Risk, Uncertainty, and Expected Returns*

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Abstract

A conditional asset pricing model with risk and uncertainty implies that the time-varying exposures of equity portfolios to the market and uncertainty factors carry positive risk premiums. The empirical results from the size, book-to-market, momentum, and industry portfolios indicate that the conditional covariances of equity portfolios with market and uncertainty predict the time-series and cross-sectional variation in stock returns. We find that equity portfolios that are highly correlated with economic uncertainty proxied by the variance risk premium (VRP) carry a significant, annualized 8 percent premium relative to portfolios that are minimally correlated with VRP.

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1 Introduction

This paper investigates whether the market price of risk and the market price of uncertainty are significantly positive and whether they predict the time-series and cross-sectional variation in stock returns. Although the literature has so far shown how uncertainty impacts optimal allocation decisions and asset prices, the results have been provided based on a theoretical model.[1] Earlier studies do not pay attention to empirical testing of whether the exposures of equity portfolios and individual stocks to uncertainty factors predict their future returns. We extend the original consumption-based asset pricing models to propose a conditional asset pricing model with time-varying market risk and economic uncertainty. According to our model, the premium on equity is composed of two separate terms; the first term compensates for the standard market risk and the second term represents additional premium for variance risk. We test whether the time-varying conditional covariances of equity returns with market and uncertainty factors predict the time-series and cross-sectional variation in future stock returns.

In this paper, economic uncertainty is proxied by the variance risk premia in the U.S. equity market. Following Britten-Jones and Neuberger (2000), Jiang and Tian (2005), and Carr and Wu (2009), we define the variance risk premium (VRP) as the difference between expected variance under the risk-neutral measure and expected variance under the objective measure.[2] We generate several proxies for financial and economic uncertainty and then compute the correlations between uncertainty variables and VRP. The first set of measures

[1]Although formal understanding of uncertainty and uncertainty aversion is poor, there exists a definition of uncertainty aversion originally introduced by Schmeidler (1989) and Epstein (1999). In recent studies, uncertainty aversion is defined for a large class of preferences and in different economic settings by Epstein and Wang (1994), Epstein and Zhang (2001), Chen and Epstein (2002), Klibanoff, Marinacci, and Mukerji (2005), Maccheroni, Marinacci, and Rustichini (2006), and Ju and Miao (2012). In addition to these theoretical papers, Ellsberg's (1961) experimental evidence demonstrates that the distinction between risk and uncertainty is meaningful empirically because people prefer to act on known rather than unknown or ambiguous probabilities.

[2]Other studies (e.g., Rosenberg and Engle (2002), Bakshi and Madan (2006), Bollerslev, Gibson, and Zhou (2011), and Bekaert, Hoerova, and Duca (2012)) interpret the difference between the implied and expected volatilities as an indicator of the representative agent's risk aversion. Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011) relate the variance risk premia to economic uncertainty risk.

can be viewed as macroeconomic uncertainty proxied by the conditional variance of the U.S. output growth and the conditional variance of the Chicago Fed National Activity Index (CFNAI). The second set of uncertainty measures is based on the extreme downside risk of financial institutions obtained from the left tail of the time-series and cross-sectional distribution of financial firms' returns. The third uncertainty variable is related to the health of the financial sector proxied by the credit default swap (CDS) index. The last uncertainty variable is based on the aggregate measure of investors' disagreement about individual stocks trading at NYSE, AMEX, and NASDAQ. We find that the variance risk premium is strongly and positively correlated with all measures of uncertainty considered in the paper. Our results indicate that VRP can be viewed as a sound proxy for financial and economic uncertainty.

Anderson, Ghysels, and Juergens (2009) introduce a model in which the volatility, skewness and higher order moments of all returns are known exactly, whereas there is uncertainty about mean returns. In their model, investors' uncertainty in mean returns is defined as the dispersion of predictions of mean market returns obtained from the forecasts of aggregate corporate profits. They find that the price of uncertainty is significantly positive and explains the cross-sectional variation in stock returns. Bekaert, Engstrom, and Xing (2009) investigate the relative importance of economic uncertainty and changes in risk aversion in the determination of equity prices. Distinct from the uncertainty that arises from disagreement among professional forecasters, Bekaert, Engstrom, and Xing (2009) focus on economic uncertainty proxied by the conditional volatility of dividend growth, and find that both the conditional volatility of cash flow growth and time-varying risk aversion are important determinants of equity returns.

Different from the aforementioned studies, we propose a conditional asset pricing model in which economic uncertainty (proxied by VRP) plays a significant role along with the standard market risk. After introducing a two-factor model with risk and uncertainty, we investigate the significance of risk-return and uncertainty-return coefficients using the time-series and cross-sectional data. Our empirical analyses are based on the size, book-to-market, momentum, and industry portfolios. We first use the dynamic conditional correlation (DCC) model of Engle (2002) to estimate equity portfolios' conditional covariances with the market portfolio and then test whether the conditional covariances predict future returns on equity portfolios. We find the risk-return coefficients to be positive and highly significant, implying a strongly positive link between expected return and market risk. Similarly, we use the DCC model to estimate equity portfolios' conditional covariances with the variance risk premia and then test whether the conditional covariances with VRP predict future returns on equity portfolios. The results indicate a significantly positive market price of uncertainty. Equity portfolios (individual stocks) that are highly correlated with uncertainty (proxied by VRP) carry a significant premium relative to portfolios that are uncorrelated or minimally correlated with VRP.

We also examine the empirical validity of the conditional asset pricing model by testing the hypothesis that the conditional alphas on the size, book-to-market, and industry portfolios are jointly zero. The test statistics fail to reject the null hypothesis, indicating that the two-factor model explains the time-series and cross-sectional variation in equity portfolios. Finally, we investigate whether the model explains the return spreads between the high-return (long) and low-return (short) equity portfolios (Small-Big for the size portfolios; Value-Growth for the book-to-market portfolios; and HiTec-Telcm for the industry portfolios). The results from testing the equality of conditional alphas for high-return and low-return portfolios provide no evidence for a significant alpha for Small-Big, Value-Growth, and HiTec-Telcm arbitrage portfolios, indicating that the two-factor model proposed in the paper provides both statistical and economic success in explaining stock market anomalies. Overall, the DCC-based conditional covariances capture the time-series and cross-sectional variation in returns on the size, book-to-market, and industry portfolios because the essential tests of the model are passed: (i) significantly positive risk-return and uncertainty-return tradeoffs; (ii) the conditional alphas are jointly zero; and (iii) the conditional alphas for high-return and low-return portfolios are not statistically different from each other.[3] These results are robust to using an alternative specification of the time-varying conditional covariances with an asymmetric GARCH model, using a larger cross-section of equity portfolios in asset pricing tests, and after controlling for a wide variety of macroeconomic variables, market illiquidity, and credit risk.[4]

Finally, we investigate the cross-sectional asset pricing performance of our model based on the 100 size and book-to-market portfolios. Using the long-short equity portfolios and the Fama and MacBeth (1973) regressions, we test the significance of a cross-sectional relation between expected returns on equity portfolios and the portfolios' conditional covariances (or betas) with VRP. Quintile portfolios are formed by sorting the 100 Size/BM portfolios based on their VRP-beta. The results indicate that the equity portfolios in highest VRP-beta quintile generate 8 percent more annual raw returns and alphas compared to the equity portfolios in the lowest VRP-beta quintile. These economically and statistically significant return differences are also confirmed by the Fama-MacBeth cross-sectional regressions, which produce positive and significant average slope coefficients on VRP-beta.

The rest of the paper is organized as follows. Section 2 presents the conditional asset pricing model with risk and uncertainty. Section 3 describes the data. Section 4 outlines the estimation methodology. Section 5 presents the empirical results. Section 6 investigates the cross-sectional asset pricing performance of our model. Section 7 concludes the paper.

[3]We find a significantly positive risk-return and uncertainty-return tradeoffs in the cross-section of momentum portfolios as well. However, the two factor model introduced in the paper rejects the hypotheses that (ii) the conditional alphas on momentum portfolios are jointly zero and (iii) the conditional alphas for winner and loser portfolios are not statistically different from each other.

[4]Alternatively, our empirical result on VRP may be interpreted as compensating for the rare disaster risk (Gabaix, 2011), jump risk (Todorov, 2010; Drechsler and Yaron, 2011), or tail risk (Bollerslev and Todorov, 2011; Kelly, 2011). Alternatively, VRP can be generated from a habit-formation model with sophisticated consumption dynamics (Bekaert and Engstrom, 2010). The finding may also be related to the expected business conditions (Campbell and Diebold, 2009) and its cross-sectional implications for stock returns (Goetzmann, Watanabe, and Watanabe, 2009).

2 Economic Motivation for VRP Factor

To guide our economic interpretation of the empirical finding in the main paper, we follow the strategy of Campbell (1993, 1996) to substitute unobservable consumption-based measures with observable market-based measures. Under a structural model with recursive preference and consumption uncertainty (Bollerslev, Tauchen, and Zhou, 2009), one can show that the two pricing factors—market return and variance risk premium—span all systematic variations in any risky assets. Our methodology basically follows Campbell, Giglio, Polk, and Turley (2014) by substituting out the consumption growth in the pricing kernel, and then we substitute the unobservable economic uncertainty with variance risk premium.

2.1 Implication from Consumption-Based Asset Pricing Model

The representative agent in the economy is endowed with Epstein-Zin-Weil recursive preferences, and has the value function V_t of her life-time utility as

$$V_t = \left[(1-\delta)\, C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t\left[V_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right]^{\frac{\theta}{1-\gamma}}, \tag{1}$$

where C_t is consumption at time t, δ denotes the subjective discount factor, γ refers to the coefficient of risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ equals the intertemporal elasticity of substitution (IES). The key assumptions are that $\gamma > 1$ and $\psi > 1$ hence $\theta < 0$. Consequently, the logarithm of the pricing kernel, $m_{t+1} \equiv \log(M_{t+1})$, may be expressed as,

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{t+1}, \tag{2}$$

where r_{t+1} is the return on the asset that pays the consumption endowment flow.

Suppose that log consumption growth and its volatility follow the joint dynamics

$$g_{t+1} = \mu_g + \sigma_{g,t} z_{g,t+1}, \tag{3}$$

$$\sigma^2_{g,t+1} = a_\sigma + \rho_\sigma \sigma^2_{g,t} + \sqrt{q_t} z_{\sigma,t+1}, \tag{4}$$

$$q_{t+1} = a_q + \rho_q q_t + \varphi_q \sqrt{q_t} z_{q,t+1}, \tag{5}$$

where $\mu_g > 0$ denotes the constant mean growth rate, $\sigma^2_{g,t+1}$ represents time-varying volatility in consumption growth, and q_t introduces the volatility uncertainty process in the consumption growth process.[5]

Let w_t denote the logarithm of the price-dividend or wealth-consumption ratio; and conjecture a solution for w_t as an affine function of the state variables, $\sigma^2_{g,t}$ and q_t,

$$w_t = A_0 + A_\sigma \sigma^2_{g,t} + A_q q_t. \tag{6}$$

One can solve for the coefficients A_0, $A_\sigma < 0$ and $A_q < 0$ using the standard Campbell and Shiller (1988) approximation $r_{t+1} = \kappa_0 + \kappa_1 w_{t+1} - w_t + g_{t+1}$. Substituting this equation into the pricing kernel (2), we get

$$m_{t+1} = \theta \log \delta + \frac{\theta}{\psi}\kappa_0 - \frac{\theta}{\psi} w_t + \frac{\theta}{\psi}\kappa_1 w_{t+1} - \gamma r_{t+1}, \tag{7}$$

without referencing consumption growth, as in Campbell, Giglio, Polk, and Turley (2014).

Suppose that asset returns have conditional joint lognormal distributions with time-varying volatility, then the risk premium on any asset i is given by

$$\mathrm{E}_t r_{i,t+1} - r_{f,t} + \frac{1}{2}\mathrm{Var}_t r_{t+1} = -\mathrm{Cov}_t[m_{t+1}, r_{i,t+1}]. \tag{8}$$

Using the pricing kernel without consumption (7), where the first three items are known at time t, we obtain the conditional asset pricing relation between the risk premium of any asset and the asset's covariances with the wealth return and time-varying shocks to future consumption:

$$\mathrm{E}_t r_{i,t+1} - r_{f,t} + \frac{1}{2}\mathrm{Var}_t r_{t+1} = \gamma \mathrm{Cov}_t[r_{i,t+1}, r_{t+1}] - \frac{\theta}{\psi}\kappa_1 \mathrm{Cov}_t[r_{i,t+1}, w_{t+1}], \tag{9}$$

where $\gamma > 1$ and $-\frac{\theta}{\psi}\kappa_1 > 0$.

One obvious advantage of Campbell (1993, 1996) is to substitute out consumption growth in the asset pricing tests, which also motivates using the market as a feasible proxy for total

[5] The parameters satisfy $a_\sigma > 0, a_q > 0$, $|\rho_\sigma| < 1$, $|\rho_q| < 1$, $\varphi_q > 0$; and $\{z_{g,t}\}$, $\{z_{\sigma,t}\}$ and $\{z_{q,t}\}$ are iid Normal$(0, 1)$ processes jointly independent with each other.

wealth. Furthermore, we substitute out the consumption growth volatility as well, using the result $\text{Var}_t r_{t+1} = \sigma_{g,t}^2 + \kappa_1^2 \left(A_\sigma^2 + A_q^2 \varphi_q^2\right) q_t$ from Bollerslev, Tauchen, and Zhou (2009). Replacing w_{t+1} with the wealth-consumption ratio equation (6), we arrive at

$$\begin{aligned} \text{E}_t r_{i,t+1} - r_{f,t} &+ \frac{1}{2}\text{Var}_t r_{t+1} + \frac{\theta}{\psi}\kappa_1 A_\sigma \text{Cov}_t[r_{i,t+1}, \text{Var}_{t+1} r_{t+2}] \\ &= \gamma \text{Cov}_t[r_{i,t+1}, r_{t+1}] + \frac{\theta}{\psi}\kappa_1 [A_\sigma \kappa_1^2 \left(A_\sigma^2 + A_q^2 \varphi_q^2\right) - A_q] \text{Cov}_t[r_{i,t+1}, q_{t+1}]. \end{aligned}$$

Overlooking the Jensen's inequality term $\text{Var}_t r_{t+1}$ and the high order term $\text{Cov}_t[r_{i,t+1}, \text{Var}_{t+1} r_{t+2}]$, we can see that the risk-return tradeoff γ before $\text{Cov}_t[r_{i,t+1}, r_{t+1}]$ is the risk-aversion coefficient and is positive. While the uncertainty-return tradeoff $\frac{\theta}{\psi}\kappa_1 [A_\sigma \kappa_1^2 \left(A_\sigma^2 + A_q^2 \varphi_q^2\right) - A_q]$ before $\text{Cov}_t[r_{i,t+1}, q_{t+1}]$ is not clearly signed, depending on the relative strength of A_σ versus A_q. In fact, even if the risk-aversion coefficient is zero, the uncertainty-return tradeoff is still non-zero in general.

Finally, since the consumption volatility-of-volatility q_t or economic uncertainty is not directly observable from the data, we follow the same spirit of Campbell et al. (2014) and substitute the unobservable uncertainty variable q_t with the readily available variance risk premium measure. Using the solution from Bollerslev, Tauchen, and Zhou (2009) linking variance risk premium (VRP_t) and economic uncertainty variable (q_t): $VRP_t = (\theta - 1)\kappa_1 \left[A_\sigma + A_q \kappa_1^2 \left(A_\sigma^2 + A_q^2 \varphi_q^2\right) \varphi_q^2\right] q_t$, we reach our final result regarding the cross-sectional pricing implications from both risk and uncertainty proxies:

$$\begin{aligned} \text{E}_t r_{i,t+1} - r_{f,t} &+ \frac{1}{2}\text{Var}_t r_{t+1} + \frac{\theta}{\psi}\kappa_1 A_\sigma \text{Cov}_t[r_{i,t+1}, \text{Var}_{t+1} r_{t+2}] \\ &= \gamma \text{Cov}_t[r_{i,t+1}, r_{t+1}] \\ &\quad + \frac{\frac{\theta}{\psi}\kappa_1 [A_\sigma \kappa_1^2 \left(A_\sigma^2 + A_q^2 \varphi_q^2\right) - A_q]}{(\theta - 1)\kappa_1 \left[A_\sigma + A_q \kappa_1^2 \left(A_\sigma^2 + A_q^2 \varphi_q^2\right) \varphi_q^2\right]} \text{Cov}_t[r_{i,t+1}, VRP_{t+1}] \\ &\equiv A \cdot \text{Cov}_t[r_{i,t+1}, r_{t+1}] + B \cdot \text{Cov}_t[r_{i,t+1}, VRP_{t+1}], \end{aligned} \tag{10}$$

where the risk-return tradeoff coefficient $A \equiv \gamma$ and the uncertainty-return tradeoff coefficient $B \equiv \frac{\frac{\theta}{\psi}\kappa_1 [A_\sigma \kappa_1^2 \left(A_\sigma^2 + A_q^2 \varphi_q^2\right) - A_q]}{(\theta - 1)\kappa_1 \left[A_\sigma + A_q \kappa_1^2 \left(A_\sigma^2 + A_q^2 \varphi_q^2\right) \varphi_q^2\right]}$. Note that the shocks to VRP and q_t are proportional to each other and of the same sign, therefore carrying the same pricing information.

Campbell (1993) shows that, in an intertemporal CAPM setting (Merton, 1973), the appropriate choices for factors relevant in cross-sectional asset pricing tests should be the current market return and any other variables that have information about the future market returns. Given the recent evidence that variance risk premium (VRP) possesses a significant forecasting power for short-term market returns, our result derived above regarding the cross-sectional asset pricing implication of VRP is not surprising at all. Although the sign of the uncertainty-return tradeoff coefficient B is not determined for general parameter setting, our empirical exercise finds it to be positive. The intuition for the positive slope coefficient B, is that investors dislike the reduced ability to hedge against a deterioration in the investment opportunity captured by VRP—which *positively* predicts future market returns. Therefore investors require a higher return premium to hold the assets or stocks that *positively* covary with VRP (Campbell, 1996).

Note that although we look at the cross-sectional pricing implication of variance risk premium, Ang, Hodrick, Xing, and Zhang (2006) investigated the cross-sectional pricing implication of the change in VXO. These two approaches are closely related but also have important differences. VXO is the option market implied volatility measure, while variance risk premium is the difference between implied and expected variances. Therefore, it is likely that, in the cross-section, VXO and VRP perform differently in terms of beta pricing. Alternatively, VRP's role for cross-sectional asset pricing may also be motivated from a systematic correlation risk factor, as in Buraschi, Trojani, and Vedolin (2014), where there is an equivalence between correlation risk premium and variance risk premium (Driessen, Maenhout, and Vilkov, 2009).

Furthermore, the literature on index option typically finds a negative volatility risk premium driven by the negative correlation between the volatility shock and shock to market returns (see, among others, Bates, 1996; Pan, 2002; Bakshi and Kapadia, 2003). In our consumption-based asset pricing model, although the shocks to consumption growth and volatility uncertainty are independent, the market return does contain a component driven by the consumption volatility uncertainty (Bollerslev, Tauchen, and Zhou, 2009). Therefore, from a market-based model perspective, variance risk premium shock carries important information about the equity risk premium—the component due to economic uncertainty. In essence, variance risk premium is a much cleaner estimate of the uncertainty premium component in equity return, hence the strong pricing power of variance risk premium for cross-sectional stock returns.[6]

2.2 Variance Risk Premia and Economic Uncertainty Measures

For the option-implied variance of the S&P500 market return, we use the end-of-month Chicago Board of Options Exchange (CBOE) volatility index on a monthly basis ($VIX^2/12$). Following earlier studies, the daily realized variance for the S&P500 index is calculated as the summation of the 78 intra-day five-minute squared log returns from 9:30am to 4:00pm including the close-to-open interval. Along these lines, we compute the monthly realized variance for the S&P500 index as the summation of five-minute squared log returns in a month. As discussed in the internet appendix (Section A), variance risk premium (VRP) at time t is defined as the difference between the ex-ante risk-neutral expectation and the objective or statistical expectation of the return variance over the $[t, t+1]$ time interval. The monthly VRP data are available from January 1990 to December 2012.

To give a visual illustration, Figure 1 plots the monthly time-series of the level and change in the variance risk premium (VRP). The VRP proxy is moderately high around the 1990 and 2001 economic recessions but much higher during the 2008 financial crisis and to a lesser degree around 1997-1998 Asia-Russia-LTCM crisis. The variance spike during October 2008 already surpasses the initial shock of the Great Depression in October 1929. The huge run-up of VRP in the fourth quarter of 2008 leads the equity market bottom reached in March 2009. The sample mean of VRP is 18.47 (in percentages squared, monthly basis), with a standard

[6] We thank a referee for suggesting this interpretation.

deviation of 21.90. Notice that the extraordinary skewness (3.76) and kurtosis (27.24) signal a highly non-Gaussian process for VRP.

According to the conditional asset pricing specification, VRP is viewed as a proxy for uncertainty. To test whether VRP is in fact associated with alternative measures of uncertainty, we generate some proxies for financial and economic uncertainty. We obtain monthly values of the U.S. industrial production index from G.17 database of the Federal Reserve Board and monthly values of the Chicago Fed National Activity Index (CFNAI) from the Federal Reserve Bank of Chicago for the period January 1990 – December 2012.[7] We use the GARCH(1,1) model of Bollerslev (1986) to estimate the conditional variance of the growth rate of industrial production and the conditional variance of the CFNAI index. These two measures can be viewed as macroeconomic uncertainty. The sample correlation between VRP and economic uncertainty variables is positive and significant; sample correlation is 53.28% with the variance of output growth and 31.01% with the variance of CFNAI index.

Our second set of uncertainty measures is based on the downside risk of financial institutions obtained from the left tail of the time-series and cross-sectional distribution of financial firms' returns (Allen, Bali, and Tang, 2012). Specifically, we obtain monthly returns for financial firms ($6000 \leq$ SIC code ≤ 6999) for the sample period January 1990 to December 2012. Then, the 1% nonparametric Value-at-Risk (VaR) measure in a given month is measured as the cut-off point for the lower one percentile of the monthly returns on financial firms.[8] For each month, we determine the one percentile of the cross-section of returns on financial firms, and obtain an aggregate 1% VaR measure of the financial system for the period 1990-2012. In addition to the cross-sectional distribution, we use the time-series daily return

[7]The CFNAI is a monthly index that determines increases and decreases in economic activity and is designed to assess overall economic activity and related inflationary pressure. It is a weighted average of 85 existing monthly indicators of national economic activity, and is constructed to have an average value of zero and a standard deviation of one. Since economic activity tends toward a trend growth rate over time, a positive index reading corresponds to growth above trend and a negative index reading corresponds to growth below trend.

[8]Assuming that we have 900 financial firms in month t, the nonparametric measure of 1% VaR is the 9th lowest observation in the cross-section of monthly returns.

distribution to estimate 1% VaR of the financial system. For each month from January 1990 to December 2012, we first determine the lowest daily returns on financial institutions over the past 1 to 12 months. The catastrophic risk of financial institutions is then computed by taking the average of these lowest daily returns obtained from alternative measurement windows. The estimation windows are fixed at 1 to 12 months, and each fixed estimation window is updated on a monthly basis. These two downside risk measures can be viewed as a proxy for uncertainty in the financial sector. The sample correlations between VRP and financial uncertainty variables are positive and significant: 48.42% with the cross-sectional VaR measure and 38.73% with the time-series VaR measure.

The third uncertainty variable is related to the health of the financial sector proxied by the credit default swap (CDS) index. We download the monthly CDS data from Bloomberg. For the sample period January 2004 – December 2012, we obtain monthly CDS data for Bank of America (BOA), Citigroup (CICN), Goldman Sachs (GS), JP Morgan (JPM), Morgan Stanley (MS), Wells Fargo (WFC), and American Express (AXP). Then, we standardized all CDS data to have zero mean and unit standard deviation. Finally, we formed the standardized CDS index (EWCDS) based on the equal-weighted average of standardized CDS values for the 7 major financial firms. For the common sample period 2004-2012, the correlation between VRP and EWCDS is positive, 44.21%, and highly significant.

The last uncertainty variable is based on the aggregate measure of investors' disagreement about individual stocks trading at NYSE, AMEX, and NASDAQ. Following Diether, Malloy, and Scherbina (2002), we use dispersion in analysts' earnings forecasts as a proxy for divergence of opinion. It is likely that investors partly form their expectations about a particular stock based on the analysts' earnings forecasts. If all analysts are in agreement about expected returns, uncertainty is likely to be low. However, if analysts provide very different estimates, investors are likely to be unclear about future returns, and uncertainty is high. The sample correlation between VRP and the aggregate measure of dispersion is about 15.23%. Overall, these results indicate that the variance risk premia is strongly and positively correlated with all measures of uncertainty considered here. Hence, VRP can be viewed as a sound proxy for financial and economic uncertainty.

3 Data

3.1 Equity Portfolios

We use the monthly excess returns on the value-weighted aggregate market portfolio and the monthly excess returns on the 10 value-weighted size, book-to-market, momentum, and industry portfolios. The aggregate market portfolio is represented by the value-weighted NYSE-AMEX-NASDAQ index. Excess returns on portfolios are obtained by subtracting the returns on the one-month Treasury bill from the raw returns on equity portfolios. The data are obtained from Kenneth French's online data library.[9] We use the longest common sample period available, from January 1990 to December 2012, yielding a total of 276 monthly observations.

Table I of the internet appendix presents the monthly raw return and CAPM Alpha differences between high-return (long) and low-return (short) equity portfolios. The results are reported for the size, book-to-market (BM), momentum (MOM), and industry portfolios for the period January 1990 – December 2012.[10] The OLS t-statistics are reported in parentheses. The Newey and West (1987) t-statistics are given in square brackets.

For the ten size portfolios, "Small" (Decile 1) is the portfolio of stocks with the smallest market capitalization and "Big" (Decile 10) is the portfolio of stocks with the biggest market capitalization. For the 1990-2012 period, the average return difference between the Small and Big portfolios is 0.31% per month with the OLS t-statistic of 1.02 and the Newey-West (1987) t-statistic of 0.99, implying that small stocks on average do not generate higher returns than

[9] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[10] Since the monthly data on variance risk premia (VRP) start in January 1990, our empirical analyses with equity portfolios and VRP are based on the sample period January 1990 - December 2012.

big stocks. In addition to the average raw returns, Table I of the internet appendix presents the intercept (CAPM alpha) from the regression of Small-Big portfolio return difference on a constant and the excess market return. The CAPM Alpha (or abnormal return) for the long-short size portfolio is 0.25% per month with the OLS t-statistic of 0.84 and the Newey-West t-statistic of 0.80. This economically and statistically insignificant Alpha indicates that the static CAPM does explain the size effect for the 1990-2012 period.

For the ten book-to-market portfolios, "Growth" is the portfolio of stocks with the lowest book-to-market ratios and "Value" is the portfolio of stocks with the highest book-to-market ratios. For the sample period January 1990 – December 2012, the average return difference between the Value and Growth portfolios is economically and statistically insignificant; 0.23% per month with the OLS t-statistic of 0.77 and the Newey-West t-statistic of 0.69, implying that value stocks on average do not generate higher returns than growth stocks. Similar to our findings for the size portfolios, the unconditional CAPM explains the value premium for the 1990-2012 period; the CAPM Alpha (or abnormal return) for the long-short book-to-market portfolio is only 0.21% per month with the OLS t-statistic of 0.69 and the Newey-West t-statistic of 0.54.

For the ten momentum portfolios, Loser (Decile 1) is the portfolio of stocks with the lowest cumulative return over the previous 11 months (skipping the past one month) and Winner (Decile 10) is the portfolio of stocks with the highest cumulative return over the previous 11 months.[11] For the 1990-2012 period, the average return difference between the Loser and Winner portfolios is 1.05% per month with the OLS t-statistic of 2.05 and the Newey-West t-statistic of 1.91, implying that winner stocks on average generate economically and statistically higher returns than loser stocks. In addition to the average raw returns, Table I of the internet appendix presents the CAPM alpha from the regression of Winner-

[11]Following Jegadeesh and Titman (1993), the momentum variable for each stock in month t is defined as the cumulative return on the stock over the previous 11 months starting 2 months ago, i.e., the cumulative return from month $t-12$ to month $t-2$.

Loser portfolio return difference on a constant and the excess market return. The CAPM alpha for the long-short momentum portfolio is 1.33% per month with the OLS t-statistic of 2.67 and the Newey-West t-statistic of 2.82. This economically and statistically significant alpha indicates that the static CAPM does not explain the momentum effect for the 1990-2012 period.

Similar to the size and value effects, the industry effect in the U.S. equity market is statistically weak over the past two decades. The average raw and risk-adjusted return differences between the high-return (HiTech) and low-return (Telcm) industry portfolios are statistically insignificant for the sample period 1990-2012.

Earlier studies starting with Fama and French (1992, 1993) provide evidence for the significant size and value premiums for the post-1963 period. Some readers may find the insignificant size and value premiums for the 1990-2012 period controversial. Hence, in Table I of the internet appendix, we examine the significance of size and book-to-market effects for the longest sample period July 1926 - December 2012 and the subsample period July 1963 - December 2012. The results indicate significant raw return difference between the Value and Growth portfolios for both sample periods and significant risk-adjusted return difference (Alpha) only for the post-1963 period. Consistent with the findings of earlier studies, we find significant raw return difference between the Small and Big stock portfolios for the 1926-2012 period, which becomes very weak for the post-1963 period. The CAPM Alpha (or abnormal return) for the long-short size portfolio is economically and statistically insignificant for both sample periods.

4 Estimation Methodology

Following Bali (2008) and Bali and Engle (2010), our estimation approach proceeds in steps.

1) We take out any autoregressive elements in returns and VRP and estimate univariate GARCH models for all returns and VRP.

2) We construct standardized returns and compute bivariate DCC estimates of the correlations between each portfolio and the market and between each portfolio and shock to VRP using the bivariate likelihood function.

3) We estimate the expected return equation as a panel with the conditional covariances as regressors. The error covariance matrix specified as seemingly unrelated regression (SUR). The panel estimation methodology with SUR takes into account heteroskedasticity and autocorrelation as well as contemporaneous cross-correlations in the error terms.

The following subsections provide details about the estimation of time-varying covariances and the estimation of time-series and cross-sectional relation between expected returns and risk and uncertainty.

4.1 Estimating Time-Varying Conditional Covariances

We estimate the conditional covariance between excess returns on equity portfolio i and the market portfolio m based on the mean-reverting dynamic conditional correlation (DCC) model:

$$R_{i,t+1} = \alpha_0^i + \alpha_1^i R_{i,t} + \varepsilon_{i,t+1} \tag{11}$$

$$R_{m,t+1} = \alpha_0^m + \alpha_1^m R_{m,t} + \varepsilon_{m,t+1} \tag{12}$$

$$E_t\left[\varepsilon_{i,t+1}^2\right] \equiv \sigma_{i,t+1}^2 = \beta_0^i + \beta_1^i \varepsilon_{i,t}^2 + \beta_2^i \sigma_{i,t}^2 \tag{13}$$

$$E_t\left[\varepsilon_{m,t+1}^2\right] \equiv \sigma_{m,t+1}^2 = \beta_0^m + \beta_1^m \varepsilon_{m,t}^2 + \beta_2^m \sigma_{m,t}^2 \tag{14}$$

$$E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1} \tag{15}$$

$$\rho_{im,t+1} = \frac{q_{im,t+1}}{\sqrt{q_{ii,t+1} \cdot q_{mm,t+1}}}, \; q_{im,t+1} = \bar{\rho}_{im} + a_1 \cdot (\varepsilon_{i,t} \cdot \varepsilon_{m,t} - \bar{\rho}_{im}) + a_2 \cdot (q_{im,t} - \bar{\rho}_{im}) \tag{16}$$

where $R_{i,t+1}$ and $R_{m,t+1}$ denote the time $(t+1)$ excess return on equity portfolio i and the market portfolio m over a risk-free rate, respectively, and $E_t[\cdot]$ denotes the expectation operator conditional on time t information. $\sigma^2_{i,t+1}$ is the time-t expected conditional variance of $R_{i,t+1}$, $\sigma^2_{m,t+1}$ is the time-t expected conditional variance of $R_{m,t+1}$, and $\sigma_{im,t+1}$ is the time-t expected conditional covariance between $R_{i,t+1}$ and $R_{m,t+1}$. $\rho_{im,t+1} = q_{im,t+1}/\sqrt{q_{ii,t+1} \cdot q_{mm,t+1}}$ is the time-t expected conditional correlation between $R_{i,t+1}$ and $R_{m,t+1}$, and $\bar{\rho}_{im}$ is the unconditional correlation. To ease the parameter convergence, we use correlation targeting assuming that the time-varying correlations mean reverts to the sample correlations $\bar{\rho}_{im}$.

We estimate the conditional covariance between the excess return on each equity portfolio i and the innovation in the variance risk premia VRP, $\sigma_{i,VRP}$, using an analogous DCC model:

$$R_{i,t+1} = \alpha^i_0 + \alpha^i_1 R_{i,t} + \varepsilon_{i,t+1} \tag{17}$$

$$VRP_{t+1} = \alpha^{VRP}_0 + \alpha^{VRP}_1 VRP_t + \varepsilon_{VRP,t+1} \tag{18}$$

$$E_t\left[\varepsilon^2_{i,t+1}\right] \equiv \sigma^2_{i,t+1} = \beta^i_0 + \beta^i_1 \varepsilon^2_{i,t} + \beta^i_2 \sigma^2_{i,t} \tag{19}$$

$$E_t\left[\varepsilon^2_{VRP,t+1}\right] \equiv \sigma^2_{VRP,t+1} = \beta^{VRP}_0 + \beta^{VRP}_1 \varepsilon^2_{VRP,t} + \beta^{VRP}_2 \sigma^2_{VRP,t} \tag{20}$$

$$E_t\left[\varepsilon_{i,t+1}\varepsilon_{VRP,t+1}\right] \equiv \sigma_{i,VRP,t+1} = \rho_{i,VRP,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{VRP,t+1} \tag{21}$$

$$\begin{aligned} \rho_{i,VRP,t+1} &= \frac{q_{i,VRP,t+1}}{\sqrt{q_{ii,t+1} \cdot q_{VRP,t+1}}}, \\ q_{i,VRP,t+1} &= \bar{\rho}_{i,VRP} + a_1 \cdot \left(\varepsilon_{i,t} \cdot \varepsilon_{VRP,t} - \bar{\rho}_{i,VRP}\right) + a_2 \cdot \left(q_{i,VRP,t} - \bar{\rho}_{i,VRP}\right) \end{aligned} \tag{22}$$

where $\sigma_{i,VRP,t+1}$ is the time-t expected conditional covariance between $R_{i,t+1}$ and VRP^{shock}_{t+1}. $\rho_{i,VRP,t+1}$ is the time-t expected conditional correlation between $R_{i,t+1}$ and VRP^{shock}_{t+1}. We use the same DCC model to estimate the conditional covariance between the market portfolio m and the shock to the variance risk premia VRP, $\sigma_{m,VRP}$.

Equations (17), (18), and (21) indicate that the shock to variance risk premia is obtained from an autoregressive of order one process. Instead of using the change in the variance risk premia, $\Delta VRP = VRP_{t+1} - VRP_t$, that restricts $\alpha^{VRP}_0 = 0$ and $\alpha^{VRP}_1 = 1$, we use a more general econometric specification to generate VRP^{shock}_{t+1}, i.e., α_0^{VRP} and α_1^{VRP} are estimated using the AR(1) specification in equation (18).

We estimate the conditional covariances of each equity portfolio with the market portfolio and with VRP^{shock} using the maximum likelihood method described in the internet appendix (Section B). Then, as discussed in the following section, we estimate the time-series and cross-sectional relation between expected return and risk and uncertainty as a panel with the conditional covariances as regressors.

At an earlier stage of the study, we use 10 equity portfolios and estimate in one step the time-varying conditional correlations as well as the parameters of time-varying conditional mean in a Multivariate GARCH-in-mean framework. To ease the parameter convergence, we use correlation targeting assuming that the time-varying correlations mean reverts to the sample correlations. To reduce the overall time of maximizing the conditional log-likelihood, we first estimate all pairs of bivariate GARCH-in-mean model and then use the median values of A, B, a_1 and a_2 as starting values along with the bivariate GARCH-in-mean estimates of variance parameters $(\beta_0, \beta_1, \beta_2)$. Even after going through these steps to increase the speed of parameter convergence, it takes a long time to obtain the full set of parameters in the Multivariate GARCH-in-mean model. Similar to the findings of Bali and Engle (2010), the results from the one-step estimation of 10 equity portfolios turned out to be similar to those obtained from the two-step estimation procedure.[12]

[12] Bali and Engle (2010) also estimate the risk aversion coefficient in two steps; first they obtain the conditional covariances with DCC and then they use the covariance estimates in the panel regression with a common slope coefficient. In this setting, since the covariance matrices implied by the DCC model are not used in estimating risk premia or in computing their standard errors, a common worry in testing asset pricing models is that time-varying covariances are measured with error. Using different samples, they show that the significance of measurement errors in covariances is small. Hence, the one-step and two-step estimation procedures generate similar slope coefficients and standard errors.

4.2 Estimating Risk-Uncertainty-Return Tradeoff

Given the conditional covariances, we estimate the portfolio-specific intercepts and the common slope estimates from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{i,t+1} \quad (23)$$

$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{m,t+1} \quad (24)$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the innovation in the variance risk premia (VRP_{t+1}^{shock}), $Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and the variance risk premia (VRP_{t+1}^{shock}), and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio.

We estimate the system of equations in (23)-(24) using a weighted least square method that allows us to place constraints on coefficients across equations. We compute the t-statistics of the parameter estimates accounting for heteroskedasticity and autocorrelation as well as contemporaneous cross-correlations in the errors from different equations. The estimation methodology can be regarded as an extension of the seemingly unrelated regression (SUR) method, the details of which are in the internet appendix (Section C).

5 Empirical Results

In this section we first present results from the 10 decile portfolios of size, book-to-market, momentum, and industry. Second, we discuss the economic significance of the two-factor conditional asset pricing model at the market level. Finally, we provide a battery of robustness checks.

5.1 Ten Decile Portfolios of Size, Book-to-Market, Momentum, and Industry

The common slopes and the intercepts are estimated using the monthly excess returns on the 10 value-weighted size, book-to-market, momentum, and industry portfolios for the sample period January 1990 to December 2012. The aggregate stock market portfolio is measured by the value-weighted CRSP index. Table 1 reports the common slope estimates (A, B), the abnormal returns or conditional alphas for each equity portfolio (α_i) and the market portfolio (α_m), and the t-statistics of the parameter estimates. The last two rows, respectively, show the Wald statistics; Wald_1 from testing the joint hypothesis $H_0 : \alpha_1 = ... = \alpha_{10} = \alpha_m = 0$, and Wald_2 from testing the equality of conditional alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; Winner vs. Loser; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

The risk aversion coefficient is estimated to be positive and highly significant for all equity portfolios: $A = 2.77$ with a t-statistic of 2.83 for the size portfolios, $A = 2.56$ with a t-statistic of 2.66 for the book-to-market portfolios, $A = 2.23$ with a t-statistic of 2.08 for the momentum portfolios, and $A = 3.48$ with a t-statistic of 2.38 for the industry portfolios.[13] These results imply a positive and significant relation between expected return and market risk.[14] Consistent with the conditional asset pricing specification, the uncertainty aversion coefficient is also estimated to be positive and highly significant for all equity portfolios: $B = 0.0037$ with a t-statistic of 3.54 for the size portfolios, $B = 0.0059$ with a t-statistic of 2.58 for the book-to-market portfolios, $B = 0.0030$ with a t-statistic of 2.17 for the momentum portfolios, and $B = 0.0062$ with a t-statistic of 2.85 for the industry portfolios. These results indicate a significantly positive market price of uncertainty in the aggregate

[13] Our risk aversion estimates ranging from 2.23 to 3.48 are very similar to the median level of risk aversion, 2.52, identified by Bekaert, Engstrom, and Xing (2009) in a different model.

[14] Although the literature is inconclusive on the direction and significance of a risk-return tradeoff, some studies do provide evidence supporting a positive and significant relation between expected return and risk (e.g., Bollerslev, Engle, and Wooldridge (1988), Ghysels, Santa-Clara, and Valkanov (2005), Guo and Whitelaw (2006), Guo and Savickas (2006), Lundblad (2007), Bali (2008), and Bali and Engle (2010)).

stock market. Equity portfolios with higher sensitivity to increases in the variance risk premia are expected to generate higher returns next period.

One implication of the conditional asset pricing model is that the intercepts (α_i, α_m) are not jointly different from zero assuming that the conditional covariances of equity portfolios with the market portfolio and the variance risk premia have enough predictive power for expected future returns. To examine the empirical validity of the conditional asset pricing model, we test the joint hypothesis $H_0 : \alpha_1 = ... = \alpha_{10} = \alpha_m = 0$. As presented in Table 1, the Wald_1 statistics for the size, book-to-market, and industry portfolios are, respectively, 16.40, 10.43, and 14.36 with the corresponding p-values of 12.69%, 49.22%, and 21.37%. The significantly positive risk and uncertainty aversion coefficients and the insignificant Wald_1 statistics indicate that the two-factor model introduced in the paper is empirically sound.

We also investigate whether the model explains the return spreads between Small and Big; Value and Growth; and HiTec and Telcm portfolios. The last row in Table 1 reports Wald_2 statistics from testing the equality of conditional alphas for high-return and low-return portfolios ($H_0 : \alpha_1 = \alpha_{10}$). These intercepts capture the monthly abnormal returns on each portfolio that cannot be explained by the conditional covariances with the market portfolio and the variance risk premia.

The first column of Table 1 shows that the abnormal return on the small-stock portfolio is $\alpha_1 = 0.53\%$ per month with a t-statistic of 1.32, whereas the abnormal return on the big-stock portfolio is $\alpha_{10} = 0.21\%$ per month with a t-statistic of 0.70. The Wald_2 statistic from testing the equality of alphas on the Small and Big portfolios is 1.07 with a p-value of 30.09%, indicating that there is no significant risk-adjusted return difference between the small-stock and big-stock portfolios. The second column provides the conditional alphas on the Value and Growth portfolios: $\alpha_1 = 0.39\%$ per month with a t-statistic of 1.01, and $\alpha_{10} = 0.78\%$ per month with a t-statistic of 1.89. The Wald_2 statistic from testing $H_0 : \alpha_1 = \alpha_{10}$ is 1.68 with a p-value of 19.49%, implying that the conditional asset pricing model explains the value premium, i.e., the risk-adjusted return difference between value and growth stocks is statistically insignificant. The last column shows that the conditional alphas on HiTec and Telcm portfolios are, respectively, 0.28% and 0.12% per month, generating a risk-adjusted return spread of 16 basis points per month. As reported in the last row, the Wald_2 statistic from testing the significance of this return spread is 0.20 with a p-value of 65.47%, yielding insignificant industry effect over the sample period 1990-2012.

We examine the empirical validity of the conditional asset pricing model for momentum portfolios by testing the hypothesis that the conditional alphas on decile portfolios are jointly zero. As reported in Table 1, the Wald_1 statistic is 22.15 (p-value = 2.33%), implying that the conditional covariances of momentum portfolios with the market and the variance risk premia do not capture the entire time-series and cross-sectional variation in expected returns of momentum portfolios. We also investigate whether the two-factor model explains the return spreads between Winner and Loser portfolios. The Wald_2 statistic from testing the equality of conditional alphas, $\alpha_1 = \alpha_{10}$, is 4.98 with a p-value of 2.56%.

Overall, the DCC-based conditional covariances capture the time-series and cross-sectional variation in returns on the size, book-to-market, and industry portfolios because the essential tests of the conditional asset pricing model are passed: (i) significantly positive risk-return and uncertainty-return tradeoffs; (ii) the conditional alphas are jointly zero; and (iii) the conditional alphas for high-return and low-return portfolios are not statistically different from each other. However, the statistically significant risk-adjusted return spread ($\alpha_{10} - \alpha_1$) between winner and loser portfolios implies failure of the conditional asset pricing model in explaining the momentum effect.

5.2 Economic Significance at the Market Level

In this section, we test whether the risk-return (A) and uncertainty-return (B) coefficients are sensible and whether the uncertainty measure is associated with macroeconomic state variables.

Specifically, we rely on equation (24) and compute the expected excess return on the market portfolio based on the estimated prices of risk and uncertainty as well as the sample averages of the conditional covariance measures:

$$E_t\left[R_{m,t+1}\right] = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) \tag{25}$$

where $\alpha_m = 0.0026$, $A = 2.77$, and $B = 0.0037$ for the 10 size portfolios; $\alpha_m = 0.0042$, $A = 2.56$, and $B = 0.0059$ for the 10 book-to-market portfolios; $\alpha_m = 0.0032$, $A = 2.23$, and $B = 0.0030$ for the 10 momentum portfolios; and $\alpha_m = 0.0026$, $A = 3.48$, and $B = 0.0062$ for the 10 industry portfolios (see Table 1). The sample averages of $Var_t\left(R_{m,t+1}\right)$ and $Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right)$ are 0.002069 and -0.7426, respectively.[15] These values produce $E_t\left[R_{m,t+1}\right] = 0.56\%$ per month when the parameters are estimated using the 10 size portfolios, $E_t\left[R_{m,t+1}\right] = 0.51\%$ per month when the parameters are estimated using the 10 book-to-market portfolios, $E_t\left[R_{m,t+1}\right] = 0.56\%$ per month when the parameters are estimated using the 10 momentum portfolios, and $E_t\left[R_{m,t+1}\right] = 0.52\%$ when the parameters are estimated using the 10 industry portfolios.

To evaluate the performance of our model with risk and uncertainty, we calculate the sample average of excess returns on the market portfolio, which is a standard benchmark for the market risk premium. The sample average of $R_{m,t+1}$ is found to be 0.53% per month for the period January 1990 – December 2012, indicating that the estimated market risk premiums of 0.51% – 0.56% are very close to the benchmark. This again shows solid performance of the two-factor model introduced in the paper.

To further appreciate the economics behind the apparent connection between the variance risk premium (VRP) and the time-series and cross-sectional variations in expected stock returns, Figure 2 plots the VRP together with the monthly growth rate of real GDP per

[15]The negative value for the conditional covariance of the market return with the VRP factor is consistent with the consumption-based asset pricing model and the negative contemporaneous correlation between the market return and the VRP factor reported by Bollerslev, Tauchen, and Zhou (2009).

capita. As seen from the figure, there is a tendency for VRP to rise in the month before a decline in GDP, while it typically narrows ahead of an increase in GDP. Indeed, the sample correlation equals -0.19 between lag VRP and current GDP (as first reported in Bollerslev et al., 2009), with a standard error of 0.06 (p-value = 0.13%). In other words, VRP as a proxy for economic uncertainty does seem to negatively relate to future macroeconomic performance.

Thus, not only the difference between the implied and expected variances positively covaries with stock returns, it also covaries negatively with future growth rates in GDP. Intuitively, when VRP is high (low), it generally signals a high (low) degree of aggregate economic uncertainty. Consequently agents tend to simultaneously cut (increase) their consumption and investment expenditures and shift their portfolios from more (less) to less (more) risky assets. This in turn results in a rise (decrease) in expected excess returns for stock portfolios that covaries more (less) with the macroeconomic uncertainty, as proxied by VRP.

As mentioned earlier in Section 2, we provide a two-factor consumption-based asset pricing model in which the consumption growth and its volatility follow the joint dynamics and hence VRP affects expected future returns. In essence, our finding of a positive significant relation between economic uncertainty measure and stock expected returns, is consistent with the consumption-based model's implication that heightened VRP does signal the worsening of macroeconomic fundamentals.

5.3 Robustness Check

We have so far provided evidence from the individual equity portfolios (10 size, 10 book-to-market, 10 momentum, and 10 industry portfolios). We now investigate whether our main findings remain intact if we use a joint estimation with all test assets simultaneously (total of 40 portfolios). Table 2 reports the parameter estimates and the t-statistics that are adjusted for heteroskedasticity and autocorrelation for each series and the cross-correlations among the error terms. As shown in the first row of Table 2, the risk aversion coefficient is estimated to be positive and highly significant for the pooled dataset: $A = 3.16$ with a t-statistic of 5.39, implying a positive and significant relation between expected return and market risk. Similar to our earlier findings, the uncertainty aversion coefficient is also estimated to be positive and highly significant for the joint estimation: $B = 0.0037$ with a t-statistic of 5.51. These results indicate a significantly positive market price of uncertainty when all portfolios are combined together. Equity portfolios with higher sensitivity to increases in VRP are expected to generate higher returns next period.

The Wald_1 and Wald_2 statistics reported in Table 2 indicate that the conditional alphas on the size, book-to-market, and industry portfolios are jointly zero and the conditional alphas for high-return (small, value, HiTech) and low-return (big, growth, Telcm) portfolios are not statistically different from each other. Hence, the DCC-based conditional covariances capture the time-series and cross-sectional variation in returns on the size, book- to-market, and industry portfolios. Similar to our earlier findings, the two-factor model with risk and uncertainty provides both statistical and economic success in explaining stock market anomalies, except momentum.

As discussed in Section 4.1, we have so far used a more general econometric specification to generate VRP_{t+1}^{shock} instead of using the change in the variance risk premia. As shown in equation (18), the shock to variance risk premia is obtained from an autoregressive of order one process. In this section, we use a simpler measure of $VRP_{t+1}^{\text{shock}} \equiv \Delta VRP_{t+1} = VRP_{t+1} - VRP_t$, that restricts $\alpha_0^{VRP} = 0$ and $\alpha_1^{VRP} = 1$ in equation (18). As presented in Table 3, the results from the change in VRP are very similar to those reported in Table 2. The risk aversion and uncertainty aversion coefficients are estimated to be positive and highly significant: $A = 3.03$ with a t-statistic of 4.65 and $B = 0.0039$ with a t-statistic of 3.41, indicating significantly positive market prices of risk and uncertainty. Consistent with our earlier findings, the Wald_1 and Wald_2 statistics reported in Table 3 indicate that the two-factor model with risk and uncertainty provides both statistical and economic success in explaining stock market anomalies, except momentum.

In Section D of the internet appendix, we provide a battery of robustness checks. There appears to be some controversy in the econometrics literature around the consistency of maximum likelihood parameter estimates generated by the DCC models.[16] To address this potential concern, in Section D.1 of the internet appendix, we use an alternative econometric methodology and estimate the conditional covariances based on the generalized conditional covariance (GCC) specification of Bali (2008). Table II of the internet appendix shows that the results from the GCC model are very similar to those reported in the paper. Second, we estimate the DCC-based conditional covariances using the Asymmetric GARCH model of Glosten, Jagannathan, and Runkle (1993). Table III of the internet appendix shows that our main findings from the Asymmetric GARCH model are very similar to those reported in Table 1.[17] Third, we examine whether the model's performance changes when we use a larger cross-section of industry portfolios. Table IV of the internet appendix shows a significantly positive market price of uncertainty in the cross-section of large number of equity portfolios; portfolios with higher correlation with the shock to VRP generate higher returns next month for the value-weighted 17-, 30-, 38-, 48-, and 49-industry portfolios. Also, the differences in conditional alphas are both economically and statistically insignificant, showing that the two-factor model introduced in the paper provides success in explaining industry effects. Fourth, we provide robustness analysis when controlling for popular macroeconomic and financial variables. Table V of the internet appendix indicates that after controlling for variables associated with business conditions, the time-varying exposures of equity portfolios to the

[16]See Aielli (2013), Caporin and McAleer (2013), and the proposed solution in Noureldin, Shephard, and Sheppard (2014).

[17]An alternative approach to estimating risk-return coefficient for the stock market portfolio is introduced by Ghysels, Santa-Clara, and Valkanov (2005). An application of the mixed data sampling (or MIDAS) approach to conditional covariances in a panel data setting represents an important direction for future research (see Ghysels, Sinko, and Valkanov, 2006; Andreou, Ghysels, and Kourtellos, 2010).

market and uncertainty factors carry positive risk premiums. Fifth, we provide results from individual stocks trading at the NYSE, AMEX, and NASDAQ. Table VI of the internet appendix reports a significantly positive market price of uncertainty for large and liquid stocks trading in the U.S. equity market. Sixth, we test whether the predictive power of the variance risk premia is subsumed by the market illiquidity and/or credit risk. Table VII of the internet appendix clearly shows that controlling for the market illiquidity and default risk individually and simultaneously does not influence the significant predictive power of the conditional covariances of portfolio returns with the market risk and VRP factors. Finally, we test whether the conditional asset pricing model with risk and uncertainty outperforms the conditional CAPM in terms of statistical fit. Table VIII of the internet appendix presents the realized monthly average excess returns on equity portfolios and the cross-section of expected excess returns generated by the one-factor conditional CAPM and the two-factor conditional asset pricing model. Clearly the newly proposed model with risk and uncertainty provides much more accurate estimates of expected returns on equity portfolios.

6 Cross-Sectional Relation between VRP-beta and Expected Returns

In this section, we investigate the cross-sectional asset pricing performance of our model by testing the significance of a cross-sectional relation between expected returns on equity portfolios and the portfolios' conditional covariances with VRP^{shock}. Following Bali (2008) and Campbell, Giglio, Polk, and Turley (2014), we use the size and book-to-market portfolios of Kenneth French as test assets. First, we estimate the DCC-based conditional covariances of 100 Size/BM portfolios with VRP^{shock} and then for each month we form quintile portfolios sorted based on the portfolios' conditional covariances (or betas) with VRP^{shock}. Since the conditional variance of VRP^{shock} is the same across portfolios, we basically sort equity portfolios based on their VRP-beta:

$$VRP_{i,t}^{beta} = \frac{\text{cov}\left[R_{i,t+1}, VRP_{t+1}^{\text{shock}}|\Omega_t\right]}{\text{var}\left[VRP_{t+1}^{\text{shock}}|\Omega_t\right]}, \tag{26}$$

where $VRP_{i,t}^{beta}$ is the VRP-beta of portfolio i in month t, $\text{cov}\left[R_{i,t+1}, VRP_{t+1}^{\text{shock}}|\Omega_t\right]$ is the conditional covariance of portfolio i with VRP_{t+1}^{shock} estimated using equation (21), and $\text{var}\left[VRP_{t+1}^{\text{shock}}|\Omega_t\right]$ is the conditional variance of VRP_{t+1}^{shock} which is constant in the cross-section of equity portfolios.

Ang, Hodrick, Xing, and Zhang (2006) test whether the exposure of individual stocks to changes in market volatility predicts cross-sectional variation in future stock returns. They first estimate the exposure of individual stocks to changes in the S&P 100 index option implied volatility (VXO). Then, they sort stocks into quintile portfolios based on these implied volatility betas. They find a negative cross-sectional relation between the volatility betas and future stock returns, that is, stocks with higher (lower) exposure to changes in the VXO generate lower (higher) returns in the next month. Motivated by Ang et al. (2006), we test whether the predictive power of $VRP_{i,t}^{beta}$ remains intact after controlling for the exposure of equity portfolios to changes in aggregate stock market volatility.

In this section, following Ang et al. (2006), we use the VXO in the estimation of the variance risk premia. We have so far used high-frequency (intraday) market returns to estimate the expected physical variance that enters the VRP, but we use low frequency returns on the market and equity portfolios to estimate the conditional covariances. To be consistent with the estimation of market variance, VRP, and conditional covariances, in this section, we define monthly realized variance of the market as the sum of squared daily returns on the S&P500 index in a month. Then, we estimate the expected physical variance by regressing one-month-ahead realized market variance on the lagged realized market variance and VXO. Since the monthly data on VXO are available from January 1986, our results in this section are based on the sample period January 1986 to December 2012.

We start cross-sectional analysis by performing univariate portfolio sorts based on $VRP_{i,t}^{beta}$. Then, we present evidence from multivariate cross-sectional regressions with market beta, $VRP_{i,t}^{beta}$, and $VXO_{i,t}^{beta}$.

Table 4 presents the average excess monthly returns of quintile portfolios that are formed by sorting the 100 Size/BM portfolios based on their VRP-beta. Q1 (Low VRP^{beta}) is the quintile portfolio of Size/BM portfolios with the lowest VRP-beta during the past month, and Q5 (High VRP^{beta}) is the quintile portfolio of Size/BM portfolios with the highest VRP-beta during the previous month. As shown in the first column of Table 4, the average excess return increases from 0.02% per month to 0.70% per month as we move from Q1 to Q5, generating an average return difference of 0.68% per month between Quintile 5 (High VRP^{beta}) and Quintile 1 (Low VRP^{beta}). This return difference is statistically significant with a Newey-West (1987) t-statistic of 4.33. In addition to the average excess returns, Table 4 also presents the intercepts (Fama-French three-factor alphas, denoted by FF3) from the regression of the average excess portfolio returns on a constant, the excess market return, a size factor (SMB), and a book-to-market factor (HML), following Fama and French (1993). As shown in the last row of Table 4, the difference in FF3 alphas between the High VRP^{beta} and Low VRP^{beta} portfolios is 0.69% per month with a Newey-West t-statistic of 4.99.

The last column of Table 4 presents the alpha of the return differential with respect to a four-factor model, following Fama and French (1993) and Carhart (1997). Besides the market, size, and book-to-market factors, it includes a fourth factor based on the return differential between stocks in the highest and lowest momentum deciles. The reason for including the fourth factor is to check whether the ability of $VRP_{i,t}^{beta}$ to predict returns can be subsumed by the tendency of these equity portfolios to co-move with the momentum factor.[18] As shown in the last row of Table 4, the difference in Fama-French-Carhart four factor alphas (denoted by FFC4) between the High $VRP_{i,t}^{beta}$ and Low $VRP_{i,t}^{beta}$ portfolios is

[18]SMB (small minus big), HML (high minus low), and MOM (winner minus loser) are described in and obtained from Kenneth Frenchs data library: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/.

0.68% per month with a Newey-West t-statistic of 4.09.

These results indicate that an investment strategy that goes long Size/BM portfolios in the highest $VRP^{beta}_{i,t}$ quintile and shorts Size/BM portfolios in the lowest $VRP^{beta}_{i,t}$ quintile produces average raw and risk-adjusted returns of 8.16% to 8.28% per annum. These return and alpha differences are economically and statistically significant at all conventional levels.

To determine whether the cross-sectional predictive power of VRP-beta is driven by the outperformance of High VRP^{beta} portfolios and/or the underperformance of Low VRP^{beta} portfolios, we compute the FF3 and FFC4 alpha of each quintile portfolio. As reported in Table 4, FF3 alpha of Q1 is -0.10% per month (t-stat. = -0.37) and FFC4 alpha of Q1 is -0.01% per month (t-stat. = -0.02), presenting economically and statistically insignificant risk-adjusted return of the short leg of the arbitrage portfolio with Low VRP-beta. When we look at the long leg of the arbitrage portfolio with High VRP-beta, the FF3 alpha of Q5 is 0.59% per month with a t-statistic of 2.67 and FFC4 alpha of Q5 is 0.67% per month with a t-statistic of 2.97. These economically and statistically significant FF3 and FFC4 alphas indicate that the significantly positive link between VRP-beta and the cross-section of portfolio returns is driven by the outperformance of individual stocks with High VRP-beta.

We now examine the cross-sectional relation between VRP-beta, Market-beta and expected returns using the Fama and MacBeth (1973) regressions. We calculate the time-series averages of the slope coefficients from the regressions of one-month-ahead portfolio returns on the conditional covariances of portfolios with the market and VRP factors, $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ and $Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right)$. The average slopes provide standard Fama-MacBeth tests for determining whether the market and/or uncertainty factors on average have non-zero premiums. Monthly cross-sectional regressions are run for the following asset pricing specification:

$$R_{i,t+1} = \lambda_{0,t} + \lambda_{1,t} \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + \lambda_{2,t} \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) + \varepsilon_{i,t+1}$$

where $R_{i,t+1}$ is the excess return on portfolio i in month $t+1$, $\lambda_{1,t}$ and $\lambda_{2,t}$ are the monthly slope coefficients on $Cov_t(R_{i,t+1}, R_{m,t+1})$ and $Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right)$, respectively. The predictive cross-sectional regressions of $R_{i,t+1}$ are run on the time-t expected conditional covariances of portfolios with the market and VRP factors.

Table 5 presents the time series averages of the slope coefficients ($\bar{\lambda}_1$, $\bar{\lambda}_2$) over the 324 months from January 1986 to December 2012 for the 100 Size/BM portfolios. The bivariate regression results produce a positive and statistically significant relation between $Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right)$ and the cross-section of portfolios returns. The average slope, $\bar{\lambda}_2$, is estimated to be 0.0250 with a Newey-West t-statistic of 2.94 for the 100 Size/BM portfolios. We also find a significantly positive link between market beta and the cross-section of expected returns. Specifically, the average slope, $\bar{\lambda}_1$, is found to be 3.38 with a t-statistic of 2.01 for the 100 Size/BM portfolios.

We now test whether significantly positive link between VRP-beta and expected returns remains intact after controlling for the negative market volatility risk premium. For each month from January 1986 to December 2012, we estimate the following cross-sectional regression specification:

$$\begin{aligned} R_{i,t+1} &= \lambda_{0,t} + \lambda_{1,t} \cdot Cov_t(R_{i,t+1}, R_{m,t+1}) + \lambda_{2,t} \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right)) \\ &\quad + \lambda_{3,t} \cdot Cov_t\left(R_{i,t+1}, VXO^{\text{shock}}_{t+1}\right) + \varepsilon_{i,t+1} \end{aligned}$$

The second row in Table 5 reports the average slope coefficients ($\bar{\lambda}_1$, $\bar{\lambda}_2$, $\bar{\lambda}_3$) for the 100 Size/BM portfolios. Similar to our finding from the bivariate regression, $\bar{\lambda}_2$ is estimated to be positive; $\bar{\lambda}_2 = 0.0292$ with a t-statistic of 3.61, implying a significantly positive uncertainty premium. Consistent with Ang et al. (2006), the average slope on implied volatility beta, $Cov_t\left(R_{i,t+1}, VXO^{\text{shock}}_{t+1}\right)$, is estimated to be negative; $\bar{\lambda}_3 = -0.0163$ with a t-statistic of -1.87. Interestingly, the average slope on market beta, $Cov_t(R_{i,t+1}, R_{m,t+1})$, is estimated to be positive but statistically insignificant; $\bar{\lambda}_1 = 0.5099$ with a t-statistic of 0.26. Overall, the results in Table 5 indicate that after controlling for the positive market risk premium and the negative market volatility risk premium, the positive link between VRP-beta and expected returns remains highly significant.

7 Conclusion

Although uncertainty is more common in decision-making process than risk, relatively little attention is paid to the phenomenon of uncertainty in empirical asset pricing literature. This paper focuses on economic uncertainty and augments the original consumption-based asset pricing models to introduce a two-factor conditional asset pricing model with time-varying market risk and uncertainty. According to the augmented asset pricing model, the premium on equity is composed of two separate terms; the first term compensates for the market risk and the second term representing a true premium for economic uncertainty. We use the conditional asset pricing model to test whether the time-varying conditional covariances of equity returns with the market and uncertainty factors predict their future returns.

Since information about economic uncertainty is too imprecise to measure with available data, we have to come up with a proxy for uncertainty that should be consistent with the investment opportunity set of risk-averse investors. Following Zhou (2010), we measure economic uncertainty with the variance risk premium (VRP) of the aggregate stock market portfolio. Different from earlier studies, we provide empirical evidence that VRP is indeed closely related to economic and financial market uncertainty. Specifically, we generate several proxies for uncertainty based on the macroeconomic variables, return distributions of financial firms, credit default swap market, and investors' disagreement about individual stocks. We show that VRP is highly correlated with all measures of uncertainty.

Based on the two-factor asset pricing model, we investigate whether the market prices of risk and uncertainty are economically and statistically significant in the U.S. equity market. Using the dynamic conditional correlation (DCC) model of Engle (2002), we estimate equity portfolios' conditional covariances with the market portfolio and VRP factors and then test whether these dynamic conditional covariances predict future returns on equity portfolios.

The empirical results from the size, book-to-market, momentum, and industry portfolios indicate that the DCC-based conditional covariances of equity portfolios with the market and VRP factors predict the time-series and cross-sectional variation in stock returns. We find the risk-return coefficients to be positive and highly significant, implying a strongly positive link between expected return and market risk. Similarly, the results indicate a significantly positive market price of uncertainty. That is, equity portfolios that are highly correlated with uncertainty (proxied by VRP) carry a significant premium relative to portfolios that are uncorrelated or minimally correlated with VRP. In addition to the size, book-to-market, momentum and industry portfolios, we investigate the significance of risk, uncertainty, and return tradeoffs using the largest 500 stocks trading at NYSE, AMEX, and NASDAQ as well as stocks in the S&P 500 index. Consistent with our findings from equity portfolios, we find significantly positive market prices of risk and uncertainty for large stocks trading in the U.S. equity market.

We also examine whether the conditional covariances with VRP could be picking up the covariances with market volatility, market illiquidity, and default risk. We find that the significantly positive link between uncertainty and future returns remains intact after controlling for market volatility, liquidity, and credit risk.

Finally, we investigate the cross-sectional asset pricing performance of our model using the long-short equity portfolios and the Fama-MacBeth regressions. The results indicate that the annual average raw and risk-adjusted returns of the equity portfolios in the highest VRP-beta quintile are about 8 percent higher than the annual average returns of the equity portfolios in the lowest VRP-beta quintile. After controlling for the market, size, book-to-market, and momentum factors of Fama-French-Carhart, the positive relation between VRP-beta and the cross-section of portfolio returns remains economically and statistically significant. Overall, we conclude that the time-varying exposures of equity portfolios to the variance risk premia predict the time-series and cross-sectional variation in stock returns.

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Table 1 Ten Decile Size, Book-to-Market, Momentum, and Industry Portfolios

This table reports the portfolio-specific intercepts and the common slope estimates from the following panel regression:

$$
\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right) + \varepsilon_{m,t+1}
\end{aligned}
$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the shock to the variance risk premia (VRP^{shock}_{t+1}), $Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and the VRP^{shock}_{t+1}, and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the ten decile size, book-to-market,momentum, and industry portfolios for the sample period from January 1990 to December 2012. The alphas (α_i) are reported for each equity portfolio and the t-statistics are presented in parentheses. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The last four rows, respectively, show the common slope coefficients (A and B), the Wald_1 statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald_2 statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; Winner vs. Loser; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

Size	α_i, α_m	*BM*	α_i, α_m	*MOM*	α_i, α_m	*Industry*	α_i, α_m
Small	0.0053	Growth	0.0039	Loser	-0.0038	NoDur	0.0053
	(1.32)		(1.01)		(-0.61)		(2.05)
2	0.0041	2	0.0046	2	0.0012	Durbl	0.0020
	(0.92)		(1.39)		(0.26)		(0.40)
3	0.0047	3	0.0054	3	0.0024	Manuf	0.0051
	(1.17)		(1.67)		(0.62)		(1.48)
4	0.0037	4	0.0062	4	0.0038	Enrgy	0.0060
	(0.96)		(1.87)		(1.15)		(1.75)
5	0.0047	5	0.0057	5	0.0032	HiTec	0.0028
	(1.24)		(1.83)		(1.04)		(0.52)
6	0.0045	6	0.0050	6	0.0033	Telcm	0.0012
	(1.28)		(1.51)		(1.13)		(0.33)
7	0.0048	7	0.0059	7	0.0043	Shops	0.0039
	(1.40)		(1.92)		(1.53)		(1.17)
8	0.0042	8	0.0056	8	0.0056	Hlth	0.0047
	(1.21)		(1.80)		(1.96)		(1.57)
9	0.0042	9	0.0067	9	0.0039	Utils	0.0046
	(1.31)		(2.02)		(1.28)		(1.83)
Big	0.0021	Value	0.0078	Winner	0.0075	Other	0.0025
	(0.70)		(1.89)		(1.74)		(0.68)
Market	0.0026	Market	0.0042	Market	0.0032	Market	0.0026
	(0.83)		(1.25)		(0.98)		(0.77)
A	2.7712	*A*	2.5585	*A*	2.2345	*A*	3.4834
	(2.83)		(2.66)		(2.08)		(2.38)
B	0.0037	*B*	0.0059	*B*	0.0030	*B*	0.0062
	(3.54)		(2.58)		(2.17)		(2.85)
Wald_1	16.40	Wald_1	10.43	Wald_1	22.15	Wald_1	14.36
	[12.69%]		[49.22%]		[2.33%]		[21.37%]
Wald_2	1.07	Wald_2	1.68	Wald_2	4.98	Wald_2	0.20
	[30.09%]		[19.49%]		[2.56%]		[65.47%]

Table 2 Results from Pooled Dataset

This table reports the portfolio-specific intercepts and the common slope estimates from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{m,t+1}$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the shock to the variance risk premia (VRP_{t+1}^{shock}), $Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and VRP_{t+1}^{shock}, and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the pooled dataset of ten decile size, book-to-market, momentum, and industry portfolios (total of 40 equity portfolios) for the sample period from January 1990 to December 2012. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. Table show the common slope coefficients (A and B), the Wald$_1$ statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald$_2$ statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; Winner vs. Loser; and HiTec vs. Telcm). The p-values of Wald$_1$ and Wald$_2$ statistics are given in square brackets.

	A	3.1557
		(5.39)
	B	0.0037
		(5.51)
Size	Wald$_1$	9.80
		[45.83%]
Small vs. Big	Wald$_2$	1.06
		[30.43%]
Book-to-Market	Wald$_1$	4.93
		[89.56%]
Value vs. Growth	Wald$_2$	0.89
		[34.55%]
Momentum	Wald$_1$	19.28
		[3.69%]
Winner vs. Loser	Wald$_2$	5.50
		[1.91%]
Industry	Wald$_1$	11.27
		[33.65%]
HiTec vs. Telcm	Wald$_2$	0.31
		[57.99%]

Table 3 Results from the Change in the Variance Risk Premia

This table reports the portfolio-specific intercepts and the common slope estimates from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, \Delta VRP_{t+1}\right) + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, \Delta VRP_{t+1}\right) + \varepsilon_{m,t+1}$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i $(R_{i,t+1})$ and the excess return on the market portfolio $(R_{m,t+1})$, $Cov_t\left(R_{i,t+1}, \Delta VRP_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the change in the variance risk premia (ΔVRP_{t+1}), $Cov_t\left(R_{m,t+1}, \Delta VRP_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and ΔVRP_{t+1}, and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the pooled dataset of ten decile size, book-to-market, momentum, and industry portfolios (total of 40 equity portfolios) for the sample period from January 1990 to December 2012. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. Table show the common slope coefficients (A and B), the Wald_1 statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald_2 statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; Winner vs. Loser; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

	A	3.0347
		(4.65)
	B	0.0039
		(3.41)
Size	Wald_1	6.70
		[75.36%]
Small vs. Big	Wald_2	0.44
		[50.67%]
Book-to-Market	Wald_1	4.36
		[92.94%]
Value vs. Growth	Wald_2	0.55
		[45.80%]
Momentum	Wald_1	21.45
		[1.82%]
Winner vs. Loser	Wald_2	5.33
		[2.09%]
Industry	Wald_1	11.27
		[33.71%]
HiTec vs. Telcm	Wald_2	0.20
		[65.42%]

Table 4 Long-Short Equity Portfolios Sorted by VRP-beta

Quintile portfolios are formed every month from January 1990 to December 2012 by sorting 100 Size/BM portfolios based on their VRP-beta (VRP^{beta}) over the past one month. Quintile 1 (Q1) is the portfolio of Size/BM portfolios with the lowest VRP^{beta} over the past one month. Quintile 5 (Q5) is the portfolio of Size/BM portfolios with the highest VRP^{beta} over the past one month. The table reports the average excess monthly returns, the 3-factor Fama-French alphas (FF3 alpha), and the 4-factor Fama-French-Carhart alphas (FFC4 alpha) on the VRP-beta sorted portfolios. The last row presents the differences in monthly returns and the differences in alphas with respect to the 3-factor and 4-factor models between Quintiles 5 and 1 and the corresponding t-statistics. Average excess returns and risk-adjusted returns are given in monthly percentage terms. Newey-West (1987) t-statistics are reported in parentheses.

	Average Excess Return	FF3 Alpha	FFC4 Alpha
Q1	0.02	-0.10	-0.01
	(0.06)	(-0.37)	(-0.02)
Q2	0.35	0.25	0.32
	(1.34)	(1.03)	(1.25)
Q3	0.45	0.34	0.41
	(1.77)	(1.42)	(1.66)
Q4	0.53	0.41	0.48
	(2.12)	(1.79)	(2.02)
Q5	0.70	0.59	0.67
	(2.94)	(2.67)	(2.97)
High-Low	0.68	0.69	0.68
	(4.33)	(4.99)	(4.09)

Table 5 Fama-MacBeth Cross-Sectional Regressions

The one-month-ahead excess returns of the 100 Size/BM equity portfolios are regressed every month on the time-varying conditional covariances: $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$, $Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right)$, and $Cov_t\left(R_{i,t+1}, VXO_{t+1}^{\text{shock}}\right)$ to test for the presence and significance of a cross-sectional relation between market beta, VRP beta, and VXO beta:

$$R_{i,t+1} = \lambda_{0,t} + \lambda_{1,t} \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + \lambda_{2,t} \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{i,t+1}$$

$$R_{i,t+1} = \lambda_{0,t} + \lambda_{1,t} \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + \lambda_{2,t} \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) + \lambda_{3,t} \cdot Cov_t\left(R_{i,t+1}, VXO_{t+1}^{\text{shock}}\right) + \varepsilon_{i,t+1}$$

This table reports the average slope coefficients ($\bar{\lambda}_1$, $\bar{\lambda}_2$, $\bar{\lambda}_3$) from the Fama and MacBeth (1973) regressions and the Newey-West (1987) t-statistics (in parentheses). The last column presents the average R^2 values from the monthly cross-sectional regressions. The sample period is from January 1990 to December 2012.

$\bar{\lambda}_1$	$\bar{\lambda}_2$	$\bar{\lambda}_3$	R^2
3.3781	0.0250		12.94%
(2.01)	(2.94)		(10.99)
0.5099	0.0292	-0.0163	16.53%
(0.26)	(3.61)	(-1.87)	(12.80)

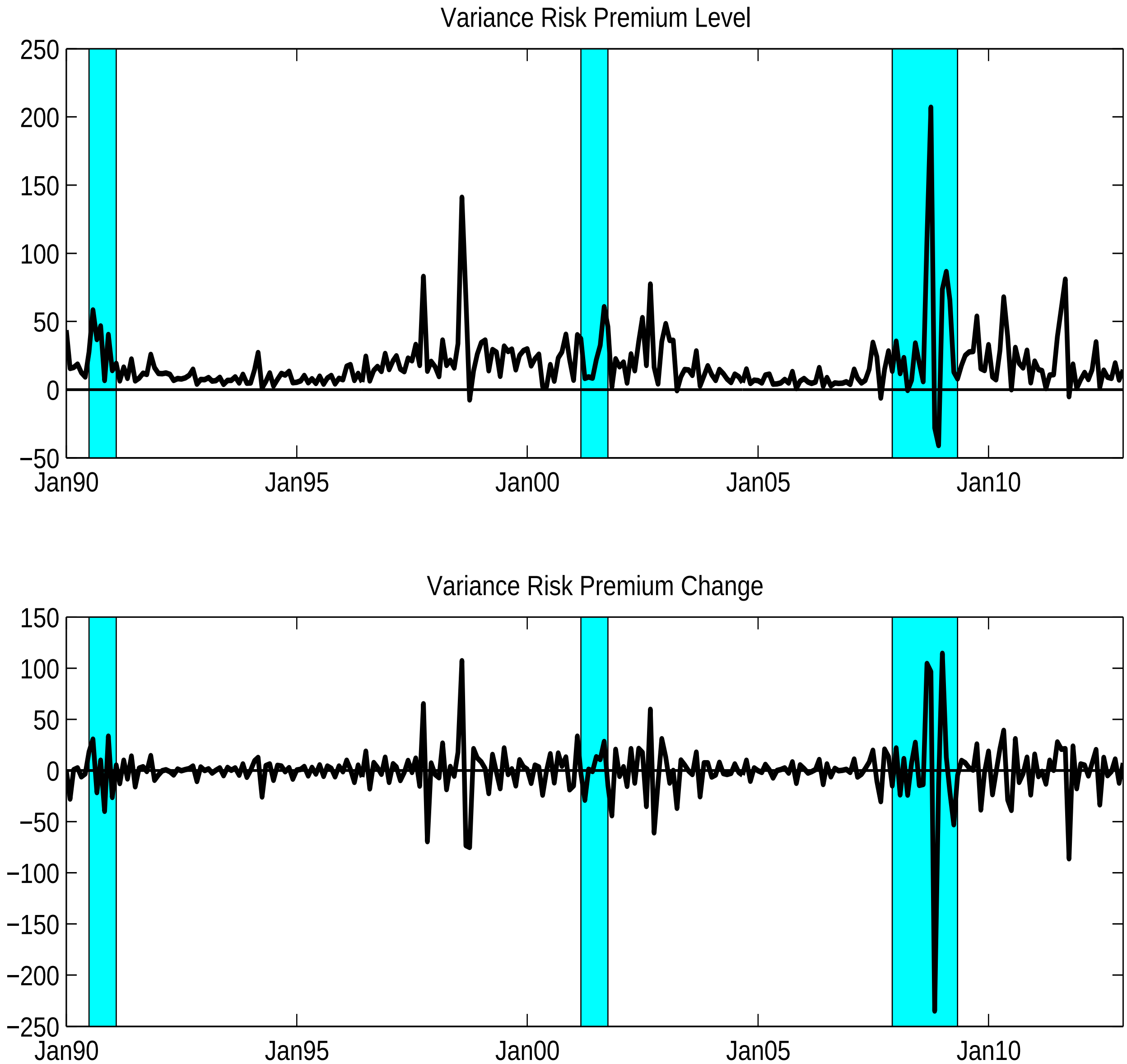

Figure 1 Variance Risk Premium Level and Change

This figure plots variance risk premium or the implied-expected variance difference (top panel) and the monthly change of variance risk premium change (bottom panel) for the S&P500 market index from January 1990 to December 2012. The variance risk premium is based on the realized variance forecast from lagged implied and realized variances. The shaded areas represent NBER recessions.

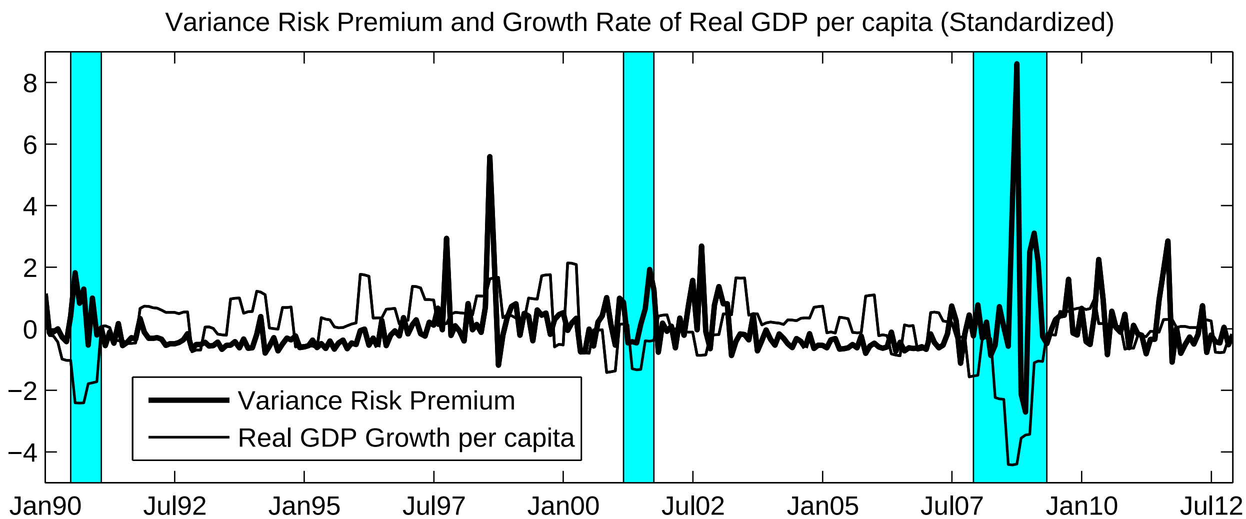

Figure 2 Variance Risk Premium and GDP Growth

The figure plots the growth rate of real GDP per capita (thin line) together with the variance risk premium (thick line) from January 1990 to December 2012. Both of the series are standardized to have mean zero and variance one. The shaded areas represent NBER recessions.

Risk, Uncertainty, and Expected Returns—Internet Appendix

A Variance Risk Premium and Empirical Measurement

The central empirical variable of this paper, as a proxy for economic uncertainty, is the market variance risk premium (VRP)—which is not directly observable but can be estimated from the difference between model-free option-implied variance and the conditional expectation of realized variance.

A.1 Variance Risk Premium: Definition and Measurement

In order to define the model-free implied variance, let $C_t(T, K)$ denote the price of a European call option maturing at time T with strike price K, and $B(t, T)$ denote the price of a time t zero-coupon bond maturing at time T. As shown by Carr and Madan (1998) and Britten-Jones and Neuberger (2000), among others, the market's risk-neutral Q expectation of the return variance σ^2_{t+1} conditional on the information set Ω_t, or the implied variance IV_t at time-t, can be expressed in a "model-free" fashion as a portfolio of European calls,

$$IV_t \;\equiv\; \mathrm{E}^Q\left[\sigma^2_{t+1}|\Omega_t\right] \;=\; 2\int_0^{\infty} \frac{C_t\left(t+1, \frac{K}{B(t,t+1)}\right) - C_t\left(t, K\right)}{K^2}\, dK, \tag{A1}$$

which relies on an ever increasing number of calls with strikes spanning from zero to infinity.[1] This equation follows directly from the classical result in Breeden and Litzenberger (1978), that the second derivative of the option call price with respect to strike equals the risk-neutral density, such that all risk neutral moments payoff can be replicated by the basic option prices (Bakshi and Madan, 2000).

In order to define the actual return variance, let p_t denote the logarithmic price of the asset. The realized variance over the discrete t to $t+1$ time interval can be measured in a "model-free" fashion by

$$RV_{t+1} \equiv \sum_{j=1}^{n} \left[p_{t+\frac{j}{n}} - p_{t+\frac{j-1}{n}}\right]^2 \longrightarrow \sigma^2_{t+1}, \tag{A2}$$

where the convergence relies on $n \to \infty$; i.e., an increasing number of within period price observations. As demonstrated in the literature (see, e.g., Andersen, Bollerslev, Diebold,

[1] Such a characterization is accurate up to the second order when there are jumps in the underlying asset (Jiang and Tian, 2005; Carr and Wu, 2009), though Martin (2011) has refined the above formulation to make it robust to jumps.

and Ebens, 2001; Barndorff-Nielsen and Shephard, 2002), this "model-free" realized variance measure based on high-frequency intraday data offers a much more accurate ex-post observation of the true (unobserved) return variance than the traditional ones based on daily or coarser frequency returns.

Variance risk premium (VRP) at time t is defined as the difference between the ex-ante risk-neutral expectation and the objective or statistical expectation at time t of the return variance at time $t+1$,

$$VRP_t \equiv \mathrm{E}^Q\left[\sigma^2_{t+1}|\Omega_t\right] - \mathrm{E}^P\left[\sigma^2_{t+1}|\Omega_t\right], \tag{A3}$$

which is not directly observable in practice.[2] To construct an empirical proxy for such a VRP concept, one needs to estimate various reduced-form counterparts of the risk neutral and physical expectations. In practice, the risk-neutral expectation $\mathrm{E}^Q\left[\sigma^2_{t+1}|\Omega_t\right]$ is typically replaced by the CBOE implied variance ($\mathrm{VIX}^2/12$) and the true variance σ^2_{t+1} is replaced by realized variance RV_{t+1}.

To estimate the objective expectation, $\mathrm{E}^P\left[\sigma^2_{t+1}|\Omega_t\right]$, we use a linear forecast of future realized variance as $RV_{t+1} = \alpha + \beta IV_t + \gamma RV_t + \epsilon_{t+1}$, with current implied and realized variances. The model-free implied variance from options market is an informationally more efficient forecast for future realized variance than the past realized variance (see, e.g., Jiang and Tian, 2005, among others), while realized variance based on high-frequency data also provides additional power in forecasting future realized variance (Andersen, Bollerslev, Diebold, and Labys, 2003). Therefore, a joint forecast model with one lag of implied variance and one lag of realized variance seems to capture the most forecasting power based on time-t available information (Drechsler and Yaron, 2011).

B DCC Model of Engle (2002)

We estimate the conditional covariances of each equity portfolio with the market portfolio and VRP ($\sigma_{im,t+1}$, $\sigma_{i,VRP,t+1}$) based on the mean-reverting DCC model of Engle (2002).

[2] The difference between option implied and GARCH type filtered volatilities has been associated in existing literature with notions of aggregate market risk aversion (Rosenberg and Engle, 2002; Bakshi and Madan, 2006; Bollerslev, Gibson, and Zhou, 2011).

Engle defines the conditional correlation between two random variables r_1 and r_2 that each has zero mean as

$$\rho_{12,t} = \frac{E_{t-1}\left(r_{1,t} \cdot r_{2,t}\right)}{\sqrt{E_{t-1}\left(r_{1,t}^2\right) \cdot E_{t-1}\left(r_{1,t}^2\right)}}, \tag{A4}$$

where the returns are defined as the conditional standard deviation times the standardized disturbance:

$$\sigma_{i,t}^2 = E_{t-1}\left(r_{i,t}^2\right), \quad r_{i,t} = \sigma_{i,t} \cdot u_{i,t}, \quad i = 1, 2 \tag{A5}$$

where $u_{i,t}$ is a standardized disturbance that has zero mean and variance one for each series. Equations (A4) and (A5) indicate that the conditional correlation is also the conditional covariance between the standardized disturbances:

$$\rho_{12,t} = \frac{E_{t-1}\left(u_{1,t} \cdot u_{2,t}\right)}{\sqrt{E_{t-1}\left(u_{1,t}^2\right) \cdot E_{t-1}\left(u_{1,t}^2\right)}} = E_{t-1}\left(u_{1,t} \cdot u_{2,t}\right). \tag{A6}$$

The conditional covariance matrix of returns is defined as

$$H_t = D_t \cdot \rho_t \cdot D_t, \text{ where } D_t = \text{diag}\left\{\sqrt{\sigma_{i,t}^2}\right\}, \tag{A7}$$

where ρ_t is the time-varying conditional correlation matrix

$$E_{t-1}\left(u_t \cdot u_t'\right) = D_t^{-1} \cdot H_t \cdot D_t^{-1} = \rho_t, \text{ where } u_t = D_t^{-1} \cdot r_t. \tag{A8}$$

Engle (2002) introduces a mean-reverting DCC model:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} \cdot q_{jj,t}}}, \tag{A9}$$

$$q_{ij,t} = \bar{\rho}_{ij} + a_1 \cdot \left(u_{i,t-1} \cdot u_{j,t-1} - \bar{\rho}_{ij}\right) + a_2 \cdot \left(q_{ij,t-1} - \bar{\rho}_{ij}\right) \tag{A10}$$

where $\bar{\rho}_{ij}$ is the unconditional correlation between $u_{i,t}$ and $u_{j,t}$. Equation (A10) indicates that the conditional correlation is mean reverting towards $\bar{\rho}_{ij}$ as long as $a_1 + a_2 < 1$.

Engle (2002) assumes that each asset follows a univariate GARCH process and writes the log likelihood function as:

$$\begin{aligned} L &= -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log|H_t| + r_t' H_t^{-1} r_t\right) \\ &= -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log|D_t| + r_t' D_t^{-1} D_t^{-1} r_t - u_t' u_t + \log|\rho_t| + u_t' \rho_t^{-1} u_t\right). \end{aligned} \tag{A11}$$

As shown in Engle (2002), letting the parameters in D_t be denoted by θ and the additional parameters in ρ_t be denoted by φ, equation (A11) can be written as the sum of a volatility part and a correlation part:

$$L(\theta,\varphi) = L_V(\theta) + L_C(\theta,\varphi). \tag{A12}$$

The volatility term is

$$L_V(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log|D_t|^2 + r_t' D_t^{-2} r_t\right), \tag{A13}$$

and the correlation component is

$$L_C(\theta,\varphi) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log|\rho_t| + u_t'\rho_t^{-1}u_t - u_t'u_t\right). \tag{A14}$$

The volatility part of the likelihood is the sum of individual GARCH likelihoods:

$$L_V(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{n}\left(\log(2\pi) + \log\left(\sigma_{i,t}^2\right) + \frac{r_{i,t}^2}{\sigma_{i,t}^2}\right), \tag{A15}$$

which is jointly maximized by separately maximizing each term. The second part of the likelihood is used to estimate the correlation parameters. The two-step approach to maximizing the likelihood is to find

$$\hat{\theta} = \arg\max\{L_V(\theta)\}, \tag{A16}$$

and then take this value as given in the second stage:

$$\hat{\varphi} = \arg\max\{L_C(\hat{\theta},\varphi)\}. \tag{A17}$$

C System of Regression Equations

Consider a system of n equations, of which the typical ith equation is

$$y_i = X_i\beta_i + u_i, \tag{A18}$$

where y_i is a $N \times 1$ vector of time-series observations on the ith dependent variable, X_i is a $N \times k_i$ matrix of observations of k_i independent variables, β_i is a $k_i \times 1$ vector of unknown coefficients to be estimated, and u_i is a $N \times 1$ vector of random disturbance terms with mean zero. Parks (1967) proposes an estimation procedure that allows the error term to be both serially and cross-sectionally correlated. In particular, he assumes that the elements of the disturbance vector u follow an AR(1) process:

$$u_{it} = \rho u_{it-1} + \varepsilon_{it};\ \ \rho_i < 1, \tag{A19}$$

where ε_{it} is serially independently but contemporaneously correlated:

$$Cov\left(\varepsilon_{it}\varepsilon_{jt}\right) = \sigma_{ij},\ \text{ for any } i, j,\ \text{ and } Cov\left(\varepsilon_{it}\varepsilon_{js}\right) = 0,\ \text{ for } s \neq t \tag{A20}$$

Equation (A18) can then be written as

$$y_i = X_i\beta_i + P_i u_i, \tag{A21}$$

with

$$P_i = \begin{bmatrix} \left(1-\rho_i^2\right)^{-1/2} & 0 & 0 & ... & 0 \\ \rho_i\left(1-\rho_i^2\right)^{-1/2} & 1 & 0 & ... & 0 \\ \rho_i^2\left(1-\rho_i^2\right)^{-1/2} & \rho & 1 & ... & 0 \\ . & & & & \\ . & & & & \\ . & & & & \\ \rho_i^{N-1}\left(1-\rho_i^2\right)^{-1/2} & \rho^{N-2} & \rho^{N-3} & ... & 1 \end{bmatrix}. \tag{A22}$$

Under this setup, Parks (1967) presents a consistent and asymptotically efficient three-step estimation technique for the regression coefficients. The first step uses single equation regressions to estimate the parameters of autoregressive model. The second step uses single equation regressions on transformed equations to estimate the contemporaneous covariances. Finally, the Aitken estimator is formed using the estimated covariance,

$$\hat{\beta} = \left(X^T\Omega^{-1}X\right)^{-1} X^T\Omega^{-1}y, \tag{A23}$$

where $\Omega \equiv E[uu^T]$ denotes the general covariance matrix of the innovation. In our application, we use the aforementioned methodology with the slope coefficients restricted to be the same for all equity portfolios and individual stocks. In particular, we use the same three-step procedure and the same covariance assumptions as in equations (A19) to (A22) to estimate the covariances and to generate the t-statistics for the parameter estimates.

In a typical panel data setting of several cross-sectional observations observed for certain period of time, OLS standard errors, which assume spherical variance-covariance matrix of the panel of errors, are inefficient due to both heteroscedasticity and cross-sectional correlations among the error terms. In other words, assuming that the errors in panel regression are cross-sectionally uncorrelated (as in the case of the OLS) can yield standard errors that are biased downwards. This bias is due to the fact that error correlations are often systematically related to the explanatory variables. To solve this problem, in addition to Parks (1967) methodology discussed above, we also used Rogers (1983, 1993) robust standard errors (the so-called *clustered* standard errors) that yield asymptotically correct standard errors for the OLS estimators under a general cross-correlation structure.

Assuming that the errors are independent across cross-sections, Rogers (1983, 1993) write the variance-covariance matrix of the coefficient estimates as $(X'X)^{-1}\sum_{t=1}^{T}[X_t'\Omega_t X_t](X'X)^{-1}$, where X is the panel of explanatory variables, Ω is the covariance matrix of the panel of errors, and X_t and Ω_t denote a single cross-section of explanatory variables and the corresponding error covariance matrix, respectively. Since $X_t'\Omega_t X_t = E[X_t'e_t e_t' X_t]$, Rogers substitutes estimated errors for true errors to get a variance estimator of regression coefficients: $(X'X)^{-1}\sum_{t=1}^{T}[X_t'\hat{e}_t\hat{e}_t X_t](X'X)^{-1}$, where e_t denotes the regression errors and $\hat{e}_t$ is the estimated errors. Rogers shows that the standard errors are consistent in T.

We should note that the results from clustered standard errors are qualitatively very similar to those reported in the paper. However, we chose to present standard errors from Parks methodology because we do not have a large number of cross-sectional observations and we use returns on equity portfolios that are serially correlated. Rogers (1983, 1993) clustered standard errors take into account heteroscedasticity and cross-sectional correlations among the error terms, but do not account for serial correlation. Since monthly equity portfolio returns are serially correlated, we think that in our setting Parks methodology is more appropriate.[3]

[3] Researchers in corporate finance and asset pricing literatures have used different solutions to fix the

D Robustness Check

In this section, we provide a battery of robustness checks.

D.1 Results from the Generalized Conditional Covariance Model

There appears to be some controversy in the econometrics literature around the consistency of QMLE parameter estimates generated by the DCC models.[4] One may wonder if the lack of consistency in the DCC models affects our main findings. To address this potential concern, we use an alternative econometric methodology and estimate the conditional covariances between excess returns on asset i and the market portfolio m based on the generalized conditional covariance (GCC) specification of Bali (2008):[5]

$$\begin{aligned}
&R_{i,t+1} = \alpha_0^i + \alpha_1^i R_{i,t} + \varepsilon_{i,t+1} \\
&R_{m,t+1} = \alpha_0^m + \alpha_1^m R_{m,t} + \varepsilon_{m,t+1} \\
&E_t\left[\varepsilon_{i,t+1}^2\right] \equiv \sigma_{i,t+1}^2 = \beta_0^i + \beta_1^i \varepsilon_{i,t}^2 + \beta_2^i \sigma_{i,t}^2 \\
&E_t\left[\varepsilon_{m,t+1}^2\right] \equiv \sigma_{m,t+1}^2 = \beta_0^m + \beta_1^m \varepsilon_{m,t}^2 + \beta_2^m \sigma_{m,t}^2 \\
&E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \beta_0^{im} + \beta_1^{im} \varepsilon_{i,t}\varepsilon_{m,t} + \beta_2^{im} \sigma_{im,t}
\end{aligned} \tag{A24}$$

where $R_{i,t+1}$ and $R_{m,t+1}$ denote the time $(t+1)$ excess return on asset i and the market portfolio m over a risk-free rate, respectively, and $E_t[\cdot]$ denotes the expectation operator conditional on time t information. In the last equation above, one-month-ahead conditional covariance, $\sigma_{im,t+1}$, is defined as a function of the last month's conditional covariance, $\sigma_{im,t}$, and the product of the last month's unexpected shocks to asset i and the market portfolio m $(\varepsilon_{i,t}\varepsilon_{m,t})$.

We estimate the conditional covariances between the excess return on each equity portfolio i and the innovation in the variance risk premia VRP, $\sigma_{i,VRP}$, using an analogous GCC

problem of correlated residuals across firms or across time in a panel data setting. Petersen (2009) examines different methods used in the two literatures and explains when the different methods yield the same (and correct) standard errors and when they diverge.

[4]See Aielli (2013), Caporin and McAleer (2013), and the proposed solution in Noureldin, Shephard, and Sheppard (2014).

[5]Following the findings of Lee and Hansen (1994), Lumsdaine (1996), and Straumann and Mikosch (2003), Francq and Zakoian (2004) provide consistency and asymptotic normality of the maximum likelihood estimator of the parameters of GCC-type GARCH processes.

model:

$$\begin{array}{l} R_{i,t+1} = \alpha_0^i + \alpha_1^i R_{i,t} + \varepsilon_{i,t+1} \\ VRP_{t+1} = \alpha_0^{VRP} + \alpha_1^{VRP} VRP_t + \varepsilon_{VRP,t+1} \\ E_t\left[\varepsilon_{i,t+1}^2\right] \equiv \sigma_{i,t+1}^2 = \beta_0^i + \beta_1^i \varepsilon_{i,t}^2 + \beta_2^i \sigma_{i,t}^2 \\ E_t\left[\varepsilon_{VRP,t+1}^2\right] \equiv \sigma_{VRP,t+1}^2 = \beta_0^{VRP} + \beta_1^{VRP} \varepsilon_{VRP,t}^2 + \beta_2^{VRP} \sigma_{VRP,t}^2 \\ E_t\left[\varepsilon_{i,t+1}\varepsilon_{VRP,t+1}\right] \equiv \sigma_{i,VRP,t+1} = \beta_0^{i,VRP} + \beta_1^{i,VRP} \varepsilon_{i,t}\varepsilon_{VRP,t} + \beta_2^{i,VRP} \sigma_{i,VRP,t} \end{array} \tag{A25}$$

We estimate the conditional covariances of each equity portfolio with the market portfolio and with the variance risk premia using the maximum likelihood method described in Bali (2008). Once we generate the conditional covariances, we estimate the system of equations given in equations (23)-(24) of the main text using the SUR methodology described in Section C of the internet appendix.

Table II of the internet appendix reports the parameter estimates and the t-statistics of the system of equations for the 10 size, 10 book-to-market, 10 momentum, and 10 industry portfolios (total of 40 portfolios) for the sample period January 1990 to December 2012. As shown in the first two rows of Table II, the risk aversion and the uncertainty aversion coefficients are estimated to be positive and highly significant for the pooled dataset: $A = 2.86$ with a t-statistic of 4.78 and $B = 0.0026$ with a t-statistic of 4.50, indicating a significantly positive market price of risk and uncertainty. Similar to our earlier findings from the DCC model, the Wald_1 and Wald_2 statistics reported in Table II indicate that the two-factor model with risk and uncertainty provides both statistical and economic success in explaining stock market anomalies, except momentum.

D.2 DCC with Asymmetric GARCH

Because the conditional variance and covariance of stock market returns are not observable, different approaches and specifications used in estimating the conditional variance and covariance could lead to different conclusions. We have so far used the bivariate GARCH(1,1) model of Bollerslev (1986) in equations (13)-(14) and (19)-(20) to obtain conditional variance and covariance estimates. In this section, we investigate whether changing these specifications influences our main findings.

The current volatility in the GARCH(1,1) model is defined as a symmetric, linear function of the last period's unexpected news and the last period's volatility. Since, in a symmetric

GARCH process, positive and negative information shocks of the same magnitude produce the same amount of volatility, the symmetric GARCH model cannot cope with the skewness of stock return distribution. If a negative return shock causes more volatility than a positive return shock of the same size, the symmetric GARCH model underpredicts the amount of volatility following negative shocks and overpredicts the amount of volatility following positive shocks. Furthermore, if large return shocks cause more volatility than a quadratic function allows, then the symmetric GARCH model underpredicts volatility after a large return shock and overpredicts volatility after a small return shock.

In this section we use an asymmetric GARCH model of Glosten, Jagannathan, and Runkle (1993) that explicitly takes account of skewed distributions and allows good news and bad news to have different impacts on the conditional volatility forecasts. To test whether such variations in the variance forecasting specification alter our conclusion, we re-estimate the DCC-based conditional covariances using the following alternative specification:

$$\begin{aligned}
&R_{i,t+1} = \alpha_0^i + \alpha_1^i R_{i,t} + \varepsilon_{i,t+1} \\
&R_{m,t+1} = \alpha_0^m + \alpha_1^m R_{m,t} + \varepsilon_{m,t+1} \\
&VRP_{t+1} = \alpha_0^{VRP} + \alpha_1^{VRP} VRP_t + \varepsilon_{VRP,t+1} \\
&E_t\left[\varepsilon_{i,t+1}^2\right] \equiv \sigma_{i,t+1}^2 = \beta_0^i + \beta_1^i \varepsilon_{i,t}^2 + \beta_2^i \sigma_{i,t}^2 + \beta_3^i \varepsilon_{i,t}^2 D_{i,t}^- \\
&E_t\left[\varepsilon_{m,t+1}^2\right] \equiv \sigma_{m,t+1}^2 = \beta_0^m + \beta_1^m \varepsilon_{m,t}^2 + \beta_2^m \sigma_{m,t}^2 + \beta_3^m \varepsilon_{m,t}^2 D_{m,t}^- \\
&E_t\left[\varepsilon_{VRP,t+1}^2\right] \equiv \sigma_{VRP,t+1}^2 = \beta_0^{VRP} + \beta_1^{VRP} \varepsilon_{VRP,t}^2 + \beta_2^{VRP} \sigma_{VRP,t}^2 + \beta_3^{VRP} \varepsilon_{VRP,t}^2 D_{VRP,t}^- \\
&E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1} \\
&E_t\left[\varepsilon_{i,t+1}\varepsilon_{VRP,t+1}\right] \equiv \sigma_{i,VRP,t+1} = \rho_{i,VRP,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{VRP,t+1} \\
&E_t\left[\varepsilon_{m,t+1}\varepsilon_{VRP,t+1}\right] \equiv \sigma_{m,VRP,t+1} = \rho_{m,VRP,t+1} \cdot \sigma_{m,t+1} \cdot \sigma_{VRP,t+1}
\end{aligned} \tag{A26}$$

where $D_{i,t}^-$, $D_{m,t}^-$, and $D_{VRP,t}^-$ are indicator functions that equals one when $\varepsilon_{i,t+1}$, $\varepsilon_{m,t+1}$, and $\varepsilon_{VRP,t+1}$ are negative and zero otherwise. The indicator function generates an asymmetric GARCH effect between positive and negative shocks. $\rho_{im,t+1}$, $\rho_{i,VRP,t+1}$, and $\rho_{m,VRP,t+1}$ are the time-t expected conditional correlations estimated using the mean-reverting DCC model of Engle (2002).

A notable point in Table III is that the main findings from an asymmetric GARCH specification of the conditional covariances are very similar to those reported in Table 1. Specifically, the risk aversion coefficients are estimated to be positive and highly significant for all equity portfolios; A is in the range of 2.53 to 3.54 with the t-statistics ranging from 2.58 to 3.11, implying a significantly positive link between expected return and risk. Similar to our results from GARCH(1,1) specification, asymmetric GARCH model of Glosten, Jagannathan, and Runkle (1993) yields positive and significant coefficient estimates on the covariance between equity portfolios and the variance risk premia. Specifically, the uncertainty aversion coefficients (B) are in the range of 0.0054 to 0.0075 with the t-statistics between 2.68 and 3.30. These results show that equity portfolios that are highly correlated with uncertainty (proxied by VRP) carry a significant premium relative to portfolios that are uncorrelated or lowly correlated with VRP.

With this alternative covariance specification, we also examine the empirical validity of the conditional asset pricing model by testing the joint hypothesis. As shown in Table III, the Wald_1 statistics for the size, book-to-market, and industry portfolios are, respectively, 16.91, 7.89, and 14.41 with the corresponding p-values of 0.11, 0.72, and 0.21. The significantly positive risk and uncertainty aversion coefficients and the insignificant Wald_1 statistics indicate that the two-factor model explains the time-series and cross-sectional variation in equity portfolios. Finally, we investigate whether the model with asymmetric GARCH specification explains the return spreads between Small and Big; Value and Growth; and HiTec and Telcm portfolios. The last row in Table III reports Wald_2 statistics from testing the equality of conditional alphas for high-return and low-return portfolios ($H_0 : \alpha_1 = \alpha_{10}$). For the size, book-to-market, and industry portfolios, the Wald_2 statistics provide no evidence for a significant conditional alpha for "Small-Big", "Value-Growth", and "HiTec-Telcm" arbitrage portfolios. Overall, the DCC-based conditional covariances from the asymmetric GARCH model captures the time-series and cross-sectional variation in returns on size, book-to-market, and industry portfolios and generates significantly positive risk-return and uncertainty-return tradeoffs.

D.3 Results from Larger Cross-Section of Industry Portfolios

Given the positive risk-return and positive uncertainty-return coefficient estimates from the three data sets and the success of the conditional asset pricing model in explaining the industry, size, and value premia, we now examine how the model performs when we use a larger cross-section of equity portfolios.

The robustness of our findings is investigated using the monthly excess returns on the value-weighted 17-, 30-, 38-, 48-, and 49-industry portfolios. Table IV reports the common slope estimates (A, B), their t-statistics in parentheses, and the Wald_1 and Wald_2 statistics along with their p-values in square brackets. For the industry portfolios, the risk aversion coefficients (A) are estimated to be positive, in the range of 2.20 to 2.78, and highly significant with the t-statistics ranging from 2.31 to 3.34. Consistent with our earlier findings from the 10 size, 10 book-to-market, and 10 industry portfolios, the results from the larger cross-section of industry portfolios (17 to 49) imply a positive and significant relation between expected return and market risk. Again similar to our findings from 10 decile portfolios, the uncertainty aversion coefficients are estimated to be positive, in the range of 0.0036 to 0.0041, and highly significant with the t-statistics ranging from 2.44 to 4.21. These results provide evidence for a significantly positive market price of uncertainty and show that assets with higher correlation with the variance risk premia generate higher returns next month.

Not surprisingly, the Wald_1 statistics for all industry portfolios have p-values in the range of 0.20 to 0.75, indicating that the two-factor asset pricing model explains the time-series and cross-sectional variation in larger number of equity portfolios. The last row shows that the Wald_2 statistics from testing the equality of conditional alphas on the high-return and low-return industry portfolios have p-values ranging from 0.44 to 0.80, implying that there is no significant risk-adjusted return difference between the extreme portfolios of 17, 30, 38, 48, and 49 industries. The differences in conditional alphas are both economically and statistically insignificant, showing that the two-factor model introduced in the paper provides success in explaining industry effects.

D.4 Controlling for Macroeconomic Variables

A series of papers argue that the stock market can be predicted by financial and/or macroeconomic variables associated with business cycle fluctuations. The commonly chosen variables include default spread (DEF), term spread (TERM), dividend price ratio (DIV), and the de-trended riskless rate or the relative T-bill rate (RREL).[6] We define DEF as the difference between the yields on BAA- and AAA-rated corporate bonds, and TERM as the difference between the yields on the 10-year Treasury bond and the 3-month Treasury bill. RREL is defined as the difference between 3-month T-bill rate and its 12-month backward moving average.[7] We obtain the aggregate dividend yield using the CRSP value-weighted index return with and without dividends based on the formula given in Fama and French (1988). In addition to these financial variables, we use some fundamental variables affecting the state of the U.S. economy: Monthly inflation rate based on the U.S. Consumer Price Index (INF); Monthly growth rate of the U.S. industrial production (IP) obtained from the G.17 database of the Federal Reserve Board; and Monthly US unemployment rate (UNEMP) obtained from the Bureau of Labor Statistics.

According to Merton's (1973) ICAPM, state variables that are correlated with changes in consumption and investment opportunities are priced in capital markets in the sense that an asset's covariance with those state variables affects its expected returns. Merton (1973) also indicates that securities affected by such state variables (or systematic risk factors) should earn risk premia in a risk-averse economy. Macroeconomic variables used in the literature are excellent candidates for these systematic risk factors because innovations in macroeconomic variables can generate global impact on firm's fundamentals, such as their cash flows, risk-adjusted discount factors, and/or investment opportunities. Following the existing literature, we use the aforementioned financial and macroeconomic variables as proxies for state variables capturing shifts in the investment opportunity set.

We now investigate whether incorporating these variables into the predictive regressions affects the significance of the market prices of risk and uncertainty. Specifically, we estimate the portfolio-specific intercepts and the common slope coefficients from the following panel

[6]See, e.g., Campbell (1987), Fama and French (1989), and Ferson and Harvey (1991) who test the predictive power of these variables for expected stock returns.

[7]The monthly data on 10-year T-bond yields, 3-month T-bill rates, BAA- and AAA-rated corporate bond yields are available from the Federal Reserve statistics release website.

regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) + \lambda \cdot X_t + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right) + \lambda \cdot X_t + \varepsilon_{m,t+1}$$

where X_t denotes a vector of lagged control variables; default spread (DEF), term spread (TERM), relative T-bill rate (RREL), aggregate dividend yield (DIV), inflation rate (INF), growth rate of industrial production (IP), and unemployment rate (UNEMP). The common slope coefficients (A, B, and λ) and their t-statistics are estimated using the monthly excess returns on the market portfolio and the ten size, book-to-market, and industry portfolios.

As presented in Table V, after controlling for a wide variety of financial and macroeconomic variables, our main findings remain intact for all equity portfolios. The common slope estimates on the conditional covariances of equity portfolios with the market factor (A) remain positive and highly significant, indicating a positive and significant relation between expected return and market risk. Similar to our earlier findings, the common slopes on the conditional covariances of equity portfolios with the uncertainty factor (B) remain significantly positive as well, showing that assets with higher correlation with the variance risk premium generate higher returns next month. Among the control variables, the growth rate of industrial production is the only variable predicting future returns on equity portfolios; λ_{IP} turns out to be positive and significant—especially for the industry portfolios. The positive relation between expected stock returns and innovations in output makes economic sense. Increases in real economic activity (proxied by the growth rate of industrial production) increase investors' expectations of future growth. Overall, the results in Table V indicate that after controlling for variables associated with business conditions, the time-varying exposures of equity portfolios to the market and uncertainty factors carry positive risk premiums.[8]

[8] We also used "expected business conditions" variable of Campbell and Diebold (2009) and our main findings remain intact for all equity portfolios. To save space, we do not report these results in the paper. They are available upon request.

D.5 Results from Individual Stocks

We have so far investigated the significance of risk, uncertainty, and return tradeoffs using equity portfolios. In this section, we replicate our analyses using individual stocks trading at NYSE, AMEX, and NASDAQ. First, we generate a dataset for the largest 500 common stocks (share code = 10 or 11) traded at NYSE/AMEX/NASDAQ. Following Shumway (1997), we adjust for stock de-listing to avoid survivorship bias.[9] Firms with missing observations on beginning-of-month market cap or monthly returns over the period January 1990 – December 2010 are eliminated. Due to the fact that the list of 500 firms changes over time as a result of changes in firms' market capitalizations, we obtain more than 500 firms over the period 1990-2010. Specifically, the largest 500 firms are determined based on their end-of-month market cap as of the end of each month from January 1990 to December 2010. There are 738 unique firms in our first dataset. In our second dataset, the largest 500 firms are determined based on their market cap at the end of December 2010. Our last dataset contains stocks in the S&P 500 index. Since the stock composition of the S&P 500 index changes through time, we rely on the most recent sample (as of December 2010). We also restrict our S&P 500 sample to 318 stocks with non-missing monthly return observations for the period January 1990 – December 2010.

Table VI presents the common slope estimates (A, B) and their t-statistics for the individual stocks in the aforementioned data sets. The risk aversion coefficient is estimated to be positive and highly significant for all stock samples considered in the paper: $A = 6.42$ with the t-statistic of 8.04 for the first dataset containing 738 stocks (largest 500 stocks as of the end of each month from January 1990 to December 2010); $A = 6.80$ with the t-statistic of 8.70 for the second dataset containing largest 500 stocks as of the end of December 2010; and $A = 6.02$ with the t-statistic of 6.79 for the last dataset containing 318 stocks with non-missing monthly return observations for the period 1990-2010. Confirming our findings

[9]Specifically, the last return on an individual stock used is either the last return available on CRSP, or the de-listing return, if available. Otherwise, a de-listing return of -100% is included in the study, except that the deletion reason is coded as 500 (reason unavailable), 520 (went to OTC), 551-573, 580 (various reason), 574 (bankruptcy), and 584 (does not meet exchange financial guidelines). For these observations, a return of -30% is assigned.

from equity portfolios, the results from individual stocks imply a positive and significant relation between expected return and market risk. Similarly, consistent with our earlier findings from equity portfolios, the uncertainty aversion coefficient is also estimated to be positive and highly significant for all data sets: $B = 0.0043$ with the t-statistic of 3.61 for the first dataset, $B = 0.0044$ with the t-statistic of 3.67 for the second dataset, and $B = 0.0046$ with the t-statistic of 3.52 for the last dataset. These results indicate a significantly positive market price of uncertainty for large stocks trading in the U.S. stock market.

D.6 Controlling for Market Illiquidity and Default Risk

Elevated variance risk premia during economic recessions and market downturns often correspond to the periods in which market illiquidity and default risk are both higher. Thus, it is natural to think that the conditional covariances of equity portfolios with market illiquidity and credit risk factors are positively linked to expected returns. In this section, we test whether the covariances with VRP_{t+1}^{shock} could be picking up covariances with illiquidity and default risk.

Following Amihud (2002), we measure market illiquidity in a month as the average daily ratio of the absolute market return to the dollar trading volume within the month:

$$ILLIQ_t = \frac{1}{n}\sum_{d=1}^{n}\frac{|R_{m,d}|}{VOLD_m}$$

where $R_{m,d}$ and $VOLD_{m,d}$ are, respectively, the daily return and daily dollar trading volume for the S&P 500 index on day d, and n is the number of trading days in month t.

First, we generate the DCC-based conditional covariances of portfolio returns with market illiquidity and then estimate the common slope coefficients (A, B_1, B_2) from the following panel regressions:

$$\begin{aligned} R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) \\ &\quad + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right) + \varepsilon_{i,t+1} \\ R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) \\ &\quad + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right) + \varepsilon_{m,t+1} \end{aligned}$$

where $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ and $Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right)$ are the time-t expected conditional covariance between the change in market illiquidity and the excess return on portfolio i and market portfolio m, respectively.

Table VII, Panel A, presents the common slope coefficients and their t-statistics estimated using the monthly excess returns on the market portfolio and the 10 size, book-to-market, and industry portfolios. The slope on $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ is found to be positive but statistically insignificant for all equity portfolios considered in the paper. A notable point in Table VII is that the slopes on $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ and $Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right)$ remain positive and highly significant after controlling for the covariances of equity portfolios with market illiquidity.

Next, we test whether the variance risk premium is proxying for default or credit risk. We use the TED spread as an indicator of credit risk and the perceived health of the banking system. The TED spread is the difference between the interest rates on interbank loans and short-term U.S. government debt (T-bills). TED is an acronym formed from T-Bill and ED, the ticker symbol for the Eurodollar futures contract.[10] The size of the spread is usually denominated in basis points (bps). For example, if the T-bill rate is 5.10% and ED trades at 5.50%, the TED spread is 40 bps. The TED spread fluctuates over time but generally has remained within the range of 10 and 50 bps (0.1% and 0.5%) except in times of financial crisis. A rising TED spread often presages a downturn in the U.S. stock market, as it indicates that liquidity is being withdrawn. The TED spread is an indicator of perceived credit risk in the general economy. This is because T-bills are considered risk-free while LIBOR reflects the credit risk of lending to commercial banks. When the TED spread increases, that is a sign that lenders believe the risk of default on interbank loans (also known as counterparty risk) is increasing. Interbank lenders therefore demand a higher rate of interest, or accept lower returns on safe investments such as T-bills. When the risk of bank defaults is considered to be decreasing, the TED spread decreases.

[10] Initially, the TED spread was the difference between the interest rates for three-month U.S. Treasuries contracts and the three-month Eurodollars contract as represented by the London Interbank Offered Rate (LIBOR). However, since the Chicago Mercantile Exchange dropped T-bill futures, the TED spread is now calculated as the difference between the three-month T-bill interest rate and three-month LIBOR.

We first estimate the DCC-based conditional covariances of portfolio returns with the TED spread and then estimate the common slope coefficients from the following SUR regressions:

$$
\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{m,t+1}
\end{aligned}
$$

where $Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right)$ and $Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right)$ are the time-t expected conditional covariance between the changes in TED spread and the excess returns on portfolio i and market portfolio m, respectively.

Table VII, Panel A, shows the common slope coefficients and their t-statistics estimated using the monthly excess returns on the market portfolio and the size, book-to-market, and industry portfolios. The slope on $Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right)$ is found to be positive for the size and book-to-market portfolios, and negative for the industry portfolios. Aside from yielding an inconsistent predictive relation with future returns, the slopes on the conditional covariances with the change in TED spread are statistically insignificant for all equity portfolios. Similar to our earlier findings, the slopes on the conditional covariances with the market risk and uncertainty factors remain positive and highly significant after controlling for the covariances with default risk.

Finally, we investigate the significance of risk and uncertainty coefficients after controlling for liquidity and credit spread simultaneously:

$$
\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{m,t+1}
\end{aligned}
$$

As shown in Panel A of Table VII, for the extended specification above, the common slope coefficient, B_2 on $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ is estimated to be positive and marginally significant for the book-to-market and industry portfolios, whereas B_2 is insignificant for the size portfolios. The covariances of equity portfolios with the change in TED spread do not predict future returns as B_3 is insignificant for all equity portfolios. Controlling for the market illiquidity and credit risk does not affect our main findings: the market risk-return and uncertainty-return coefficients (A and B_1) are both positive and highly significant for all equity portfolios. Equity portfolios that are highly correlated with VRP_{t+1}^{shock} carry a significant premium relative to portfolios that are uncorrelated or minimally correlated with VRP_{t+1}^{shock}.

We have so far provided evidence from the individual equity portfolios (10 size, 10 book-to-market, and 10 industry portfolios). We now investigate whether our main findings remain intact if we use a joint estimation with all test assets simultaneously (total of 30 portfolios). Panel B of Table VII reports the parameter estimates and the t-statistics that are adjusted for heteroskedasticity and autocorrelation for each series and the cross-correlations among the error terms. As shown in the first row of Panel B, the risk aversion coefficient is estimated to be positive and highly significant for the pooled dataset: $A = 2.31$ with the t-statistic of 2.64, implying a positive and significant relation between expected return and market risk. Similar to our earlier findings, the uncertainty aversion coefficient is also estimated to be positive and highly significant for the joint estimation: $B = 0.0053$ with the t-statistic of 3.72. These results indicate a significantly positive market price of uncertainty when all portfolios are combined together. Equity portfolios with higher sensitivity to increases in VRP are expected to generate higher returns next period.

The last three rows in Panel B of Table VII provide evidence for a positive and marginally significant relation between $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ and future returns, indicating that the conditional covariances of equity portfolios with the market illiquidity are positively linked to expected returns. However, the insignificant relation between $Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right)$ and portfolio returns remains intact for the joint estimation as well. A notable point in Panel B is that controlling for the market illiquidity and default risk individually and simultaneously does not influence the significant predictive power of the conditional covariances of portfolio returns with the market risk and VRP factors.

D.7 Relative Performance of the Conditional Asset Pricing Model with Risk and Uncertainty

We now assess the relative performance of the newly proposed model in predicting the cross-section of expected returns on equity portfolios. Specifically, we test whether the conditional asset pricing model with the market and uncertainty factors outperforms the conditional CAPM with the market factor in terms of statistical fit. The goodness of fit of an asset pricing model describes how well it fits a set of realized return observations. Measures of goodness of fit typically summarize the discrepancy between observed values and the values expected under the model in question. Hence, we focus on the cross-section of realized average returns on equity portfolios (as a benchmark) and the portfolios' expected returns implied by the two competing models.

Using equation (23), we compute the expected excess return on equity portfolios based on the estimated prices of risk and uncertainty (A, B) and the sample averages of the conditional covariance measures, $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ and $Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right)$:

$$E_t\left[R_{i,t+1}\right] = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right). \tag{A27}$$

Table VIII of the online appendix presents the realized monthly average excess returns on the size, book-to-market, and industry portfolios and the cross-section of expected excess returns generated by the Conditional CAPM and the two-factor conditional asset pricing models. Clearly the newly proposed model with risk and uncertainty provides much more accurate estimates of expected returns on the size, book-to-market, and industry portfolios. Especially for the size and industry portfolios, expected returns implied by the two-factor model with the market and VRP factors are almost identical to the realized average returns. The last row in Table VIII reports the Mean Absolute Percentage Errors (MAPE) for the two competing models:

$$MAPE = \frac{|\text{Realized} - \text{Expected}|}{\text{Expected}}, \tag{A28}$$

where "Realized" is the realized monthly average excess return on each equity portfolio and "Expected" is the expected excess return implied by equation (A27). For the conditional

CAPM with the market factor, MAPE equals 5.20% for the size portfolios, 5.37% for the book-to-market portfolios, and 6.32% for the industry portfolios. Accounting for the variance risk premium improves the cross-sectional fitting significantly: MAPE reduces to 0.61% for the size portfolios, 1.66% for the book-to-market portfolios, and 0.55% for the industry portfolios.

Figure 1 of the internet appendix provides a visual depiction of the realized and expected returns for the size, book-to-market, and industry portfolios. It is clear that the two-factor model with uncertainty nails down the realized returns of the size, book-to-market, and industrial portfolios, while the conditional CAPM systematically over-predicts these portfolio returns. Overall, the results indicate superior performance of the conditional asset pricing model introduced in the paper.

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Table I Monthly Raw Returns and CAPM Alphas for the Long-Short Equity Portfolios

This table presents the monthly raw return and CAPM Alpha differences between high-return (long) and low-return (short) equity portfolios. The results are reported for the size, book-to-market (BM), and industry portfolios for the sample periods July 1926 – December 2012, July 1963 – December 2012, and January 1990 – December 2012. The OLS t-statistics are reported in parentheses. The Newey-West t-statistics are given in square brackets.

	July 1926 – December 2012				July 1963 – December 2012				January 1990 – December 2012		
Portfolio	Long-Short	Return Diff.	Alpha	Portfolio	Long-Short	Return Diff.	Alpha	Portfolio	Long-Short	Return Diff.	Alpha
10 Size	Small-Big	0.0056	0.0025	10 Size	Small-Big	0.0033	0.0025	10 Size	Small-Big	0.0031	0.0025
		(2.39)	(1.11)			(1.67)	(1.25)			(1.02)	(0.84)
		[2.46]	[1.28]			[1.33]	[1.00]			[0.99]	[0.80]
10 BM	Value-Growth	0.0053	0.0025	10 BM	Value-Growth	0.0051	0.0050	10 BM	Value-Growth	0.0023	0.0021
		(2.62)	(1.32)			(2.63)	(2.61)			(0.77)	(0.69)
		[2.67]	[1.28]			[2.29]	[2.17]			[0.69]	[0.54]
10 MOM	Winner-Loser	0.0120	0.0153	10 MOM	Winner-Loser	0.0136	0.0147	10 MOM	Winner-Loser	0.0105	0.0133
		(4.83)	(6.53)			(4.71)	(5.15)			(2.05)	(2.67)
		[5.01]	[7.80]			[4.26]	[5.28]			[1.91]	[2.82]
10 Industry	Durbl-Telcm	0.0024	-0.0013	10 Industry	Enrgy-Utils	0.0025	0.0013	10 Industry	HiTech-Telcm	0.0045	0.0020
		(1.27)	(-0.81)			(1.35)	(0.71)			(1.35)	(0.63)
		[1.26]	[-0.78]			[1.41]	[0.71]			[1.49]	[0.60]

Table II Results from the Generalized Conditional Covariance

This table reports the portfolio-specific intercepts and the common slope estimates from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{m,t+1}$$

where the conditional variance of the market and the conditional covariances are estimated with the generalized conditional covariance (GCC) specification of Bali (2008). The parameters in the panel regression and their t-statistics are estimated using monthly excess returns on the market portfolio and the pooled datasets of ten decile size, book-to-market, momentum, and industry portfolios (total of 40 equity portfolios) for the sample period from January 1990 to December 2012. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. Table show the common slope coefficients (A and B), the Wald_1 statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald_2 statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; Winner vs. Loser; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

	A	2.8562
		(4.78)
	B	0.0026
		(4.50)
Size	Wald_1	9.22
		[51.11%]
Small vs. Big	Wald_2	0.88
		[34.85%]
Book-to-Market	Wald_1	4.46
		[92.43%]
Value vs. Growth	Wald_2	0.78
		[37.60%]
Momentum	Wald_1	19.67
		[3.25%]
Winner vs. Loser	Wald_2	5.35
		[2.07%]
Industry	Wald_1	11.39
		[32.80%]
HiTec vs. Telcm	Wald_2	0.33
		[56.38%]

Table III Results from Asymmetric GARCH Model

This table reports the portfolio-specific intercepts and the common slope estimates from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right) + \varepsilon_{m,t+1}$$

where the conditional variance and covariances are estimated using the asymmetric GARCH model of Glosten, Jagannathan, and Runkle (1993). The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the ten decile size, book-to-market, and industry portfolios for the sample period from January 1990 to December 2010. The alphas (α_i) are reported for each equity portfolio and the t-statistics are presented in parentheses. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The last four rows, respectively, show the common slope coefficients (A and B), the Wald_1 statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald_2 statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

Size	α_i, α_m	*BM*	α_i, α_m	*Industry*	α_i, α_m
Small	0.0052	Growth	0.0035	NoDur	0.0051
	(1.23)		(0.87)		(1.94)
2	0.0037	2	0.0047	Durbl	0.0028
	(0.85)		(1.35)		(0.57)
3	0.0040	3	0.0052	Manuf	0.0055
	(0.99)		(1.55)		(1.61)
4	0.0030	4	0.0064	Enrgy	0.0064
	(0.75)		(1.85)		(1.85)
5	0.0038	5	0.0056	HiTec	0.0029
	(0.97)		(1.71)		(0.52)
6	0.0037	6	0.0050	Telcm	0.0004
	(1.05)		(1.48)		(0.11)
7	0.0041	7	0.0057	Shops	0.0036
	(1.19)		(1.76)		(1.04)
8	0.0034	8	0.0058	Hlth	0.0043
	(0.97)		(1.74)		(1.37)
9	0.0036	9	0.0066	Utils	0.0042
	(1.11)		(1.92)		(1.58)
Big	0.0012	Value	0.0081	Other	0.0030
	(0.38)		(1.88)		(0.81)
Market	0.0018	Market	0.0033	Market	0.0028
	(0.57)		(1.20)		(0.82)
A	3.2927	A	2.5303	A	3.5369
	(3.11)		(2.62)		(2.58)
B	0.0054	B	0.0060	B	0.0075
	(3.12)		(2.68)		(3.30)
Wald_1	16.91	Wald_1	7.89	Wald_1	14.41
	[0.11]		[0.72]		[0.21]
Wald_2	1.48	Wald_2	1.99	Wald_2	0.46
	[0.22]		[0.16]		[0.50]

Table IV Results from Larger Cross-Section of Industry Portfolios

This table presents the common slope estimates (A, B) from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{i,t+1}$$

$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right) + \varepsilon_{m,t+1}$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP_{t+1}^{\text{shock}}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the shock to the variance risk premia (VRP_{t+1}^{shock}), $Cov_t\left(R_{m,t+1}, VRP_{t+1}^{\text{shock}}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and VRP_{t+1}^{shock}, and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the 17, 30, 38, 48, and 49 industry portfolios for the sample period from January 1990 to December 2010. The alphas (α_i) are reported for each equity portfolio and the t-statistics are presented in parentheses. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The last four rows, respectively, show the common slope coefficients (A and B), the Wald_1 statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald_2 statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

17-industry portfolios		30-industry portfolios		38-industry portfolios		48-industry portfolios		49-industry portfolios	
A	2.6399	A	2.1975	A	2.2988	A	2.3271	A	2.7840
	(2.31)		(2.52)		(2.47)		(2.97)		(3.34)
B	0.0041	B	0.0036	B	0.0035	B	0.0041	B	0.0041
	(2.44)		(2.98)		(2.45)		(3.47)		(4.21)
Wald_1	16.41	Wald_1	35.11	Wald_1	30.89	Wald_1	57.20	Wald_1	52.04
	[0.56]		[0.28]		[0.75]		[0.20]		[0.39]
Wald_2	0.58	Wald_2	0.06	Wald_2	0.32	Wald_2	0.53	Wald_2	0.13
	[0.44]		[0.80]		[0.57]		[0.47]		[0.72]

Table V Controlling for Macroeconomic Variables

This table presents the common slope estimates from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) + \lambda \cdot X_t + \varepsilon_{i,t+1}$$
$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right) + \lambda \cdot X_t + \varepsilon_{m,t+1}$$

where X_t denotes a vector of lagged control variables; default spread (DEF), term spread (TERM), relative T-bill rate (RREL), aggregate dividend yield (DIV), inflation rate (INF), growth rate of industrial production (IP), and unemployment rate (UNEMP). The common slope coefficients (A, B, and λ) and their t-statistics are estimated using the monthly excess returns on the market portfolio and the ten size, book-to-market, and industry portfolios for the sample period January 1990 to December 2010. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The last two rows the Wald_1 statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald_2 statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

	Size	Book-to-Market	Industry
A	4.2630	2.5763	4.0421
	(3.32)	(2.40)	(2.74)
B	0.0057	0.0051	0.0066
	(2.85)	(2.25)	(2.96)
λ_{DEF}	-0.3804	-0.0739	0.6243
	(-0.50)	(-0.09)	(1.02)
λ_{TERM}	-0.1964	-0.5366	-0.5405
	(-0.64)	(-1.69)	(-2.17)
λ_{RREL}	0.2330	0.1834	0.0104
	(0.68)	(0.52)	(0.04)
λ_{DIV}	0.0489	0.0228	0.0314
	(1.33)	(0.60)	(1.05)
λ_{INF}	0.0270	0.7158	-0.1862
	(0.04)	(0.93)	(-0.31)
λ_{IP}	0.7433	0.8689	1.1941
	(1.77)	(2.01)	(3.51)
λ_{UNEMP}	0.0031	0.0047	0.0026
	(1.13)	(1.61)	(1.15)
Wald_1	16.96	7.97	14.78
	[0.11]	[0.72]	[0.19]
Wald_2	1.46	1.63	0.67
	[0.23]	[0.20]	[0.41]

Table VI Results from Individual Stocks

This table presents the common slope estimates (A, B) from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) + \varepsilon_{i,t+1}$$

$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right) + \varepsilon_{m,t+1}$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the shock to the variance risk premia VRP^{shock}_{t+1}, $Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and the VRP^{shock}_{t+1}, and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the largest 500 stocks trading at NYSE, AMEX, and NASDAQ, and 318 stocks in the S&P 500 index for the sample period from January 1990 to December 2010. First, the largest 500 firms is determined based on their end-of-month market cap as of the end of each month from January 1990 to December 2010. Due to the fact that the list of 500 firms changes over time as a result of changes in firms' market capitalizations, there are 738 unique firms in our first dataset. In our second dataset, the largest 500 firms is determined based on their market cap at the end of December 2010. Our last dataset contains stocks in the S&P 500 index. Since the stock composition of the S&P 500 index changes through time, we rely on the most recent sample. We also restrict our S&P 500 sample to 318 stocks with non-missing monthly return observations for the period January 1990 – December 2010. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios.

Largest 500 Stocks		*Largest 500 Stocks*		*Largest 500 Stocks*	
end-of-month		*as of December 2010*		*S&P 500 Index*	
A	6.4237	A	6.8014	A	6.0243
	(8.04)		(8.70)		(6.79)
B	0.0043	B	0.0044	B	0.0046
	(3.61)		(3.67)		(3.52)

Table VII Controlling for Market Illiquidity and Default Risk

This table presents the common slope estimates (A, B_1, B_2, B_3) from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right) + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{i,t+1}$$

$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP^{\text{shock}}_{t+1}\right) + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{m,t+1}$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP^{\text{shock}}_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the shock to the variance risk premia (VRP^{shock}_{t+1}), $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the change in market illiquidity ($\Delta ILLIQ_{t+1}$), $Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the change in TED spread (ΔTED_{t+1}), and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. In Panel A, the parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the 10 decile size, book-to-market, and industry portfolios for the sample period from January 1990 to December 2010. In Panel B, the results are generated using a joint estimation with all test assets simultaneously (total of 30 portfolios). The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and the cross-correlations among the portfolios.

Panel A. Results from 10 Equity Portfolios

10 Equity Portfolios	A	B_1	B_2	B_3
Size	6.2227	0.0069	1.2423	
	(2.47)	(3.07)	(1.29)	
Size	3.6465	0.0052		0.6372
	(2.84)	(2.09)		(0.91)
Size	5.7826	0.0057	0.4347	1.1582
	(2.48)	(2.12)	(0.69)	(1.17)
Book-to-Market	5.3065	0.0062	2.2003	
	(2.66)	(2.65)	(1.34)	
Book-to-Market	2.5695	0.0056		0.3148
	(2.24)	(2.37)		(0.54)
Book-to-Market	6.4767	0.0079	2.8237	0.3247
	(2.13)	(2.90)	(1.69)	(0.61)
Industry	7.8266	0.0080	2.5677	
	(2.35)	(3.16)	(1.52)	
Industry	3.1868	0.0071		-0.7625
	(2.17)	(2.88)		(-1.11)
Industry	9.2805	0.0102	3.5064	-1.0014
	(2.69)	(3.49)	(1.99)	(-1.43)

Table VII (continued)

Panel B. Results from 30 Equity Portfolios

A	B_1	B_2	B_3
2.3110	0.0053		
(2.64)	(3.72)		
3.2552	0.0060	0.6796	
(2.82)	(4.03)	(1.94)	
2.1153	0.0055		-0.0477
(2.41)	(3.49)		(-0.11)
3.0967	0.0062	0.6497	-0.0844
(2.72)	(3.78)	(1.95)	(-0.20)

Table VIII Relative Performance of the Two-Factor Model with VRP

This table presents the realized monthly average excess returns on the size, book-to-market, and industry portfolios and the cross-section of expected excess returns generated by the conditional CAPM with the market factor and the two-factor conditional asset pricing model with the market and VRP factors. The last row reports the Mean Absolute Percentage Errors (MAPE) for the two competing models.

	Realized Return Benchmark	**Two-Factor Model with VRP**	**Conditional CAPM**
Size	Average Excess Returns	Expected Excess Returns	Expected Excess Returns
Small	0.8464%	0.8461%	0.8742%
2	0.7737%	0.7677%	0.8110%
3	0.7690%	0.7647%	0.8093%
4	0.6632%	0.6637%	0.7032%
5	0.7525%	0.7550%	0.7943%
6	0.7055%	0.7025%	0.7406%
7	0.7409%	0.7379%	0.7749%
8	0.6837%	0.6810%	0.7221%
9	0.6670%	0.6643%	0.7000%
Big	0.4479%	0.4598%	0.4789%
MAPE		**0.61%**	**5.20%**

	Realized Return Benchmark	**Two-Factor Model with VRP**	**Conditional CAPM**
Book-to-Market	Average Excess Returns	Expected Excess Returns	Expected Excess Returns
Growth	0.5286%	0.5327%	0.5645%
2	0.5614%	0.5658%	0.5961%
3	0.6140%	0.6039%	0.6488%
4	0.6752%	0.6559%	0.6960%
5	0.6119%	0.6017%	0.6423%
6	0.5439%	0.5547%	0.5803%
7	0.6014%	0.5979%	0.6360%
8	0.5885%	0.5956%	0.6233%
9	0.6827%	0.6666%	0.7133%
Value	0.8221%	0.7994%	0.8564%
MAPE		**1.66%**	**5.37%**

	Realized Return Benchmark	**Two-Factor Model with VRP**	**Conditional CAPM**
Industry	Average Excess Returns	Expected Excess Returns	Expected Excess Returns
Telcm	0.2727%	0.2747%	0.3280%
Utils	0.4712%	0.4727%	0.4965%
Other	0.4965%	0.4910%	0.5366%
Durbl	0.5313%	0.5315%	0.5513%
Shops	0.5954%	0.5912%	0.6247%
Hlth	0.6138%	0.6088%	0.6478%
NoDur	0.6110%	0.6152%	0.6534%
Manuf	0.7172%	0.7206%	0.7474%
Enrgy	0.7606%	0.7643%	0.7824%
HiTec	0.8358%	0.8350%	0.8466%
MAPE		**0.55%**	**6.32%**

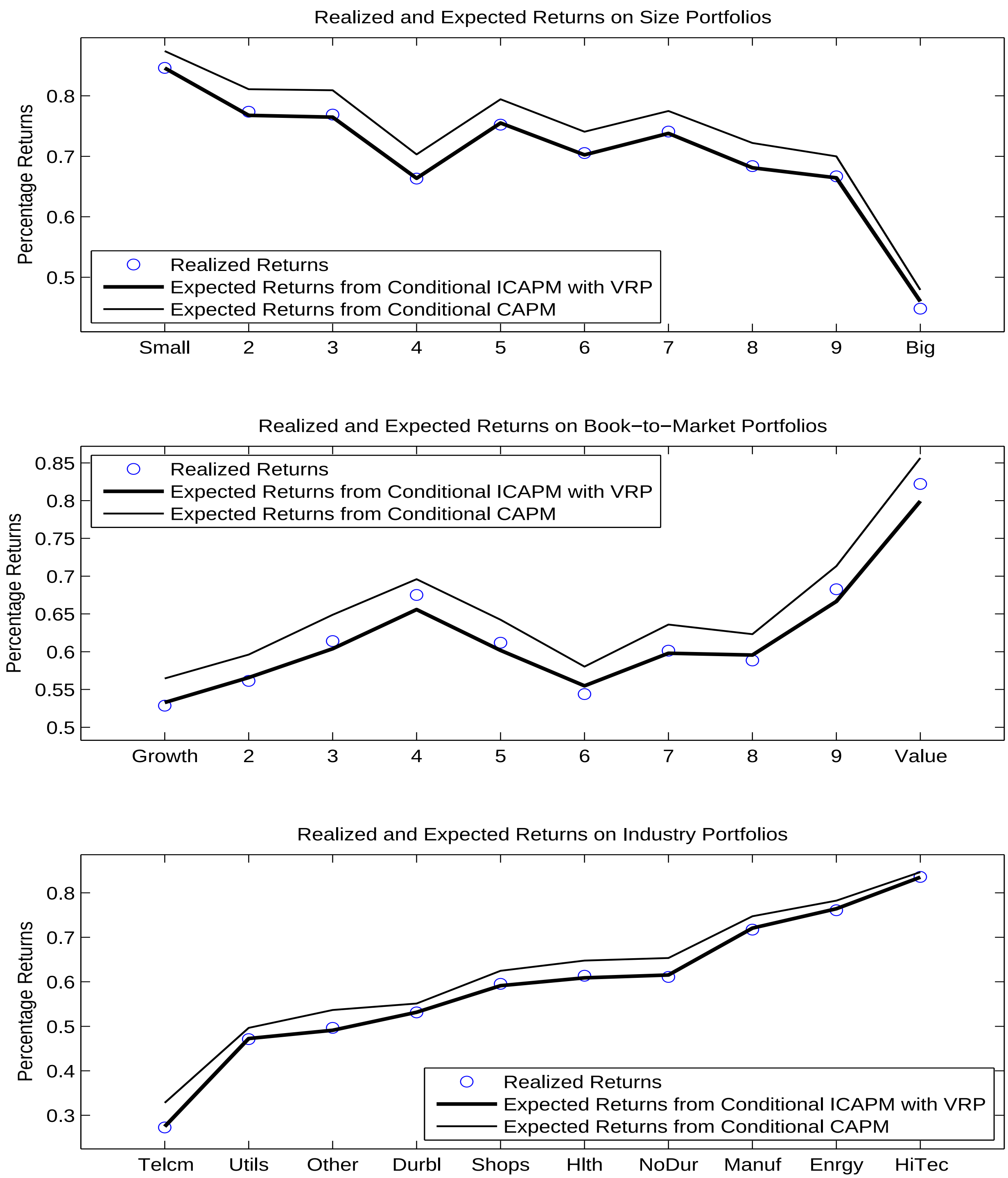

Figure 1 Relative Performance of the Conditional ICAPM with Uncertainty

This figure plots the realized monthly average excess returns on the size (top panel), book-to-market (middle panel), and industry portfolios (bottom panel) and the cross-section of expected excess returns generated by the Conditional CAPM with the market factor and the Conditional ICAPM with the market and VRP factors. The results indicate superior performance of the conditional asset pricing model introduced in the paper.