. Therefore, we use their values for τ_2 (0.768), and calibrate τ_3 such that the average value of τ_1 from the model matches the estimated value from Gouveia and Strauss (1994).[24]

[21] Analytically, holding the income distribution constant, we can show that $\partial RIE/\partial\tau_1$ is negative. By construction, $\partial RIE/\partial\tau_0$ is zero holding the income distribution constant, so the negative correlation between RIE and the tax revenue in the τ_0-adjustment specification is solely driven by changes in the income distribution.

[22] The time series of τ_1, τ_2, and τ_3 are reported in Gouveia and Strauss (1999), while that of τ_0 is obtained from our own estimation (see below and Appendix A.3). For completeness we also report the correlations for τ_2 and τ_3, although these two models do not pass our first criterion for model selection.

[23] The correlation coefficients are 0.4535 and 0.3760, respectively. We use the same detrending methods (HP-filtering for the real log series of Y and $T - Tr$ and quadratic detrending for the progressivity measure, i.e., the difference in the average marginal tax rates) and the same sample period (1966-1989) as in Table 2. We find similar results when we create the alternative progressivity measure using other income groups (1%-99%, 5%-90%, and 10%-90%).

[24] Note that the estimation in Gouveia and Strauss (1994) is carried out on annual federal income tax data, whereas our model frequency is quarterly. Given the nonlinear nature of the tax function (equation (3.1)), this may raise a time aggregation issue. We therefore checked the implied tax function from simulated annual income and annual tax payment data from our model (aggregated from simulated quarterly observations). The results from this estimation are very close to those from the annual data.

Table 3
Wealth distribution.

	% of wealth held by top					Fraction with wealth< 0	Gini coefficient
	1%	5%	10%	20%	30%		
Model	31%	59%	71%	80%	86%	10%	0.78
K&S	24%	54%	72%	87%	91%	11%	0.81
Data	30%	51%	64%	79%	88%	11%	0.79

Notes: The wealth distribution in the data is taken from Krusell and Smith (1998). Household wealth in our model is the sum of physical capital and government bonds.

Table 4
Business cycle moments.

A: Data (1960 I - 2007 IV)	Y	T-Tr	G	(T-Tr/Y)	(G/Y)	(B/Y)
Standard deviation	0.0149	0.0543	0.0134	0.0123	0.0083	0.0772
Autocorrelation	0.8616	0.8134	0.7823	0.9045	0.9573	0.9945
Corr(Y,X)	1	0.7242	0.0992	0.4791	-0.3826	-0.0472
Corr(G,X)	0.0992	0.0352	1	0.0345	0.4806	-0.0281

B: Model simulation	Y	T-Tr	G	(T-Tr/Y)	(G/Y)	(B/Y)
Standard deviation	0.0235	0.0414	0.0123	0.0063	0.0086	0.0403
	(0.0018)	(0.0043)	(0.0025)	(0.0009)	(0.0012)	(0.0151)
Autocorrelation	0.5840	0.5870	0.6978	0.8183	0.8252	0.9732
	(0.0561)	(0.0558)	(0.0582)	(0.0575)	(0.0546)	(0.0341)
Corr(Y,X)	1	0.9892	-0.0012	0.6941	-0.6436	-0.1822
	(0)	(0.0043)	(0.1294)	(0.1053)	(0.0939)	(0.1685)
Corr(G,X)	-0.0012	0.1316	1	0.2499	0.3805	-0.0089
	(0.1294)	(0.1296)	(0)	(0.1121)	(0.1079)	(0.0577)

Notes: In Panel A, $Y, T - Tr$ and G are HP-filtered (with a smoothing parameter of 1600) real log series of output, tax revenue net of transfers and government purchases, respectively. $(T - Tr)/Y$, G/Y and B/Y are linearly detrended output ratios of tax revenue net of transfers, government purchases and federal government debt, respectively. The data sources are documented in Appendix A.2. In Panel B, all variables are defined and filtered the same way as those in Panel A. The reported numbers are the average values from 1,000 independent simulations of the same length as the data (192 quarters). We show the standard deviations across these simulations in parentheses.

For the consumption tax rate and the linear part of the income tax function, we follow standard procedures and calculate the time series of the corresponding tax rates from the quarterly NIPA data (see, e.g., Fernández-Villaverde et al., 2015 and Mendoza et al., 1994). We then take the time-series average values to obtain the following tax rates: $\tau_c = 8.14\%$ and $\tau_0 = 5.25\%$; see Appendix A.3 for the details.

3.3. The wealth distribution and business cycle moments

In this section, we examine the wealth distribution and the business cycle moments, focusing on the fiscal variables, generated by our calibrated model. For our model to be a suitable laboratory for the experiment of eliminating fiscal volatility, and for producing reliable quantitative answers to our welfare and distributional questions, it should broadly match these aspects of the data.

Table 3 compares the long-run wealth distribution generated by our model with both the data and the model results in Krusell and Smith (1998). From Table 3, we see that our wealth distribution is a good match for the U.S. wealth distribution, especially for those in the top 1 percent.[25]

Table 4 provides the results of a comparison between the key business cycle moments generated by the model and those from the data. This comparison includes output, tax revenue, and government purchases volatility and persistence. We calculate the same moments for the output ratios of tax revenue, government purchases and federal government debt. Finally, we examine the co-movements of these series with output and government purchases.

From Table 4, we see that our baseline model is successful in matching most of the business cycle moments, with the exception of output volatility (which is about 70% larger in the model). We checked that, even without fiscal volatility, as in Krusell and Smith (1998), the model produces higher output fluctuations than found in the data, while the introduction of fiscal volatility does not contribute substantially to the volatility of output. To check whether our welfare results are affected

[25] While Krusell and Smith (1998) exogenously fix the share of households in each extreme $\tilde{\beta}$ state at 10%, we use this share as a parameter to be calibrated to target the top 1% wealth share. This calibration makes that share 1.75%.

by this feature of the model, we conduct a robustness check where we recalibrate the aggregate productivity process so that the model matches the output volatility in the data. The results remain unchanged.

4. Computation

4.1. Stochastic steady state

To compute the model's equilibrium with two aggregate shocks, we use the approximate aggregation technique proposed by Krusell and Smith (1998).[26] This technique assumes that households act as if only a limited set of moments of the wealth distribution matters for predicting the future of the economy, and that the aggregate result of their actions is consistent with their perceptions of how the economy evolves. However, in contrast to Krusell and Smith (1998), we find that higher moments of the wealth distribution are necessary in our model with progressive taxation. That is, the accurate description of our economy's evolution requires a combination of average physical capital and the Gini coefficient of the wealth distribution.

Furthermore, the optimization problem in our model requires households to know the endogenous tax parameter, τ_1. We therefore approximate the function Θ, as defined in equation (2.14), with a parameterized function of the same moments that represent the wealth distribution.[27]

We can now state the following functional forms for H_Γ and Θ:

$$\begin{aligned} log(K') =& a_0(z,G) + a_1(z,G)log(K) + a_2(z,G)B + a_3(z,G)(log(K))^2 + a_4(z,G)B^2 \\ & + a_5(z,G)B^3 + a_6(z,G)log(K)B + a_7(z,G)Gini(a), \qquad (4.1) \\ Gini(a') =& \tilde{a}_0(z,G) + \tilde{a}_1(z,G)log(K) + \tilde{a}_2(z,G)B + \tilde{a}_3(z,G)(log(K))^2 + \tilde{a}_4(z,G)B^2 \\ & + \tilde{a}_5(z,G)B^3 + \tilde{a}_6(z,G)log(K)B + \tilde{a}_7(z,G)Gini(a), \qquad (4.2) \\ \tau_1 =& b_0(z,G) + b_1(z,G)log(K) + b_2(z,G)B + b_3(z,G)(log(K))^2 + b_4(z,G)B^2 \\ & + b_5(z,G)B^3 + b_6(z,G)log(K)B + b_7(z,G)Gini(a), \qquad (4.3) \end{aligned}$$

where K denotes the average physical capital, and $Gini(a)$ denotes the Gini coefficient of the wealth distribution.[28] We compute the equilibrium using a fixed-point iteration procedure from the parameters in equations (4.1)-(4.3) onto themselves; see Online Appendix B.1 for the details of the computational algorithm and Online Appendix B.2 for the estimated equilibrium laws of motions.

A check of the one-step-ahead forecast accuracy yields R^2s above 0.999993 for H_Γ (equations (4.1) and (4.2)), and above 0.99998 for Θ (equation (4.3)). However, as den Haan (2010) points out, high $R2$-statistics are not necessarily indicative of multi-step-ahead forecast accuracy. Hence, we also examine the 10-year ahead forecast errors of our model. This check shows that our forecast errors are small and unbiased; see Online Appendix B.2 for the details.

4.2. Transition-path equilibrium

To study the welfare effects of eliminating fiscal volatility, we start with the ergodic distribution of the two-shock equilibrium. From time $t=1$, we let G_t follow its deterministic conditional mean along the transition path until it converges to G_m. While we do not take a stance on how this stabilization is brought about (Lucas, 1987 and Krusell et al., 2009), we do note that, in contrast to stabilizing aggregate productivity shocks, the G_t-process is arguably under more direct government control.

As stated, during the transition periods G_t follows a time-dependent deterministic conditional-mean process until it converges to G_m, i.e.,

$$G_t = [\mathbb{1}(G_1 = G_l), \mathbb{1}(G_1 = G_m), \mathbb{1}(G_1 = G_h)]\, \Pi_{GG'}^{t-1} [G_l, G_m, G_h]^T \qquad (4.4)$$

where $\Pi_{GG'}$ is the transition probability matrix of the G-process in the two-shock economy discussed in Appendix A. Note that, depending on G_1, the G_t-paths will have different dynamics. For example, if $G_1 = G_m$, G_t will stay at G_m for all $t \geq 1$, and the economy will immediately transition to its long-run G level. However, if the economy starts the transition away from G_m, G_t converges to G_m over time through the deterministic process described in (4.4). In this case, the counterfactual economy will go through transitional dynamics to eventually reach the productivity-shock-only stochastic steady state.

Recall the assumption that the government purchases process is independent from other stochastic processes, which implies that none of the other exogenous stochastic processes changes during or after the elimination of the fiscal shocks.

[26] The solution method for the stochastic steady state of the model with only aggregate productivity shocks is the same, except that $G_t = G_m, \forall t$.

[27] This is in the same spirit as the bond price treatment in Krusell and Smith (1997).

[28] These specific functional forms perform best among a large set of (relatively parsimonious) functional forms tested.

Therefore, our counterfactual economy features aggregate productivity shocks both during and after the transition. This creates a new technical challenge in addition to those present in previous transition path analyses of heterogeneous-agent economies (e.g., Huggett, 1997). While these studies model a deterministic aggregate economy along the transition path, our stochastic setting with aggregate uncertainty produces an exponentially higher number of possible aggregate paths as the transition period lengthens. This feature precludes computation of the equilibrium for all possible realizations of aggregate shocks.

To address this challenge, we extend the approximate aggregation technique to the transition-path setting: that is, we postulate that time-dependent prediction functions govern the evolution of the economy on the transition path, through the following set of laws of motions:

$$\Gamma_{t+1} = H^{trans}_{\Gamma,t}\big(\Gamma_t, B_t, z_t\big), \tag{4.5}$$

$$\tau_{1,t} = \Theta^{trans}_{t}\big(\Gamma_t, B_t, z_t\big), \tag{4.6}$$

where t denotes an arbitrary period along the transition path. At the end of the transition path, the laws of motions converge to those in our one-shock equilibrium. Consequently, solving for the transition-path equilibrium is equivalent to finding the appropriate approximations for (4.5) and (4.6), such that the realized evolution of the economy is consistent with the postulated evolution; see Online Appendix B.3 for the details of the algorithm. We find that the same functional forms we use for the stochastic steady state economy yield accurate predictions also for the transition-path equilibrium. That is, for every period on the transition path, we achieve a similar forecast accuracy as in the stochastic steady state two-shock economy; see Online Appendix B.4 for the details.

5. Results

Following Lucas (1987), we measure the welfare costs of fiscal volatility as the proportional change in a household's life-time consumption (Consumption Equivalent Variation or λ), such that:

$$E_1[\sum_{t=1}^{\infty} \beta_t u((1+\lambda)c_t, G_t)] = E_1[\sum_{t=1}^{\infty} \beta_t u(\tilde{c}_t, \tilde{G}_t)], \tag{5.1}$$

where c_t is consumption in the baseline economy with G_t-fluctuations, while $\tilde{c}_t$ is consumption in the counterfactual economy with a deterministic $\tilde{G}_t$-process.

5.1. Baseline results

To obtain our baseline results, we first calculate welfare gains conditional on wealth, employment status and time preference for every sample economy in the transition-path computation,[29] using the value functions from our two-shock and transition-path equilibria.[30] We then average these across the sample economies, including all possible values of G_1, the government purchases level when fiscal volatility is eliminated. The results, presented in Table 5, can thus be interpreted as the *ex-ante* expected welfare gains from eliminating fiscal volatility.

The results in Table 5 show that the aggregate welfare gain, i.e., the average welfare change across the whole population, is about 0.03%, comparable in size to the results in Lucas (1987). We further find that the welfare gains increase with wealth and patience while employment status does not affect the welfare changes. In the next sub-section, we examine the mechanisms affecting the welfare gains along the wealth dimension.

5.2. The mechanisms

Our analyses show that the increasing-with-wealth welfare gain pattern is the result of three interacting channels: a direct utility channel, an income risk channel, and an average factor price channel. The direct utility channel isolates the utility gains resulting from household risk aversion with respect to government purchases fluctuations. In the income risk channel, two types of fiscal risk arising from tax rate fluctuations coexist: an after-tax-wage risk and an after-tax-rate-of-return risk. These risks have different distributional effects through the precautionary saving behavior of households and the risk exposure of households' resources. Finally, the average factor price channel reflects changes in average factor prices along the transition path.

In the following sub-sections, we discuss each channel in turn. We can exactly and quantitatively separate the direct utility channel from the other two. Although an exact quantitative separation of the income risk channel from the average factor price channel is not feasible as they are intertwined in the economy, we can illustrate the distinct ways of how they work.

---

[29] To start the transition-path simulation, we draw a large set (16,000) of independent joint distributions over $(a, \varepsilon, \tilde{\beta})$ from the simulation of the two-shock equilibrium; see Online Appendix B.3 for the details.

[30] The right side of (5.1) is the value function from the transition-path equilibrium. Given the log-log utility assumption in the baseline calibration, the left side of (5.1) can be expressed using the value function from the two-shock equilibrium and λ; see Online Appendix B.5 for the details of the derivation.

Table 5
Expected welfare gains λ (%).

	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.0293	0.0289	0.0295	0.0296	0.0293	0.0290	0.0287	0.0313	0.0371
$\varepsilon = 1$	0.0293	0.0288	0.0294	0.0296	0.0293	0.0290	0.0287	0.0313	0.0371
$\varepsilon = 0$	0.0294	0.0291	0.0297	0.0297	0.0294	0.0291	0.0287	0.0312	0.0371
$\tilde{\beta} = \tilde{\beta}_l$	0.0277	0.0278	0.0277	0.0276	0.0275	0.0274	0.0268	0.0272	0.0314
$\tilde{\beta} = \tilde{\beta}_m$	0.0292	0.0300	0.0299	0.0296	0.0293	0.0290	0.0285	0.0302	0.0356
$\tilde{\beta} = \tilde{\beta}_h$	0.0360	0.0329	0.0327	0.0326	0.0326	0.0326	0.0336	0.0377	0.0440

Notes: The wealth groups are presented in ascending order from left to right. The welfare number for a particular combination of ε (or $\tilde{\beta}$) and a wealth group is calculated as follows: we first draw a large set (16,000) of independent joint distributions over $(a, \varepsilon, \tilde{\beta})$ from the simulation of the two-shock equilibrium. These distributions are used to start the computation of the transition-path equilibria. For each sample economy, we then find all the individuals that fall into a particular wealth×employment status or wealth×preference category, and calculate their welfare gain according to equation (5.1). We then take the average over the individuals in a particular category to find the welfare numbers for a given sample economy. To arrive at the numbers in this table, we finally take the average across all the 16,000 samples.

Table 6
Expected welfare gains from private consumption changes, λ_c (%).

	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.0082	0.0081	0.0085	0.0085	0.0082	0.0079	0.0076	0.0101	0.0159
$\varepsilon = 1$	0.0082	0.0081	0.0085	0.0085	0.0082	0.0079	0.0076	0.0101	0.0159
$\varepsilon = 0$	0.0083	0.0083	0.0087	0.0086	0.0083	0.0080	0.0076	0.0101	0.0159
$\tilde{\beta} = \tilde{\beta}_l$	0.0072	0.0073	0.0073	0.0072	0.0071	0.0069	0.0064	0.0068	0.0110
$\tilde{\beta} = \tilde{\beta}_m$	0.0082	0.0089	0.0088	0.0085	0.0082	0.0079	0.0074	0.0091	0.0145
$\tilde{\beta} = \tilde{\beta}_h$	0.0142	0.0111	0.0109	0.0108	0.0108	0.0108	0.0118	0.0159	0.0222

Notes: The welfare numbers in this table are calculated as those in Table 5, using (5.3) instead of (5.1).

5.2.1. The direct utility channel

Since a household's utility over G is strictly concave, eliminating fluctuations in G leads to a direct increase in expected lifetime utility. To isolate this direct utility gain, we first compute a λ_c such that:

$$E_1[\sum_{t=1}^{\infty} \beta_t u((1+\lambda_c)c_t, G_t)] = E_1[\sum_{t=1}^{\infty} \beta_t u(\tilde{c}_t, G_t)], \tag{5.2}$$

where c_t, $\tilde{c}_t$, and G_t are defined in the same way as before. Note that, λ_c is by definition insulated from any utility change caused by direct changes in the G-process, since the stochastic G-process now enters both sides of equation (5.2). Therefore, λ_c represents welfare changes that result solely from changes in private consumption profiles. The difference between λ and λ_c thus characterizes the direct utility channel.

Furthermore, with a separable flow utility function, λ_c can be computed using the following simpler equation:

$$E_1[\sum_{t=1}^{\infty} \beta_t log((1+\lambda_c)c_t)] = E_1[\sum_{t=1}^{\infty} \beta_t log(\tilde{c}_t)]. \tag{5.3}$$

The results, presented in Table 6, show positive, albeit smaller welfare changes when fiscal volatility is eliminated (after the gain from the direct utility channel is subtracted). Thus, we conclude that the direct utility channel is quantitatively important for the overall level of welfare changes, but, distributionally, the other two channels are the ones that matter.

In addition to the direct utility channel, fluctuations in government purchases can contribute to the welfare of households through affecting factor prices (pre-tax labor and capital income) and individual income tax rates, both of which determine households' after-tax income. The government purchases process can directly change individual income tax rates due to the aggregate tax revenue rule. Indirectly, the government purchases process influences the amount of physical capital (hence factor prices) in the economy, through changes in the split of aggregate wealth between capital and government bonds (due to the effect of government spending on government debt), and also through changes in the saving behavior of households facing changes in the tax rate process.

The distributional effects from these channels show the importance of capturing realistic household heterogeneity and its interaction with a realistically calibrated progressive tax system. Even though all the households are facing the same tax schedule, depending on where they belong in the income distribution and also depending on the cyclicality of the progressivity in the income tax system, their household-specific tax rate risks are, in principle, differentially impacted by

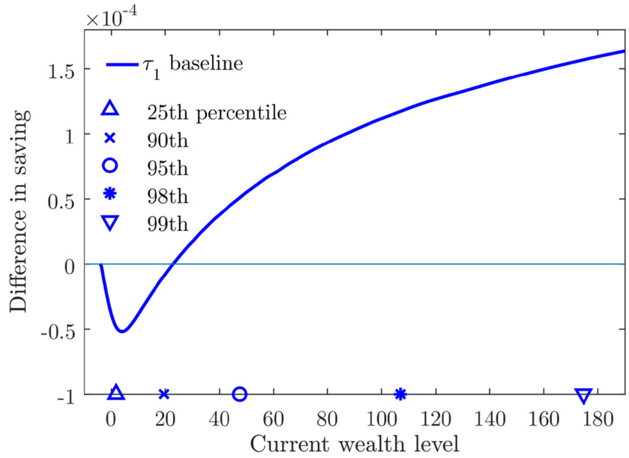

Fig. 1. Policy function comparison - saving.
Notes: This figure shows the difference between the first-period policy function for saving from the transition equilibrium (with $G_1 = G_m$) and that from the two-shock equilibrium (with $G_1 = G_m$), evaluated at $z = z_g$, $\varepsilon = 1$, $\tilde{\beta} = \tilde{\beta}_m$, and the long-run averages of $(K, B, Gini)$ conditional on $G_1 = G_m$ and $z = z_g$. Note that, under our baseline calibration, the average quarterly output turns out to be close to one (1.11). Hence, the wealth levels can roughly be interpreted as ratios to quarterly gross income per household.

the aggregate productivity and fiscal risk. Heterogeneous exposures to tax rate risks may then in turn also shape the general equilibrium effect. Whether this actually matters for average welfare is ex-ante an open question. We also computed a representative-agent version of the model and found that the λ_c welfare gains from eliminating government purchases fluctuations are 0.0075, that is, somewhat smaller than in the heterogeneous agent case but not substantially so.

In the following two subsections, we separately consider the volatility and the level effects on households' after-tax income from fluctuations in government purchases. We denote the volatility effect as the *income risk channel*, and the level effect as the *average factor price channel*.

5.2.2. The income risk channel

Fluctuations in government purchases lead to more volatile after-tax income through both tax rates and factor prices. The distributional welfare implications of eliminating this after-tax income risk are, however, not straightforward. This is because the two components of after-tax income risk, labor income risk and rate-of-return risk (or capital income risk), have opposite distributional effects.

On the one hand, the effect of eliminating after-tax labor income uncertainty depends on a household's (heterogeneous) degree of self-insurance against labor income risks. As in other Bewley-type incomplete market economies, our households engage in precautionary saving. Wealthier households can better insure themselves against after-tax labor income risk. As a result, wealth-poor households should benefit more from the elimination of this uncertainty. Hereafter, we refer to this as the *precautionary saving effect*.

On the other hand, the tax-rate uncertainty induced by the G-shocks also creates a rate-of-return risk on after-tax capital income. This rate-of-return risk makes households' intertemporal transfer of resources riskier. In our model with a realistic incomplete financial market, wealth-rich households' financial wealth, which is subject to the rate-of-return risk, accounts for a larger share of their expected life-time resources than is the case for the wealth-poor. Therefore, the wealth-rich households have more exposure to the rate-of-return risk, and they should benefit more from the elimination of fiscal volatility. Hereafter, we refer to this as the *rate-of-return risk effect*. In Online Appendix C, we employ a partial equilibrium model, to build up the intuition further and illustrate the distributional consequences of both the precautionary saving and rate-of-return risk effects.

The precautionary saving and rate-of-return risk effects, in turn, have different effects on saving behavior. The wealth-poor, whose saving is mainly driven by the precautionary saving motive, have less incentives to save with a reduction in their after-tax labor income uncertainty, and hence reduce their saving after the elimination of fiscal volatility. By contrast, the wealth-rich, for whom the rate-of-return risk is the more important factor in their saving decision, may increase their saving. Fig. 1 confirms this conjecture showing that agents reduce their saving in the first period of the transition-path equilibrium compared to the two-shock equilibrium until approximately the 90th wealth percentile, whereas above this threshold, the wealth-rich increase their saving after the elimination of fiscal volatility.[31]

[31] The policy function difference for saving is evaluated at $G_1 = G_m$, $z = z_g$, $\varepsilon = 1$, $\tilde{\beta} = \tilde{\beta}_m$, and the long-run averages of $(K, B, Gini)$ conditional on $G_1 = G_m$ and $z = z_g$. However, similar patterns hold for other combinations of state variables. The comparison also looks similar when the policy functions from other periods on the transition path are used.

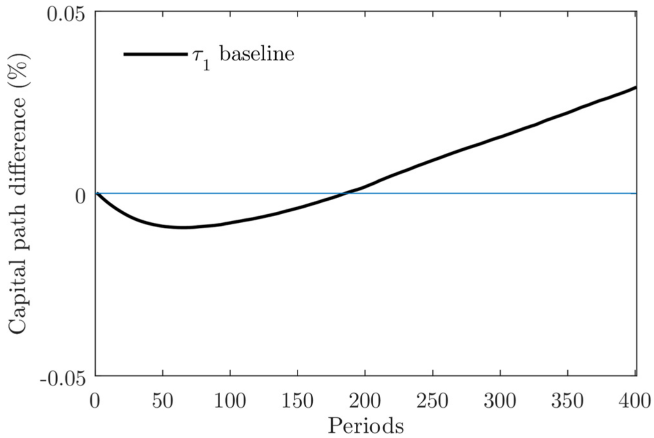

Fig. 2. Expected aggregate capital path comparison.
Notes: This figure shows the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium. We use the same 16,000 sample economies and the same sequences of z-shocks (for both the transition and the two-shock aggregate capital paths) as in the transition-path computation and then take the average. The G-shock sequences in the two-shock simulations are constructed in such a way that the cross-sectional joint distribution of (z, G)-shocks in each period is close to the invariant joint distribution.

In short, the income risk channel is an amalgam of the aforementioned two competing effects. As will be made clear in Section 6.1, through alternative counterfactual tax adjustment mechanisms, the distributional effects from this channel depend on how the tax rate volatility burden is distributed in a given tax system. Note that when τ_1 is adjusted to satisfy the aggregate tax revenue rule as in the baseline case, the wealth-rich face significant uncertainty in after-tax returns from their savings, because τ_1 determines the top marginal tax rates, which renders the rate-of-return risk effect strong in the baseline case. The rate-of-return risk effect, accompanied by an average factor price effect that initially also favors the wealth-rich, as we will show in the next subsection, results in the increasing-with-wealth welfare gain pattern in Table 6.

5.2.3. *The average factor price channel*

We next examine the average factor price channel. In our model with a representative neoclassical firm, factor price changes follow aggregate capital stock changes. If the aggregate capital stock drops after the elimination of fiscal volatility, then pre-tax capital returns, all else equal, will increase relative to wages. Because wealth-rich (wealth-poor) households have higher (lower) capital income shares, the wealth-rich (wealth-poor) households will benefit (lose) from this relative factor price change. As a result, changes in the aggregate capital stock will have distributional effects.

To examine the direction of the average factor price channel for our baseline scenario, we compute the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium. The results in Fig. 2 show that the expected aggregate capital path in the transition-path equilibrium falls at first slightly below, then returns to, and finally goes above of that in the two-shock equilibrium.

The differential saving adjustment in the cross section after the elimination of fiscal volatility examined in Fig. 1 explains the aggregate capital adjustment pattern in Fig. 2. In particular, in response to the elimination of fiscal volatility, the majority wealth-poor households decrease their saving while the wealth-rich households increase their saving. What is more, simulation results show that the pace of saving adjustments is faster for the poor. Therefore, aggregate capital drops at first and gradually increases.

To further illustrate the average factor price channel, we examine the welfare gains from eliminating fiscal volatility *conditional* on G_1, government purchases at the time the policy change is instituted. The results in Table 7 reveal similar overall increasing-with-wealth welfare gain patterns. However, the slope of the welfare gains is steeper (flatter) when $G_1 = G_h$ ($G_1 = G_l$), compared to that from the case with $G_1 = G_m$. We trace the causes of those differences to the average factor price channel.[32] Fig. 3 plots the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium, conditional on G_1. The differences in capital path adjustment are consistent with the welfare patterns across different G_1 cases. For $G_1 = G_h$, the aggregate capital declines more after the elimination of fiscal volatility, benefiting the wealth-rich more (compared to the case with $G_1 = G_m$). By contrast, for $G_1 = G_l$, the reduction in the aggregate capital occurs for a much shorter period of time, which benefits the wealth-poor more (compared to the case with $G_1 = G_m$).

In sum, in our baseline setup, the elimination of fiscal volatility favors the wealthy distributionally, because the rate-of-return risk effect as part of the income risk channel, and, at least initially, the average factor price effect favor the wealth-rich.

[32] The reduction in the volatility of the after-tax income is similar across different G_1 values.

Table 7
Expected welfare gains from private consumption, λ_c (%), conditional on G_1.

	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
$G_1 = G_l$									
All	0.0135	0.0141	0.0144	0.0142	0.0138	0.0133	0.0124	0.0130	0.0179
$\varepsilon = 1$	0.0135	0.0141	0.0144	0.0142	0.0138	0.0133	0.0124	0.0130	0.0179
$\varepsilon = 0$	0.0137	0.0143	0.0146	0.0144	0.0139	0.0134	0.0124	0.0130	0.0178
$\tilde{\beta} = \tilde{\beta}_l$	0.0131	0.0134	0.0133	0.0131	0.0128	0.0125	0.0111	0.0096	0.0126
$\tilde{\beta} = \tilde{\beta}_m$	0.0134	0.0149	0.0147	0.0142	0.0138	0.0133	0.0122	0.0120	0.0163
$\tilde{\beta} = \tilde{\beta}_h$	0.0185	0.0176	0.0172	0.0169	0.0167	0.0166	0.0166	0.0194	0.0248
$G_1 = G_m$									
All	0.0085	0.0084	0.0088	0.0088	0.0085	0.0082	0.0078	0.0102	0.0160
$\varepsilon = 1$	0.0085	0.0084	0.0088	0.0088	0.0085	0.0082	0.0078	0.0102	0.0160
$\varepsilon = 0$	0.0086	0.0085	0.0089	0.0089	0.0086	0.0083	0.0078	0.0102	0.0160
$\tilde{\beta} = \tilde{\beta}_l$	0.0075	0.0076	0.0076	0.0075	0.0073	0.0072	0.0066	0.0070	0.0111
$\tilde{\beta} = \tilde{\beta}_m$	0.0084	0.0092	0.0091	0.0088	0.0085	0.0082	0.0076	0.0093	0.0146
$\tilde{\beta} = \tilde{\beta}_h$	0.0143	0.0112	0.0111	0.0110	0.0110	0.0110	0.0119	0.0160	0.0223
$G_1 = G_h$									
All	0.0025	0.0015	0.0020	0.0023	0.0022	0.0021	0.0023	0.0068	0.0137
$\varepsilon = 1$	0.0025	0.0015	0.0020	0.0023	0.0022	0.0021	0.0023	0.0068	0.0137
$\varepsilon = 0$	0.0025	0.0017	0.0022	0.0023	0.0022	0.0021	0.0023	0.0068	0.0137
$\tilde{\beta} = \tilde{\beta}_l$	0.0008	0.0007	0.0007	0.0008	0.0008	0.0009	0.0012	0.0036	0.0091
$\tilde{\beta} = \tilde{\beta}_m$	0.0024	0.0024	0.0024	0.0023	0.0022	0.0021	0.0021	0.0059	0.0124
$\tilde{\beta} = \tilde{\beta}_h$	0.0097	0.0042	0.0043	0.0044	0.0045	0.0047	0.0066	0.0123	0.0195

Notes: The welfare numbers in this table are calculated as in Table 6, but separately for $G_1 = G_l, G_m, G_h$, using 8,000 simulations for $G_1 = G_m$ and 4,000 simulations each for $G_1 = G_l, G_h$.

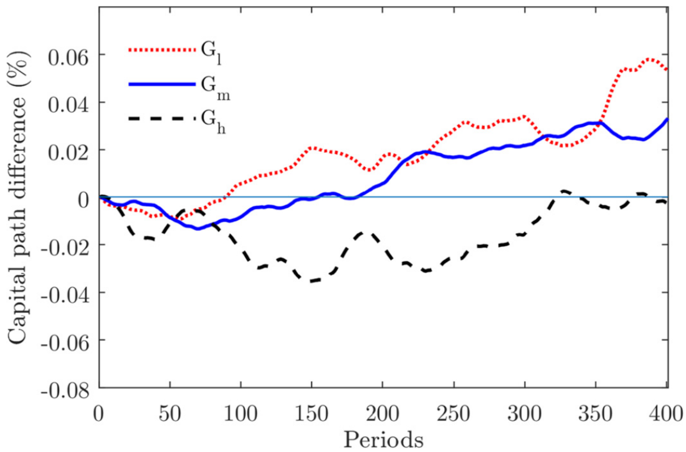

Fig. 3. Expected aggregate capital path comparison, conditional on G_1.
Notes: This figure shows the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium conditional on G_1. We use the same 16,000 sample economies and the same sequences of z-shocks (for both the transition and the two-shock aggregate capital paths) as in the transition-path computation and then average by G_1: 8,000 simulations for $G_1 = G_m$, and 4,000 simulations each for $G_1 = G_l, G_h$. Note that, due to our conditioning on G_1 and the subsequent smaller sample sizes, the expected aggregate capital paths in Fig. 3 are more volatile compared to those in Fig. 2.

6. Alternative specifications and additional experiments

In this section, we examine the welfare and distributional consequences of eliminating fiscal volatility under the following alternative model specifications: different adjustments to the progressive tax function, other fiscal regimes, different flow utility functions, and alternative TFP and labor income processes. In addition, we examine our results when we double fiscal volatility, as well as when the elimination of fiscal volatility is accompanied by a sudden change in the level of government purchases. In each case, we re-calibrate parameter values when necessary to preserve target moment-data consistency. We summarize the welfare change results in terms of λ_c in Table 8. Table D.1 in Online Appendix D.1 reports the corresponding λ-measures.

Table 8
Expected welfare gains from private consumption, λ_c (%), under different cases.

	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
Baseline	0.0082	0.0081	0.0085	0.0085	0.0082	0.0079	0.0076	0.0101	0.0159
Different Tax Function Adjustment									
Adjusting τ_0	0.0084	0.0083	0.0086	0.0087	0.0084	0.0082	0.0077	0.0091	0.0135
Adjusting τ_2	0.0088	0.0093	0.0095	0.0095	0.0093	0.0091	0.0080	0.0051	0.0065
Adjusting τ_3	0.0087	0.0095	0.0097	0.0096	0.0093	0.0090	0.0078	0.0028	0.0030
Other Fiscal Regimes									
Balanced Budget	0.0112	0.0109	0.0109	0.0108	0.0107	0.0107	0.0110	0.0159	0.0242
Linear Tax	0.0072	0.0067	0.0068	0.0070	0.0070	0.0070	0.0071	0.0103	0.0163
Lump-sum Tax	0.0073	0.0070	0.0070	0.0070	0.0069	0.0068	0.0071	0.0129	0.0204
Linear Capital Tax	0.0051	0.0057	0.0057	0.0057	0.0056	0.0051	0.0043	0.0028	0.0041
Benabou Tax Function	0.0071	0.0072	0.0076	0.0076	0.0073	0.0070	0.0064	0.0072	0.0102
Non-separable Utility Function									
Substitute	0.0008	0.0015	0.0028	0.0028	0.0003	0.0020	-0.0017	-0.0019	0.0014
Complement	0.0278	0.0258	0.0281	0.0284	0.0236	0.0305	0.0285	0.0322	0.0294
Alternative TFP and labor income processes									
Constant TFP	0.0089	0.0084	0.0089	0.0090	0.0087	0.0085	0.0084	0.0119	0.0191
Demand Externality	0.0090	0.0090	0.0095	0.0095	0.0092	0.0089	0.0084	0.0093	0.0127
Richer Income Process	0.0054	0.0058	0.0059	0.0059	0.0057	0.0052	0.0046	0.0050	0.0107
Superstar Income	0.0062	0.0069	0.0069	0.0069	0.0066	0.0060	0.0053	0.0049	0.0086
Higher UI rate	0.0081	0.0077	0.0088	0.0082	0.0080	0.0078	0.0075	0.0105	0.0169
Additional Experiments									
Double Volatility	-0.0070	-0.0070	-0.0074	-0.0073	-0.0070	-0.0067	-0.0063	-0.0089	-0.0148
Sudden Change	0.0108	0.0125	0.0123	0.0118	0.0111	0.0104	0.0092	0.0100	0.0158

6.1. Alternative specifications

Tax function adjustments Recall that, in our baseline specification, the top marginal rate of the progressive income tax (τ_1) is determined endogenously to satisfy the government's tax revenue response rule (equation (2.6)), while the linear tax rate (τ_0) and the tax function parameters τ_2 and τ_3 in the progressive tax function are fixed. Although we have argued that a fluctuating τ_1 can best represent the cyclicality of the progressivity of the U.S. tax system, here we consider the following three alternative adjustments in the tax function: adjusting τ_0, the linear part in the income tax function, and adjusting τ_2 and τ_3, the tax parameters that govern the progressivity of the tax system.

The average and distributional welfare results from the case with τ_0-adjustment turn out to be quite similar to those from the baseline (row 2 of Table 8). To understand this outcome, we first note that the distributional implications are pretty similar when either τ_0 and τ_1 is adjusted. To be more precise, in the first case, all households face the same tax rate changes in terms of absolute magnitude; while in relative terms, the tax rate change is decreasing with income levels. In the second case, all households face the same percentage change in their tax rates; while in terms of absolute magnitude, the tax rate change is increasing with income levels. Regardless of the specific differences, the wealth-rich households, whose marginal tax rates are close to the upper bound $\tau_0 + \tau_1$, face a similar rate-of-return risk in both cases. The analogues of Figs. 1 and 2 look essentially the same for the τ_0-adjustment case (see Fig. 4), which confirms our intuition.

The results in row 3 and 4 of Table 8 show that adjustment through τ_2 or τ_3 yields similar overall welfare gains as in the baseline case. However, unlike in the baseline scenario, the welfare gain for the top 5% of households is *smaller* than the average welfare gain in both cases. First note that the average factor price channel cannot explain the lower welfare gain for the wealthy. In each of the two cases, the expected aggregate capital stock decreases (more than in the baseline case) after the elimination of fiscal volatility (see Panel A in Fig. 4), which leads to lower wages and a higher pre-tax capital rate-of-return, favoring the wealth-rich households. The difference is rather due to the fact that in these cases the rate-of-return risk effect plays a limited role for the wealth-rich households. A fluctuating τ_2 or τ_3 does not generate substantial tax-rate uncertainty for the very rich households as their marginal tax rate is close to the upper bound, a (constant) $\tau_0 + \tau_1$. By contrast, it is the tax rates for the middle of the income distribution that respond the most to changes in τ_2 and τ_3. Therefore the wealth-rich households do not benefit much from the fiscal volatility reduction in these cases.

Panel (B) in Fig. 4 compares the changes in saving behavior for the τ_2-adjustment and the τ_3-adjustment cases with those of the baseline case. In both the τ_2-adjustment and the τ_3-adjustment cases, more households decrease their savings compared to the baseline case, which is consistent with the aggregate capital adjustment path comparisons in Panel (A). In particular, there is little (no) saving increase from the wealth-rich households in the τ_2-adjustment case (τ_3-adjustment case), which confirms that the rate-of-return effect is rather limited.

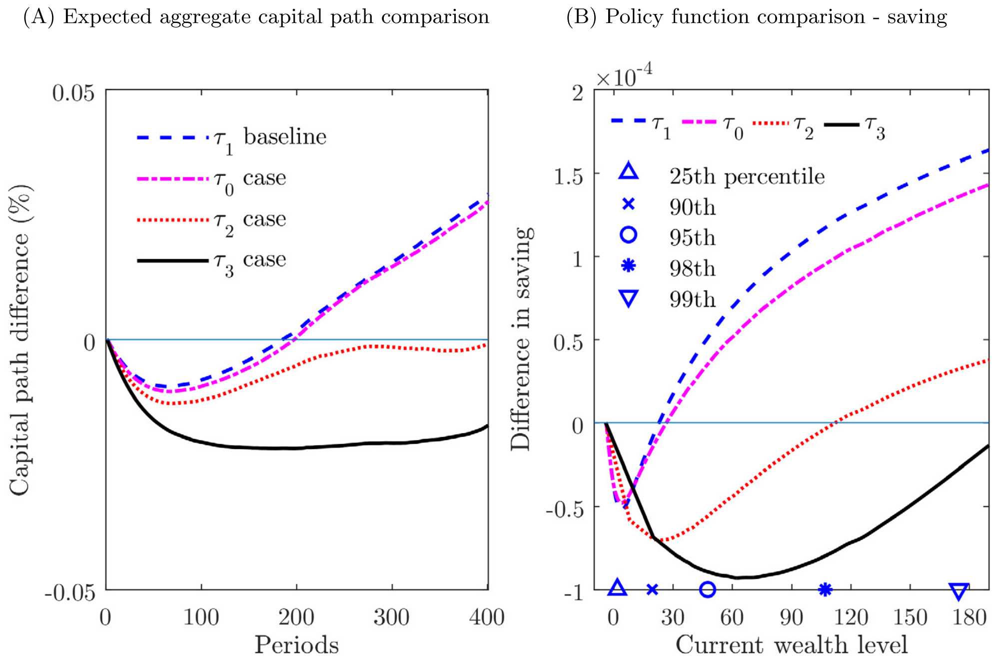

Fig. 4. Comparing various tax adjustment cases with the baseline.

Notes: Panel (A) compares the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium in the baseline scenario with those from the τ_0-, τ_2-, and τ_3-adjustment cases. The calculation of each case is done in exactly the same manner as that of Fig. 2. Panel (B) compares the difference between the first-period policy function for saving from the transition equilibrium and that from the two-shock equilibrium in the baseline scenario with those from the τ_0-, τ_2-, and τ_3-adjustment cases. The calculation of each case is done in exactly the same manner as that of Fig. 1. The wealth percentiles in Panel (B) are from the baseline model (they are, however, very similar across different tax adjustment specifications).

The different welfare gain patterns in the baseline scenario vis-à-vis the τ_2- and τ_3-adjustment scenarios have important policy implications: the distributional effects of eliminating fiscal volatility depend on which wealth group experiences the tax volatility burden that is caused by government purchases shocks. In the U.S. this group appears to be the wealth-rich.

The comparison between the baseline case and the τ_2- and τ_3-adjustment cases also allows us to gauge the quantitative importance of the income risk channel. The average factor price change in the baseline case is more favorable to the wealth-poor than those in the τ_2- and τ_3-adjustment cases. Nonetheless, the welfare gain is increasing across wealth in the baseline case while it is decreasing in the latter cases. Therefore, it appears that the income risk channel is the main driver behind the opposite distributional effects of the baseline case vis-à-vis the τ_2- and τ_3-adjustment cases.

Other fiscal regimes We next present distributional welfare results under five additional fiscal regimes. These analyses will shed additional light on the mechanisms behind the welfare effects of the elimination of fiscal volatility. In our first regime, a balanced budget scenario, we dispense with the tax revenue response rule (equation (2.6)) and assume that government spending is financed exclusively through tax revenue. In our next two regimes, a linear (lump-sum) tax scenario, we keep the tax revenue response rule but change the progressive tax system to a linear (lump-sum) tax, by setting $\tau_1 = 0$ ($\tau_0 = 0$ and $\tau_2 = -1$). The linear tax rate (the lump-sum tax amount) are then endogenously determined to satisfy the aggregate tax revenue response rule. In our fourth regime, we assume that capital income is taxed at a constant tax rate and only labor income is subject to the progressive tax function (equation (2.9)). In our final fiscal regime, we use the functional form from Benabou (2002) and Heathcote et al. (2017) in lieu of equation (2.9) to model the progressive income tax system.

We present the results of this set of analyses in rows 5 to 9 of Table 8. In the balanced budget regime, the welfare gain is larger than in the baseline case across the wealth distribution, even though the distributional effects are similar. This implies that allowing the government to borrow helps to smooth out tax revenue changes (and tax rate fluctuations) caused by government spending shocks, which in turn reduces the welfare cost of fiscal volatility.

Turning next to the linear tax case, we find that the welfare gains are very similar to those in the baseline τ_1-adjustment case, both in terms of magnitude and pattern, as shown in row 6 of Table 8. Indeed, the mechanisms work in a similar way: in a linear tax regime, cyclical adjustments in tax rates cause after-tax rate-of-return uncertainty, especially for the wealth-rich. Consequently, the elimination of this uncertainty benefits them. Indeed, when we compare the saving policy function between the two-shock and the transition-path equilibrium, we find an almost identical pattern as that in the baseline case (see Fig. 5).

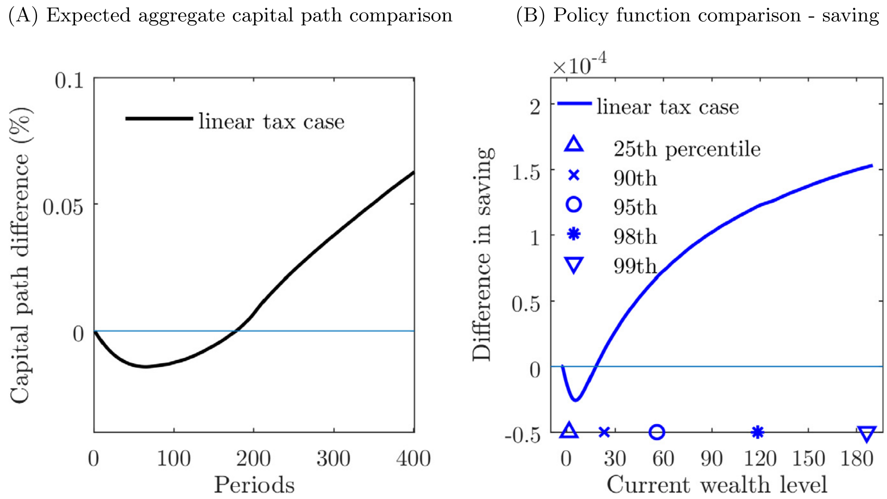

Fig. 5. Analysis of the linear tax case.

Notes: Panel (A) shows the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium in the linear tax case. The calculation is done in exactly the same manner as that of Fig. 2. Panel (B) shows the difference between the first-period policy function for saving from the transition equilibrium and that from the two-shock equilibrium in the linear tax case. The calculation of each case is done in exactly the same manner as that of Fig. 1. The wealth percentiles in Panel (B) are from the linear tax model, calculated in the same way as those in Fig. 1.

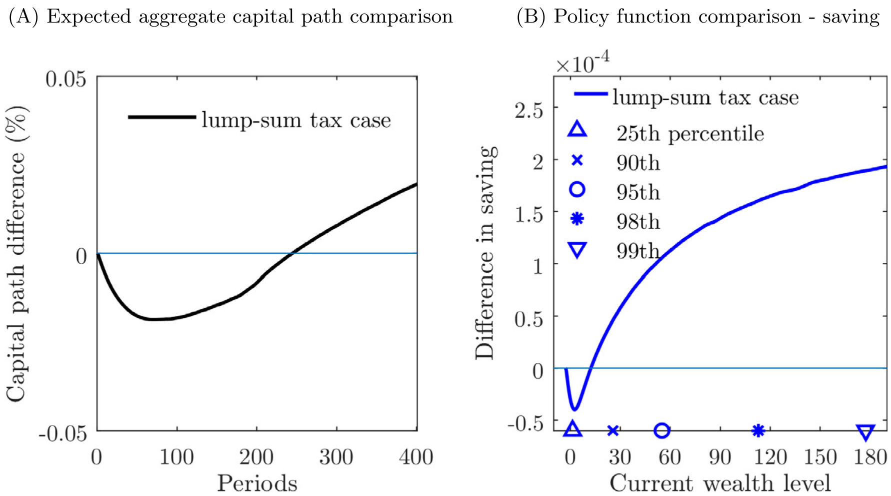

Fig. 6. Analysis of the lump-sum tax case.

Notes: Panel (A) shows the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium in the lump-sum tax case. The calculation is done in exactly the same manner as that of Fig. 2. Panel (B) shows the difference between the first-period policy function for saving from the transition equilibrium and that from the two-shock equilibrium in the lump-sum tax case. The calculation of each case is done in exactly the same manner as that of Fig. 1. The wealth percentiles in Panel (B) are from the lump sum tax model, calculated in the same way as those in Fig. 1.

With regard to the lump-sum tax case, our results (row 7 of Table 8) show that the welfare gains from eliminating fiscal volatility are again increasing in wealth, even more so than in the baseline case.[33] Indeed, Panel (A) of Fig. 6 shows that the expected aggregate capital stock in the lump-sum tax case decreases for almost 300 periods after the elimination of volatility and thus longer (and initially steeper with a shallower rebound) than in the baseline case, which implies that the average factor price channel favors the wealth-rich very strongly here. Overall, since there is no distortion when taxes are lump-sum, the average factor price channel is particularly powerful here (and without a direct impact on after-tax capital

[33] We note that the lump-sum tax is imposed only on employed households to avoid negative after-tax incomes. This treatment is slightly different from all the other cases, where tax payments are zero if and only if $y \leq 0$. With lump sum taxes, this could potentially introduce a discontinuity in the budget constraint of the unemployed at $y = 0$ (for the employed, it must hold that $y - T \geq 0$ at the borrowing limit, and thus, *a fortiori*, for all employed households).

returns and wages in this case we also have a less powerful income risk channel): the higher return makes capital more attractive as a saving vehicle after the elimination of volatility (see Panel B of Fig. 6), leading to benefits for the wealth-rich.

In our baseline model, we subject both labor and capital income to the same progressive tax function on total income. We view this as a good first pass because, at the household level in the U.S., the following capital income types are indeed taxed like labor income: interests, dividends, and short-term capital gains. The two exceptions are long-term capital gains and the corporate income tax. In our one-sector neoclassical model it is difficult to capture capital gains. In addition, they are a relatively small part of GDP, approximately 3%, and even long-term capital gains are subject to progressive tax schedules (see Tax Foundation, 2013 and York, 2019). Also the corporate income tax is a relatively small fraction of total tax revenue (10% in our sample). Nevertheless, in the next extension, we fix the tax rate on capital income to a constant flat-rate of 36.7%, as in Fernández-Villaverde et al. (2015). Given the brief discussion of capital income taxation in the U.S. above, this is clearly an extreme assumption but, together with the baseline specification, it maps out two polar opposites and thus the likely range of the effects. We then subject the labor income to the same progressive tax function (equation 2.9) as in the baseline.[34] We let τ_1 endogenously adjust to satisfy the aggregate tax revenue response rule. The welfare gains in this case (row 8 of Table 8) are smaller in magnitude than those in the baseline and are *decreasing* along the wealth dimension. This is consistent with our analysis of the mechanisms in Section 5.2.2: the overall welfare gain in the baseline model comes from both the precautionary saving effect (decreasing along the wealth dimension) and the rate-of-return risk effect (increasing along the wealth dimension). Now with the rate-of-return risk effect shut down, we see smaller average welfare numbers which are decreasing in wealth.

To model the progressive U.S. federal income tax system, some recent papers (for instance, Holter et al., 2019, Guner et al., 2016, and Heathcote et al., 2017) use the following tax function from Benabou (2002):

$$\tau^y(y_t) = y_t - \lambda_1 y_t^{1-\lambda_2}, \tag{6.1}$$

where λ_1 determines the overall tax level and λ_2 governs the progressivity of the tax system. We thus examine the robustness of our results when we use this functional form. Following Heathcote et al. (2017), we let λ_1 be the endogenous cyclical tax adjustment variable and keep λ_2 fixed. We calibrate λ_2 to be 0.047 based on the Residual Income Elasticity reported in Gouveia and Strauss (1999) for 1989, the year on which we base the calibration of the fixed tax parameters in the progressive tax function in the baseline model. Under this specification, the distribution of the tax rate adjustments is similar to that in the τ_0-adjustment case, as in both cases the adjustments are made through the overall level of the tax rates. As a result, the welfare gains (row 9 of Table 8) also resemble those from the τ_0-adjustment case (row 2 of Table 8).

Non-separable utility Recall that our baseline specification assumes a separable flow utility function in private and public consumption ($\rho = 1$), which implies that government purchases volatility affects the household decisions only indirectly, through equilibrium tax rate changes. By contrast, if public and private consumption are non-separable, then this volatility has a direct effect on the consumption-saving decision, since the government purchases level affects the marginal utility of private consumption.[35] We thus examine two alternative specifications where public consumption is an Edgeworth substitute (complement), $\rho = 0.5$ ($\rho = 1.5$), to private consumption.[36]

The results in rows 10 and 11 of Table 8 show that when G and c are complements (substitutes), the welfare gains from the elimination of fiscal volatility are larger (smaller) than in the baseline scenario. This is because the positive conditional comovement between G and taxes in the estimated tax revenue response rule makes volatility in G more costly when c and G are complements.[37] Since households face higher tax rates (lower disposable income) when G and the marginal utility of private consumption are high, the utility gain from fiscal volatility elimination in the case of complements is larger than in the separable case. An analogous argument applies when G and c are substitutes.

Alternative TFP and labor income processes Recall that our baseline scenario adopts the same TFP and unemployment processes as used in Krusell and Smith (1998). However, these choices produce an output volatility in the model that is 70% larger than that in the data (Section 3.3). To examine whether this difference affects our welfare results, we match the output volatility in the data by keeping TFP constant (at $z = 1$), but allowing unemployment rate fluctuations. The results in row 12 in Table 8 are very similar to those from the baseline model, suggesting that the excess output volatility in our model does not influence our welfare results.

Note that in our baseline model the impact fiscal multiplier is zero by construction. We next show, in an admittedly somewhat reduced-form way, that our results are not driven by the fiscal multiplier of zero coming from our modeling

[34] The labor income process is based on the superstar income case discussed later in this section. We did not implement this extension based on our baseline because without capital income in the progressive tax function, our baseline homogeneous labor income case would practically make the progressive tax system lump-sum.

[35] See Fiorito and Kollintzas (2004) for an overview of utility specifications for public consumption.

[36] To calculate λ_c with a non-separable utility function, we calculate the left-hand side of (5.2) as a discounted sum of flow utilities under various values of λ_c, using the equilibrium policy functions. We then find a value of λ_c that satisfies the equation numerically, using a bisection search.

[37] In the estimated aggregate tax revenue response rule, the tax-output ratio responds to the government-purchases-output ratio with a coefficient of $\rho_{T,G} = 0.484$; see Appendix A.2 for the details.

choices (flexible prices in combination with inelastic labor supply). Following Krueger et al. (2016), we implement a model specification with a demand externality where output is an increasing function of the sum of private and public consumption. To be specific, we assume the following aggregate production function:

$$Y_t = z_t (C_t + G_t)^{\Omega} K_t^{\alpha} L_t^{1-\alpha}, \tag{6.2}$$

where $\Omega > 0$ determines the strength of the effect of demand on output. We calibrate Ω to be 0.43, which gives us a fiscal multiplier on impact around 0.5, a value within the range reported in Ramey (2011).[38] The welfare numbers in row 13 of Table 8 show that the welfare gains in the economy with a positive fiscal multiplier are similar to those from the baseline model. This is because the positive fiscal multiplier has two offsetting effects. On the one hand, a positive multiplier amplifies the impact of government purchases shocks on economic volatility, which by itself would lead to higher welfare gains from eliminating fiscal volatility. On the other hand, the positive multiplier acts as partial insurance to the same shocks by increasing the effective TFP when tax rates are higher.[39] Moreover, our distributional result that welfare gains are increasing in wealth continues to hold.

Our baseline specification, to ease the computational burden, also does not allow for any labor income heterogeneity conditional on being employed. However, a realistically richer income process might matter for our welfare results. It can affect the implications of the progressive tax system as well as the degree of precautionary saving motive. Moreover, it provides an additional channel through which wealth inequality is generated in the model (Krueger et al., 2016). To examine whether this feature affects our welfare results, we introduce individual-specific productivity shocks as in McKay and Reis (2016). The idiosyncratic productivity process is assumed to be independent from any other processes and determines the labor earnings for employed households.[40] The welfare numbers in row 14 of Table 8 indicate that the richer labor income dynamics does not change the main message of the baseline model. The average welfare change is slightly smaller than that in the baseline case, but the wealth-rich still benefit more from the fiscal volatility elimination.

The richer income process case above still relies on heterogeneous discount factors to generate realistic wealth inequality; one might wonder whether our results would stay the same if the wealth distribution were purely driven by precautionary savings. To examine this question, we consider a model with a homogeneous discount factor and an income process that allows for “superstars,” as in Castañeda et al. (2003), Heathcote (2005), and Kindermann and Krueger (2014). In particular, we add a superstar state to the rich income process above.[41] The welfare results from this case are shown in row 15 of Table 8 and both the aggregate magnitudes and the welfare pattern are similar to those in the richer-income-dynamics model with preference heterogeneity. This suggests that preference heterogeneity being a (partial) source of wealth inequality does not drive our results.

Finally, in our baseline specification, we set the UI benefit to be 10% of the market wage. To examine whether our result is sensitive with respect to the generosity of the UI system, we implement a specification where the replacement rate of the UI benefit is 50% instead of 10%. This produces almost the same welfare numbers (row 16 of Table 8) as those in the baseline. Though unemployment is a significant risk in our model, this risk is almost orthogonal to the welfare costs of fiscal volatility as unemployed households pay only a negligible amount of taxes unless their wealth level is very high.

6.2. Additional experiments

Transition to a higher level of fiscal volatility As mentioned, one topic that has received some debate is how permanently heightened fiscal policy volatility might impact aggregate economic activity and welfare. To address this question, we let the economy transit to a level of fiscal volatility which has twice the variance of government purchases than that in our baseline economy.[42] Online Appendix D.2 provides the details of the computational implementation of this experiment.

The penultimate row in Table 8 shows the welfare changes from this magnified volatility experiment. As in the baseline experiment, higher fiscal volatility leads to a welfare loss for every wealth group, with wealth-rich households experiencing a larger loss. Overall, the numbers suggest that, at least for the range between zero and twice the pre-crisis level of fiscal volatility, the welfare effects of fiscal volatility are roughly symmetric.

[38] We obtain this multiplier by estimating a VAR using simulated data, where $log(G_t)$ is ordered first then followed by $log(T_t - TR_t)$ and $log(Y_t)$.

[39] We indeed find that the partial insurance effect dominates when the fiscal multiplier is smaller. The average welfare gain from eliminating fiscal volatility is by 30% smaller than in the baseline when the impact fiscal multiplier is 0.3 instead of 0.5.

[40] In particular, building on McKay and Reis (2016), we calibrate the idiosyncratic productivity process to follow the following discretized Markov chain: grids take values from [0.49, 0.90, 1.61], a transition can only happen between adjacent grids, and it happens with a probability of 0.02.

[41] This superstar state can only be reached from the high-income state, and households at the superstar state can only fall back to the high-income state (this setup is similar to that in Kindermann and Krueger, 2014). In terms of calibration, among the three regular income states, transition still can only happen between adjacent states with the same probability of 0.02. We choose the probability of leaving the superstar state to imply a 50-year expected duration. The superstar state productivity, which is 9.5 times larger than the next highest productivity, and the superstar state entering probability are chosen together to match the top 1% and top 5% wealth share. This calibration implies a one-percent superstar population share and a 15 percent top 1% income share.

[42] To put this exercise into historical context, had we started our data sample in 1950I instead of 1960I to include the Korean War period when estimating the government purchases process, its variance would be about 1.5 times larger than that used in the baseline model.

Table 9
Conditional expected welfare gains from private consumption, λ_c (%), sudden change.

$G_1 = G_l$	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	-0.3749	-0.5121	-0.4826	-0.4469	-0.4198	-0.3922	-0.3030	0.0078	0.2184
$\varepsilon = 1$	-0.3748	-0.5134	-0.4833	-0.4471	-0.4200	-0.3924	-0.3036	0.0074	0.2182
$\varepsilon = 0$	-0.3762	-0.5054	-0.4764	-0.4455	-0.4179	-0.3897	-0.2952	0.0132	0.2212
$\tilde{\beta} = \tilde{\beta}_l$	-0.4978	-0.5347	-0.5213	-0.4932	-0.4636	-0.4369	-0.3232	-0.0537	0.1555
$\tilde{\beta} = \tilde{\beta}_m$	-0.3786	-0.4904	-0.4732	-0.4465	-0.4198	-0.3924	-0.3090	-0.0032	0.2056
$\tilde{\beta} = \tilde{\beta}_h$	-0.0521	-0.4158	-0.3934	-0.3644	-0.3380	-0.3097	-0.1665	0.0754	0.2759
$G_1 = G_h$	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.4010	0.5452	0.5142	0.4767	0.4472	0.4174	0.3243	0.0119	-0.1873
$\varepsilon = 1$	0.4008	0.5463	0.5149	0.4768	0.4474	0.4176	0.3249	0.0123	-0.1870
$\varepsilon = 0$	0.4026	0.5389	0.5088	0.4756	0.4455	0.4151	0.3166	0.0064	-0.1904
$\tilde{\beta} = \tilde{\beta}_l$	0.5268	0.5653	0.5513	0.5209	0.4893	0.4610	0.3417	0.0673	-0.1356
$\tilde{\beta} = \tilde{\beta}_m$	0.4045	0.5244	0.5055	0.4763	0.4472	0.4176	0.3300	0.0210	-0.1772
$\tilde{\beta} = \tilde{\beta}_h$	0.0825	0.4525	0.4280	0.3968	0.3689	0.3393	0.1938	-0.0442	-0.2321

Sudden change in the level of government purchases In this experiment, we examine the consequences of a concomitant sudden change in the government purchases level by letting government purchases move to and stay at their unconditional mean value, G_m, immediately after the elimination of fiscal volatility. We view this and the baseline scenario, where government purchases gradually converge to their long-run level, as two extreme ways of how fiscal volatility can be eliminated.

From the results in the last row of Table 8, we see that the unconditional welfare gains with a sudden change in the level of government purchases are very similar to those in the baseline case. However, the results in Table 9 show that the welfare changes conditional on $G_1 = G_l$ and $G_1 = G_h$ are one order of magnitude larger than those in the baseline case (Table 7). For the $G_1 = G_l$-case, the welfare changes are increasing in the wealth level, while the opposite pattern holds for the case of $G_1 = G_h$. However, these patterns are not driven by the elimination of fiscal volatility *per se*. The sudden change in the level of government purchases (and hence taxation) leads to a faster aggregate capital stock adjustment and a larger effect on welfare. For instance in the $G_1 = G_l$-case, the sudden increase in government purchases leads to a faster decrease in aggregate capital, output, and average welfare. However, since lower aggregate capital levels (higher pre-tax rates of return) favor the wealth-rich capital income earners, the welfare change pattern increases with wealth. Following a similar intuition, the distributional effect for the $G_1 = G_h$-case is reversed.

7. Conclusion

The recent recession, the economy's slow recovery, and political turmoil have sparked a debate over the economic effects of fiscal volatility and uncertainty. In this study, we quantify the welfare effects of fiscal volatility and their distribution in a neoclassical stochastic growth environment with incomplete markets. In our model, aggregate uncertainty arises from both productivity and government purchases shocks. Government spending is financed by a progressive tax system, modeled to match important features of the U.S. tax system. We calibrate the model to U.S. data and evaluate the welfare and distributional consequences of eliminating government purchases shocks.

Our baseline results show that the welfare costs of fiscal volatility are fairly small on average. However, distributionally, the welfare costs are increasing in wealth. This distributional implication follows because the cyclicality of the overall progressivity of the U.S. tax system implies a strong role for top marginal tax rates in the cyclical adjustment of the tax system to satisfy the government budget constraint.

While our study provides insight into the impact of eliminating fiscal volatility, it should be viewed as a first step towards a more comprehensive analysis of the welfare and distributional implications of fiscal volatility. Future research could explore how our results change if nominal frictions that cause relative price distortions are added to the model. There is also no role in our model for a direct influence of government purchases on the unemployment process and thus cyclical idiosyncratic risk. Including this feature in a future quantitative analysis would require the development of a statistical model of how government purchases influence idiosyncratic unemployment processes, but such a model is elusive in the literature. Instead, since government purchases appear to be independent of the cycle in U.S. post-war quarterly frequency data, we have also used this assumption in the model. Furthermore, we have chosen to place exogenous volatility fundamentally on the level of government purchases, while the volatility of individual tax rates is derived from our model. Among fiscal data, we view the official aggregate data on government purchases as cleanest and least subject to construction choices, but recognize that the data on tax rates collected in Mertens and Montiel Olea (2018) could provide an alternative route. Finally, we model government purchases as a symmetric autoregressive process. However, future research could examine fiscal uncertainty in an economy facing the risk of very large government purchases as a very rare and dramatic event.

Appendix A. Estimation of the fiscal parameters

For the calibration, we use quarterly data from 1960I to 2007IV.

A.1. Government purchases process

We first construct a real government purchases (G) series by deflating the “Government consumption expenditures and gross investment” series (from NIPA table 3.9.5, line 1) with the GDP deflator (from NIPA table 1.1.9, line 1). We then estimate an AR(1) process for the linearly detrended real log(G) series. The estimated AR(1) coefficient is 0.9603 and the standard deviation of the innovation term is 0.0096. We use the Rouwenhorst method (see Rouwenhorst (1995)) to approximate this zero-mean AR(1) process with a three-state Markov Chain. This gives us a transition probability matrix, and a grid in the form $(-m,0,m)$, where m represents the percentage deviation from the middle grid point. The middle grid point of the G-process, G_m, is then calibrated to match the time series average of nominal G over nominal GDP from U.S. national accounting data (nominal GDP is from NIPA table 1.1.5, line 1), 20.86%. The grid for G is given by (G_l, G_m, G_h), where $G_h = (1+m)G_m$ and $G_l = (1-m)G_m$, and the discretized G-process on $[0.2205, 0.2319, 0.2433]$ has the following transition matrix:

$$\begin{bmatrix} 0.9607 & 0.0389 & 0.0004 \\ 0.0195 & 0.9611 & 0.0195 \\ 0.0004 & 0.0389 & 0.9607 \end{bmatrix}.$$

A.2. Fiscal rule

A.2.1. Methodology and estimation results

We first estimate the fiscal rule separately at two levels of government: the federal government level and the state/local level, allowing for debt only at the federal level.[43] We then construct a composite rule, using the share of federal government purchases in total government purchases.

The empirical specification for the federal fiscal rule is based on Bohn (1998) and Davig and Leeper (2011) and takes the following form:

$$\frac{T_t^F - Tr_t^F}{Y_t} = \rho_{T,0}^F + \rho_{T,B}^F \frac{B_t}{Y_t} + \rho_{T,Y}^F log(\frac{Y_t}{\bar{Y}_t}) + \rho_{T,G}^F \frac{G_t^F}{Y_t}, \tag{A.1}$$

where:

Y_t: Nominal GDP (Line 1 of NIPA table 1.1.5).
$T_t^F - Tr_t^F$: Federal government current receipts (Line 1 of NIPA table 3.2) minus federal government transfer expenditure (Line 25 of NIPA table 3.2).
B_t: Market value of privately held gross federal debt at the beginning of a quarter: data are from the Federal Reserve Bank of Dallas (http://www.dallasfed.org/research/econdata/govdebt.cfm).
$\bar{Y}_t$: Nominal CBO potential GDP: data are from the CBO website (http://www.cbo.gov/publication/42912).
G_t^F: Nominal federal government consumption expenditures and gross investment (Line 23 of NIPA table 1.1.5).

At the state and local level, we drop the debt-to-GDP ratio term, yielding the following equation for the state and local level:

$$\frac{T_t^{SL} - Tr_t^{SL}}{Y_t} = \rho_{T,0}^{SL} + \rho_{T,Y}^{SL} log(\frac{Y_t}{\bar{Y}_t}) + \rho_{T,G}^{SL} \frac{G_t^{SL}}{Y_t}, \tag{A.2}$$

where:

$T_t^{SL} - Tr_t^{SL}$: State and local government receipts (Line 1 of NIPA table 3.3) minus state and local government transfer expenditure (Line 24 of NIPA table 3.3).
G_t^{SL}: Nominal state and local government consumption expenditures and gross investment (Line 26 of NIPA table 1.1.5).

We then linearly detrend all ratio variables, except for $log(Y_t/\bar{Y}_t)$, before estimating equations (A.1) and (A.2).
Table 10 summarizes the estimation results.

[43] When we estimate one equation, using the sum of federal and the state-local level data, the estimation result implies a non-stationary government debt process.

Table 10
Estimated coefficients of the fiscal rule.

	constant	B_t/Y_t	$log(Y_t/\bar{Y}_t)$	G_t/Y_t
Federal	-0.009	0.017	0.321	0.146
	(0.001)	(0.003)	(0.032)	(0.096)
State and local	0.001	–	-0.039	0.771
	(0.001)	–	(0.015)	(0.063)

A.2.2. The composite fiscal rule

The composite fiscal rule used in our model is given by:

$$\begin{aligned}\frac{T_t - Tr_t}{Y_t} &= \rho_{T,0} + \rho_{T,B}\frac{B_t}{Y_t} + \rho_{T,Y}log(\frac{Y_t}{\bar{Y}_t}) + \rho_{T,G}\frac{G_t}{Y_t} \\ &= \rho_{T,0} + \rho^F_{T,B}\frac{B_t}{Y_t} + (\rho^F_{T,Y} + \rho^{SL}_{T,Y})log(\frac{Y_t}{\bar{Y}_t}) + (\gamma^F\rho^F_{T,G} + (1-\gamma^F)\rho^{SL}_{T,G})\frac{G_t}{Y_t},\end{aligned} \quad (A.3)$$

where γ^F is calibrated as the average share of federal government purchases within total government purchases: 0.46. This yields the following fiscal rule parameters:

$$\rho_{T,B} = 0.017, \quad \rho_{T,Y} = 0.282, \quad \rho_{T,G} = 0.484.$$

We use $\rho_{T,0}$ to match the average debt-to-GDP ratio in the data: 30%.

A.3. Consumption and income tax parameters

For the consumption tax function and the linear part of the income tax function, we use the average tax rate calculated from the data.

To be specific, the average tax rate on consumption is defined as:

$$\tau_c = \frac{TPI - PRT}{PCE - (TPI - PRT)}, \quad (A.4)$$

where the numerator is taxes on production and imports (TPI, NIPA table 3.1, line 4) minus state and local property taxes (PRT, NIPA table 3.3, line 8). The denominator is personal consumption expenditures (PCE, NIPA table 1.1.5, line 2) net of the numerator. We calculate the average $\tau_{c,t}$ over our sample period as our τ_c parameter: 8.14%.

For income taxes, we use the state level tax revenue to approximate the linear part:

$$\tau_0 = \frac{PIT + CT + PRT}{\text{Taxable Income}}, \quad (A.5)$$

where PIT (NIPA table 3.3, line 4) is state income tax, CT (NIPA table 3.3, line 10) is state tax on corporate income, and PRT (NIPA table 3.3, line 8) is state property taxes. Note that we exclude the social insurance contribution in the numerator since we do not have social security expenditures in the model. The denominator is GDP minus consumption of fixed capital (NIPA table 1.7.5, line 6), since our model has a depreciation allowance for capital income. Averaging $\tau_{0,t}$ from 1960I to 2007IV yields $\tau_0 = 5.25\%$.

Appendix B. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.red.2020.04.001.

References

Aiyagari, S. Rao, 1994. Uninsured idiosyncratic risk and aggregate saving. The Quarterly Journal of Economics 109 (3), 659–684.
Angeletos, George-Marios, Calvet, Laurent-Emmanuel, 2006. Idiosyncratic production risk, growth and the business cycle. Journal of Monetary Economics 53 (6), 1095–1115.
Arellano, Cristina, Bai, Yan, Kehoe, Patrick J., 2019. Financial frictions and fluctuations in volatility. Journal of Political Economy 127 (5), 2049–2103.
Auclert, Adrien, 2019. Monetary policy and the redistribution channel. The American Economic Review 109 (6), 2333–2367.
Auray, Stéphane, Fuller, David L., Lkhagvasuren, Damba, 2019. Unemployment insurance take-up rates in an equilibrium search model. European Economic Review 112, 1–31.
Bachmann, Rüdiger, Bayer, Christian, 2013. 'Wait-and-see' business cycles? Journal of Monetary Economics 60 (6), 704–719.
Bachmann, Rüdiger, Bayer, Christian, 2014. Investment dispersion and the business cycle. The American Economic Review 104 (4), 1392–1416.
Bachmann, Rüdiger, Bai, Jinhui, 2013a. Politico-economic inequality and the comovement of government purchases. Review of Economic Dynamics 16 (4), 565–580.
Bachmann, Rüdiger, Bai, Jinhui, 2013b. Public consumption over the business cycle. Quantitative Economics 4 (3), 417–451.
Baker, Scott R., Bloom, Nicholas, Davis, Steven J., 2016. Measuring economic policy uncertainty. The Quarterly Journal of Economics 131 (4), 1593–1636.

Basu, Susanto, Bundick, Brent, 2017. Uncertainty shocks in a model of effective demand. Econometrica 85 (3), 937–958.
Benabou, Roland, 2002. Tax and education policy in a heterogeneous-agent economy: what levels of redistribution maximize growth and efficiency? Econometrica 70 (2), 481–517.
Bhandari, Anmol, Evans, David, Golosov, Mikhail, Sargent, Thomas J., 2017a. Fiscal policy and debt management with incomplete markets. The Quarterly Journal of Economics 132 (2), 617–663.
Bhandari, Anmol, Evans, David, Golosov, Mikhail, Sargent, Thomas J., 2017b. Public debt in economies with heterogeneous agents. Journal of Monetary Economics 91, 39–51.
Bhandari, Anmol, Evans, David, Golosov, Mikhail, Sargent, Thomas J., 2018. Inequality, Business Cycles and Monetary-Fiscal Policy. NBER Working Paper 24710.
Bi, Huixin, Leeper, Eric M., Leith, Campbell, 2013. Uncertain fiscal consolidations. The Economic Journal 123 (566), F31–F63.
Bizer, David S., Judd, Kenneth L., 1989. Taxation and uncertainty. The American Economic Review 79 (2), 331–336.
Bloom, Nicholas, 2009. The impact of uncertainty shocks. Econometrica 77 (3), 623–685.
Bloom, Nicholas, Floetotto, Max, Jaimovich, Nir, Saporta-Eksten, Itay, Terry, Stephen J., 2018. Really uncertain business cycles. Econometrica 86 (3), 1031–1065.
Böhm, Christoph, 2015. Household Balance Sheets, Default, and Fiscal Policy at the Zero Lower Bound. Working Paper. University of Michigan.
Bohn, Henning, 1998. The behavior of U.S. public debt and deficits. The Quarterly Journal of Economics 113 (3), 949–963.
Born, Benjamin, Pfeifer, Johannes, 2014. Policy risk and the business cycle. Journal of Monetary Economics 68, 68–85.
Brinca, Pedro, Holter, Hans A., Krusell, Per, Malafry, Laurence, 2016. Fiscal multipliers in the 21st century. Journal of Monetary Economics 77, 53–69.
Castañeda, Ana, Díaz-Giménez, Javier, Ríos-Rull, José-Víctor, 2003. Accounting for the U.S. earnings and wealth inequality. Journal of Political Economy 111 (4), 818–857.
Chun, Young Jun, 2001. The redistributive effect of risky taxation. International Tax and Public Finance 8 (4), 433–454.
Coibion, Olivier, Gorodnichenko, Yuriy, Kueng, Lorenz, Silvia, John, 2017. Innocent bystanders? Monetary policy and inequality. Journal of Monetary Economics 88, 70–89.
Conesa, Juan Carlos, Krüger, Dirk, 2006. On the optimal progressivity of the income tax code. Journal of Monetary Economics 53 (7), 1425–1450.
Croce, M. Max, Kung, Howard, Nguyen, Thien T., Schmid, Lukas, 2012. Fiscal policies and asset prices. The Review of Financial Studies 25 (9), 2635–2672.
Davig, Troy, Leeper, Eric M., 2011. Monetary-fiscal policy interactions and fiscal stimulus. European Economic Review 55 (2), 211–227.
den Haan, Wouter J., 2010. Assessing the accuracy of the aggregate law of motion in models with heterogeneous agents. Journal of Economic Dynamics and Control 34 (1), 79–99.
Drautzburg, Thorsten, Fernández-Villaverde, Jesús, Guerrón-Quintana, Pablo, 2017. Political Distribution Risk and Aggregate Fluctuations. NBER Working Paper 23647.
Dyrda, Sebastian, Pedroni, Marcelo Zouain, 2017. Optimal Fiscal Policy in a Model with Uninsurable Idiosyncratic Shocks. Mimeo.
Fernández-Villaverde, Jesús, Guerrón-Quintana, Pablo A., Kuester, Keith, Rubio-Ramírez, Juan, 2015. Fiscal volatility shocks and economic activity. The American Economic Review 105 (11), 3352–3384.
Ferriere, Axelle, Navarro, Gaston, 2017. The Heterogeneous Effects of Government Spending: It's All About Taxes. Mimeo.
Fiorito, Riccardo, Kollintzas, Tryphon, 2004. Public goods, merit goods, and the relation between private and government consumption. European Economic Review 48 (6), 1367–1398.
Gilchrist, Simon, Sim, Jae W., Zakrajšek, Egon, 2014. Uncertainty, Financial Frictions, and Investment Dynamics. NBER Working Paper 20038.
Giorgi, Giacomo De, Gambetti, Luca, 2012. The Effects of Government Spending on the Distribution of Consumption. UFAE and IAE Working Paper 905.12.
Gomes, Francisco, Michaelides, Alexander, Polkovnichenko, Valery, 2013. Fiscal policy and asset prices with incomplete markets. The Review of Financial Studies 26 (2), 531–566.
Gornemann, Nils, Kuester, Keith, Nakajima, Makoto, 2016. Doves for the Rich, Hawks for the Poor? Distributional Consequences of Monetary Policy. Mimeo.
Gouveia, Miguel, Strauss, Robert P., 1994. Effective federal individual income tax functions: an exploratory empirical analysis. National Tax Journal 47 (2), 317–339.
Gouveia, Miguel, Strauss, Robert P., 1999. Effective tax functions for the US individual income tax: 1966-1989. In: Proceedings 92nd Annual Conference on Taxation. National Tax Association-Tax Institute of America, pp. 155–165.
Guner, Nezih, Lopez-Daneri, Martin, Ventura, Gustavo, 2016. Heterogeneity and government revenues: higher taxes at the top? Journal of Monetary Economics 80, 69–85.
Hagedorn, Marcus, Manovskii, Iourii, Mitman, Kurt, 2019. The Fiscal Multiplier. NBER Working Paper 25571.
Heathcote, Jonathan, 2005. Fiscal policy with heterogeneous agents and incomplete markets. The Review of Economic Studies 72 (1), 161–188.
Heathcote, Jonathan, Storesletten, Kjetil, Violante, Giovanni L., 2009. Quantitative macroeconomics with heterogeneous households. Annual Review of Economics 1 (1), 319–354.
Heathcote, Jonathan, Storesletten, Kjetil, Violante, Giovanni L., 2017. Optimal tax progressivity: an analytical framework. The Quarterly Journal of Economics 132 (4), 1693–1754.
Hedlund, Aaron, Karahan, Fatih, Mitman, Kurt, Ozkan, Serdar, 2016. Monetary Policy, Heterogeneity, and the Housing Channel. Conference Presentation at the NBER Summer Institute.
Holter, Hans A., Krueger, Dirk, Stepanchuk, Serhiy, 2019. How does tax progressivity and household heterogeneity affect Laffer curves? Quantitative Economics 10 (4), 1317–1356.
Huggett, Mark, 1993. The risk-free rate in heterogeneous-agent incomplete-insurance economies. Journal of Economic Dynamics and Control 17 (5–6), 953–969.
Huggett, Mark, 1997. The one-sector growth model with idiosyncratic shocks: steady states and dynamics. Journal of Monetary Economics 39 (3), 385–403.
Kaplan, Greg, Violante, Giovanni L., 2014. A model of the consumption response to fiscal stimulus payments. Econometrica 82 (4), 1199–1239.
Kaplan, Greg, Moll, Benjamin, Violante, Giovanni L., 2018. Monetary policy according to HANK. The American Economic Review 108 (3), 697–743.
Kelly, Bryan, Pastor, Lubos, Veronesi, Pietro, 2016. The price of political uncertainty: theory and evidence from the option market. The Journal of Finance 71 (5), 2417–2480.
Kindermann, Fabian, Krueger, Dirk, 2014. High Marginal Tax Rates on the Top 1%? Lessons from a Life Cycle Model with Idiosyncratic Income Risk. NBER Working Paper 20601.
Kopecky, Karen, Suen, Richard, 2010. Finite state Markov-chain approximations to highly persistent processes. Review of Economic Dynamics 13 (3), 701–714.
Krueger, D., Mitman, K., Perri, F., 2016. Macroeconomics and household heterogeneity. In: Taylor, John B., Uhlig, Harald (Eds.), Handbook of Macroeconomics, Vol. 2. Elsevier, pp. 843–921. Chapter 11.
Krusell, Per, Smith, Anthony A., 1997. Income and wealth heterogeneity, portfolio choice, and equilibrium asset returns. Macroeconomic Dynamics 1 (02), 387–422.
Krusell, Per, Smith, Anthony A., 1998. Income and wealth heterogeneity in the macroeconomy. Journal of Political Economy 106 (5), 867–896.
Krusell, Per, Smith, Anthony A., 1999. On the welfare effects of eliminating business cycles. Review of Economic Dynamics 2 (1), 245–272.

Krusell, Per, Mukoyama, Toshihiko, Sahin, Aysegul, Smith, Anthony A., 2009. Revisiting the welfare effects of eliminating business cycles. Review of Economic Dynamics 12 (3), 393–402.
Li, Rong, 2013. The Distributional Effects of Government Spending Shocks. Working Paper. Ohio State University.
Lucas Jr., Robert E., 1987. Models of Business Cycles. Basil Blackwell, New York.
Lucas Jr., Robert E., 2003. Macroeconomic priorities. The American Economic Review 93 (1), 1–14.
McKay, Alisdair, Reis, Ricardo, 2016. The role of automatic stabilizers in the U.S. business cycle. Econometrica 84 (1), 141–194.
Mendoza, Enrique G., Razin, Assaf, Tesar, Linda L., 1994. Effective tax rates in macroeconomics: cross-country estimates of tax rates on factor incomes and consumption. Journal of Monetary Economics 34 (3), 297–323.
Mertens, Karel, Montiel Olea, José Luis, 2018. Marginal tax rates and income: new time series evidence. The Quarterly Journal of Economics 133 (4), 1803–1884.
Mukoyama, Toshihiko, Sahin, Aysegul, 2006. Costs of business cycles for unskilled workers. Journal of Monetary Economics 53 (8), 2179–2193.
Mumtaz, Haroon, Zanetti, Francesco, 2013. The impact of the volatility of monetary policy shocks. Journal of Money, Credit, and Banking 45 (4), 535–558.
Mumtaz, Haroon, Surico, Paolo, 2018. Policy uncertainty and aggregate fluctuations. Journal of Applied Econometrics 33 (3), 319–331.
Musgrave, Richard A., Thin, Tun, 1948. Income tax progression: 1929-1948. Journal of Political Economy 56 (6), 498–514.
Nodari, Gabriela, 2014. Financial regulation policy uncertainty and credit spreads in the US. Journal of Macroeconomics 41, 122–132.
Pastor, Lubos, Veronesi, Pietro, 2012. Uncertainty about government policy and stock prices. Journal of Finance 64 (4), 1219–1264.
Pastor, Lubos, Veronesi, Pietro, 2013. Political uncertainty and risk premia. Journal of Financial Economics 113 (3), 520–545.
Piketty, Thomas, Saez, Emmanuel, 2007. How progressive is the U.S. Federal Tax System? A historical and international perspective. Journal of Economic Perspectives 21 (1), 3–24.
Ramey, Valerie A., 2011. Can government purchases stimulate the economy? Journal of Economic Literature 49 (3), 673–685.
Richter, Alexander W., Throckmorton, Nathaniel A., 2015. The consequences of an unknown debt target. European Economic Review 78, 76–96.
Röhrs, Sigrid, Winter, Christoph, 2017. Reducing government debt in the presence of inequality. Journal of Economic Dynamics and Control 82, 1–20.
Rouwenhorst, Geert K., 1995. Asset pricing implications of equilibrium business cycle models. In: Cooley, Thomas F. (Ed.), Frontiers of Business Cycle Research. Princeton University Press, Princeton, pp. 294–330. Chapter 10.
Skinner, Jonathan, 1988. The welfare cost of uncertain tax policy. Journal of Public Economics 37 (2), 129–145.
Stokey, Nancy L., 2016. Wait-and-see: investment options under policy uncertainty. Review of Economic Dynamics 21 (Supplement C), 246–265.
Stone, Chad, Chen, William, 2014. Introduction to Unemployment Insurance. Center on Budget and Policy Priorities, Washington, DC. Available at http://www.cbpp.org/cms/index.cfm?fa=view&id=1466.
Tax Foundation, 2013. Federal Capital Gains Tax Rates, 1988-2013. Tax Foundation, Washington, DC. Available at https://taxfoundation.org/federal-capital-gains-tax-rates-1988-2013/.
York, Erica, 2019. An Overview of Capital Gains Taxes. Tax Foundation, Washington, DC. Available at http://taxfoundation.org/capital-gains-taxes.